Principles of Wireless Sensor Networks

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Lecture 4

Physical Layer

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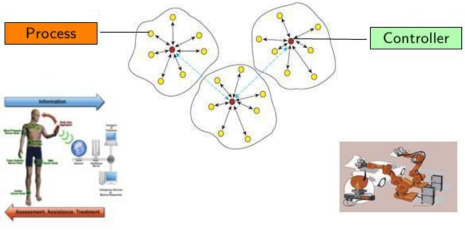
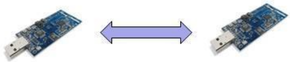
Where we are

Application
Presentation
Session
Transport
Routing
MAC
Phy





- How messages are successfully transmitted and received over the wireless channel?
- Aim: modeling the probability to successfully receive messages as function of the radio power, modulations, coding, and channel attenuations normally experienced in WSNs

Today's learning goals

- How bits of messages are transmitted over a channel?
- What is the probability to successfully receive messages over AWGN channels?
- What is the probability to successfully receive messages over fading channels?

Outline

- Basics of modulation theory
- Probability of error over AWGN channels
- Probability of error over fading + AWGN channels

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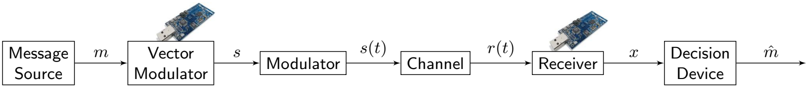
- Basics of modulation theory
- Probability of error over AWGN channels
  - BPSK
  - Amplitude Modulation
- Probability of error over fading + AWGN channels

Digital modulations



$$s\left(t\right)=\sum_{k=0}^{\infty}a_k\left(t\right)g\left(t-kT_s\right)$$ modulated signal transmitted by the node's antenna

$$G\left(f\right)\triangleq\int_{0}^{T_s}a_0 g\left(t\right)e^{-2\pi f t}dt$$ spectrum of the signal over a symbol

$$\Phi_s\left(f\right)\triangleq\frac{\left|G\left(f\right)\right|^2}{T_s}$$ power spectral density of the signal

T_s symbol duration

Example: Binary Phase Shift Keying (BPSK) modulation

$$\text{Message Source} \xrightarrow{m} \text{Vector Modulator} \xrightarrow{s} \text{Modulator} \xrightarrow{s(t)} \text{Channel} \xrightarrow{r(t)} \text{Receiver} \xrightarrow{x} \text{Decision Device} \xrightarrow{\hat{m}}$$

$$s\left(t\right) = \sum_{k=0}^{\infty} a_k\left(t\right) g\left(t - kT_s\right)$$

$$a_k\left(t\right) = \begin{cases} \cos\left(2\pi f_c t\right) & \text{if bit 1 at symbol time } k, \\ \cos\left(2\pi f_c t + \pi\right) & \text{if bit 0 at symbol time } k. \end{cases}$$

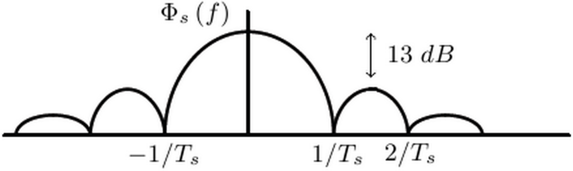
Modulation of the bits

$$g\left(t\right) = \sqrt{\frac{E}{T_s}}\,,\ 0 \le t \le T_s \qquad P_t = \frac{E}{T_s}$$

transmit power

Example: BPSK spectral density

The spectral density tells how the signal is spread over the frequencies



$$G(f) = -\sqrt{\frac{E}{T_s}}\frac{e^{j\pi f T_s} - e^{-j\pi f T_s}}{j2\pi f} = -\sqrt{\frac{E}{T_s}}T_s\frac{\sin(\pi f T_s)}{\pi f T_s} = -\sqrt{\frac{E}{T_s}}T_s \mathrm{sinc}(f T_s)$$

$$\Phi_s(f) = \frac{E}{T_s}\mathrm{sinc}^2(f T_s)$$

In this example, the BPSK signal is roughly/mostly spread between $f_c - 1/T_s$ and $f_c + 1/T_s$

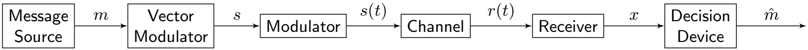
Probability of successful bit reception

Now, we would like to compute the probability that a bit is received successfully when it is transmitted by a modulation over an AWGN channel

Outline

- Basics of modulation theory
- Probability of error over AWGN channels
  - BPSK
  - Amplitude Modulation
- Probability of error over fading + AWGN channels

BPSK probability of error in AWGN channels

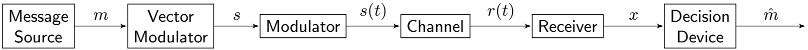


- Assume that the transmitted signal is received corrupted by an Additive White Gaussian Noise (AWGN)
- Assume that there is no fading, namely $A = 1$

$$r(t) = s(t) + n_0(t)$$

$$n_0(t) \in N\left(0, \sigma^2 = \frac{N_0}{2T_s}\right)$$

BPSK detection in AWGN wireless channels



- After a matched filter, the demodulator in the receiver produces a signal

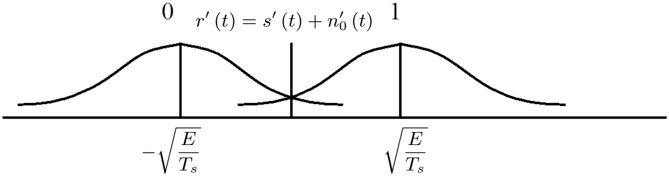
$$r'(t) = s'(t) + n_0'(t)$$

$$s'(t) = \begin{cases} \sqrt{\dfrac{E}{T_s}} & \text{if bit 1 was transmitted} \\ -\sqrt{\dfrac{E}{T_s}} & \text{if bit 0 was transmitted.} \end{cases}$$

$$n_0'(t) \in N\left(0, \sigma^2 = \frac{N_0}{2T_s}\right)$$

- If $r'(t) \geq 0$ the detector decides for bit 1
- If $r'(t) < 0$ the detector decides for bit 0
- Given the AWGN, what is the error in this detection?

