Department of Mathematics

SF1625
Calculus 1
Year 2015/2016

Problem set for seminar 3

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do. **At this seminar there will be a hand-in.** Solve the problems 1-4 below and write down the solution with one problem per sheet of paper. Write name and birthdate on each sheet. When the seminar begins you will be told what problem to hand in. This will take place at the beginning of the seminar, so **don't be late!** Before you begin working on the problems below you should work on the recommended exercises in the text book Calculus by Adams and Essex (8:th edition):

Ch 3.1: uppg 3, 9, 23. Ch 3.2: uppg 3, 5, 9, 15, 25, 29. Ch 3.3: uppg 3, 5, 7, 9, 19, 21, 31, 33, 43, 51, 59. Ch 3.4: 1, 3, 5, 9, 11, 17, 23, 25. Ch 3.5: 1, 3, 5, 7, 13, 19, 21, 23, 35. Ch 3.7: 1, 3, 5, 7, 9, 13, 15, 21, 25, 29. Ch 18.6: uppg 1, 3, 5, 7

PROBLEMS TO SOLVE BEFORE THE SEMINAR

Uppgift 1. Differentiate these expressions with respect to the variable x and state in each case at what points the derivative exists.

A. $xe^{-x^2/2}$.

B. $x \ln x - x$

C. $\ln \sqrt{1+x^2}$.

D. $e^{-|x|}$

E. $\arctan \frac{1}{x}$

F. $\arcsin \sqrt{x}$

Uppgift 2. A. Compute the inverse of $f(x) = 2 + 4\ln(1+6x)$. State the domain of definition and the range of the inverse.

B. Show using the derivative that $f(x) = 4\arcsin\sqrt{x} + 2\arcsin\sqrt{1-x}$ is invertible. Determine the maximum value of the inverse.

Uppgift 3. Solve these differential equations.

A. $y''(t) - 3y'(t) + 2y(t) = 0$
B. $y''(t) - 3y'(t) + 2y(t) = 10$
C. $y''(t) - 4y'(t) + 4y(t) = 0$
D. $y''(t) - 4y'(t) + 4y(t) = t$

Uppgift 4. When the heating of a house breaks down the temperature drops according to Newton's law of cooling: the rate of chage in temperature is proportional to the difference in temperature between indoors and outdoors. Suppose it is -10° C outdoors. Indoors the temperature is at first 20° C but after one hour it has dropped to 17° C. After how many hours is the indoors temperature below 0° C?

PROBLEMS TO DISCUSS AT THE SEMINAR

Here are some extra problems. You don't have to write solutions in advance.

- Determine $\arcsin(-1/2)$, $\arccos(-1/2)$, $\arctan\sqrt{3}$ and $\ln(1/\sqrt{e})$
- Compute $\arcsin(\sin(3\pi/4))$ and $\cos(\arcsin(1/5))$
- Simplify:

$$\ln e^x \qquad \ln\frac{1}{e} \qquad \ln 18 - 2\ln 3 \qquad e^{\ln\pi} \qquad \ln(e^{2x}\cdot e^{-3x})$$

- Find an equation for the tangent to $y = e^x$ at the point where the x-coordinate is 0. Use the tangent to find an approximate value of $e^{1/10}$.
- On what intervals is $f(t) = te^{-t^2}$ strictly increasing and strictly decreasing, respectively? Is f invertible?
- Find an equation for the tangent to $y = \arctan x$ at the point where the x-coordinate is 1.
- Sketch the graph $y = f(x)$ for $f(x) = \arctan x + \arctan(1/x)$, using the derivative.
- A curve is defined by

$$\arctan(xy) = \frac{\pi}{4}e^{x-y}.$$

Find an equation for the tangent to the curve at the point $(1, 1)$.

- Solve the initial value problem

$$\begin{cases} y''(t) - 6y'(t) + 9y(t) = 18 \\ y(0) = 2, \quad y'(0) = 1 \end{cases}$$

- Solve the initial value problem

$$\begin{cases} y''(t) + y(t) = \sin t \\ y(0) = 0, \quad y'(0) = 0 \end{cases}$$

- Compute $\cos(\arctan x)$, $\sin(\arctan x)$ och $\cos\left(\arccos\frac{4}{5} + \arcsin\frac{5}{13}\right)$
- Is there an x such that $\arctan(\tan x) \neq x$?
- For all x, $\tan(\arctan x) = x$. Use implicit differentiation to find the derivative of $\arctan x$.