**Department of Mathematics**

SF1625
Calculus 1
Year 2015/2016

# Problem set for seminar 2

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do. **At this seminar there will be a written test.** In the test you will be asked to solve a problem like one of the problems below or like one of the recommended exercises from the text book Calculus by Adams och Essex (8:th edition):

Ch 2.1: uppg 5, 7. Ch 2.2: uppg 1, 3, 11, 26, 27, 40, 41, 42, 43, 44, 45, 47. Ch 2.3: uppg 1, 7, 11, 17, 25, 33, 35, 47. Ch 2.4: uppg 3, 5, 11, 18, 23, 30, 31, 37. Ch 2.5: uppg 13, 15, 23, 29, 31, 35, 45, 62. Ch 2.6: uppg 3, 9. Ch 2.7: uppg 1, 3, 11, 13, 23, 29. Ch 2.8: uppg 5, 13, 21, 27. Ch 2.9: uppg 3, 9, 13. Ch 2.9: uppg 3, 9, 13. Ch 2.11: uppg 5, 7, 13, 16, 17, 18, 19.

---

## PROBLEMS TO SOLVE BEFORE THE SEMINAR

**Uppgift 1.** Differentiate these functions with respect to $x$ and state for what $x$ they are differentiable. Do some of these functions fail to be differentiable at some points of their domains of definition?

A. $f(x) = \tan^2 x$

B. $g(x) = \dfrac{ax+b}{cx+d}$

C. $h(x) = 2\sin\sqrt{x}$

D. $k(x) = |x|\cos x$

E. $r(x) = \sqrt{1+x}$

F. $s(x) = \dfrac{\sin x}{1+\cos x}$

**Uppgift 2.** Let $g(x) = x^2 \sin x$.

A. Determine the domain of definition of $g$.
B. At what points is $g$ continuous?
C. Compute $g'(x)$.
D. At what points is $g$ differentiable?

**Uppgift 3.** Let $h(t) = |1 + t|(1 + 2 \sin t)^5$.

A. Determine the domain of definition of $h$.
B. At what points is $h$ continuous?
C. Compute $h'(t)$.
D. At what points is $h$ differentiable?

**Uppgift 4.** Find equations for the tangent and the normal at the point $(2, 16)$ to the curve $y = x^4$.

**Uppgift 5.** On what intervals is $f(x) = x^4 - 4x^3 + 4x^2$ strictly increasing? Strictly decreasing?

**Uppgift 6.** Show using the derivative that $f(x) = 2 \sin^2 x + \cos 2x$ is constant.

**Uppgift 7.** Let $f(x) = \sin x^2$. Compute $f^{(n)}(0)$ for $n = 1, 2, 3$.

**Uppgift 8.** Find an equation for the tangent to $y = \tan x$ at the point on the curve where $x = \pi/4$. Can you use this to find an approximate value of $\tan(\pi/5)$?

**Uppgift 9.** Find an equation for the tangent to $x^3 + y^3 + y + x = 0$ at the point $(-1, 1)$. Hint: implicit differentiation.

**Uppgift 10.** Theory: Show using the definition that $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$

**Uppgift 11.** Theory: Prove the product rule!

**Uppgift 12.** Theory: Prove that a function differentiable at $a$ must be continuous at $a$. Give an example showing that the converse is not true.

## EXTRA PROBLEMS TO DISCUSS AT THE SEMINAR

Here are some extra problems. You don't have to write down solutions in advance.

- Give an example of a function with domain of definition $\mathbb{R}$, that is neither continuous nor differentiable at $x = 1$.
- Give an example of a function with domain of definition $\mathbb{R}$, that is continuous but not differentiable at $x = 1$.
- Is there a function that is differentiable but not continuous at $x = 1$?
- Let $h(x) = |x| - |x + 1|$. Compute $h'(x)$ and state at what points $h$ is differentiable. What does the graph look like at points where $h$ is not differentiable?
- Let $U(t)$ be the Heaviside function given by

  $$U(t) = \begin{cases} 1 & \text{om } t \geq 0 \\ 0 & \text{om } t < 0 \end{cases}$$

  Compute $U'(t)$. At what points is $U$ differentiable?
- Let $U$ be as in the previous exercise and put $f(t) = (U(t-\pi) - U(t-3\pi)) \sin t$. Make a simple sketch of $y = f(x)$. At what points is $f$ continuous? Compute $f'(t)$. At what points is $f$ differentiable? What does the graph look like at points where $h$ is not differentiable?