BPSK probability of error

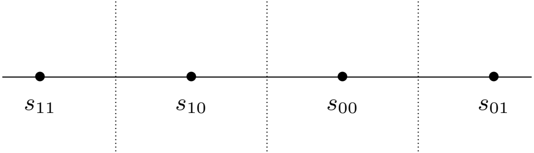


$$\mathrm{P}_{e,0|1} \triangleq \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(x-\sqrt{\frac{E}{T_s}}\right)^2}{2\sigma^2}} dx = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

$$\mathrm{P}_{e,1|0} \triangleq \int_{0}^{-\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(x+\sqrt{\frac{E}{T_s}}\right)^2}{2\sigma^2}} dx = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

$$\mathrm{P}_{e} \triangleq \frac{1}{2} P_{e,0|1} + \frac{1}{2} P_{e,1|0} = Q\left(\sqrt{\frac{2E}{N_0}}\right) \qquad \mathrm{SNR} \triangleq \frac{E}{N_0}$$

Amplitude modulation, AM



- BPSK is a simple example of binary Amplitude Modulation
- By adding more amplitude values, more general modulations are possible
- Modulations are characterized by constellation points
- Every point (symbol) is associated to a signal with specific amplitude
- Every signal is then associated to a code-word of bits

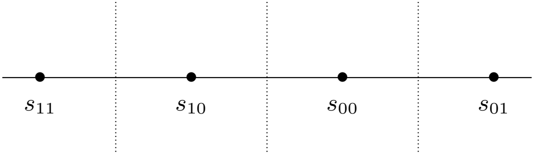
Probability of error for AM

- The distance between the constellation points determines the probability that a symbol is detected with error
- It is possible to show that

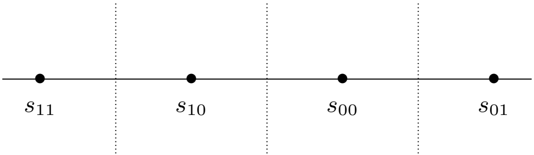
$$\mathrm{P}_e \simeq N_{d_{\min}} Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right)$$

$d_{\min}$ minimum distance among points

$N_{d_{\min}}$ average number of neighbors at minimum distance



Exercise: 4-AM (or 4-PAM)



$$s_{00} = \sqrt{\frac{E}{5}} g(t) \qquad s_{01} = 3\sqrt{\frac{E}{5}} g(t)$$

$$s_{11} = -3\sqrt{\frac{E}{5}} g(t) \qquad s_{10} = -\sqrt{\frac{E}{5}} g(t)$$

$$g(t) = \sqrt{\frac{1}{T_s}} \qquad 0 \leq t \leq T_s$$

What is the probability that a bit is erroneously received?

Solution

$$d_{\min}^2 = 4\frac{E}{5} \qquad N_{\min} = 1.5$$

- The probability of symbol error is

$$\mathrm{P}_e \simeq 1.5Q\left(\sqrt{\frac{2E}{5N_0}}\right) = 1.5Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$$

- Since per every symbol there are two bits, the probability of a bit in error is

$$\mathrm{P}_b \simeq 0.75Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$$

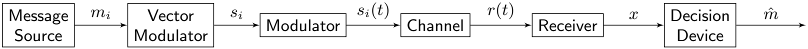
$$E_b = \frac{E}{\log_2 M} = \frac{E}{2}$$ Energy per bit

4-Quadrature Amplitude Modulation

- Let's consider a more general amplitude modulation that is used in
  - TskyMotes
  - IEEE 802.15.4 standard (which we study next lecture)

- 4-Quadrature Amplitude Modulation - 4QAM

4-QAM

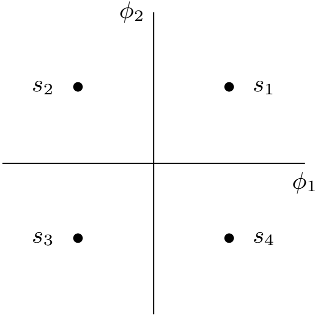


$$
\begin{aligned}
s_i\left(t\right) &= \frac{2E}{T_s}\cos\left(2\pi f_c t + \frac{\left(2i-1\right)\pi}{4}\right) \quad 0 \le t \le T_s \qquad\qquad i = 1, ..., 4 \\
&= \frac{2E}{T_s}\cos\left(\frac{\left(2i-1\right)\pi}{4}\right)\cos\left(2\pi f_c t\right) - \frac{2E}{T_s}\sin\left(\frac{\left(2i-1\right)\pi}{4}\right)\sin\left(2\pi f_c t\right) \quad 0 \le t \le T_s
\end{aligned}
$$

Signal space by two basis functions

$$\phi_1\left(t\right) = \frac{2}{T_s}\cos\left(2\pi f_c t\right) \quad 0 \le t \le T_s$$

$$\phi_2\left(t\right) = \frac{2}{T_s}\sin\left(2\pi f_c t\right) \quad 0 \le t \le T_s$$



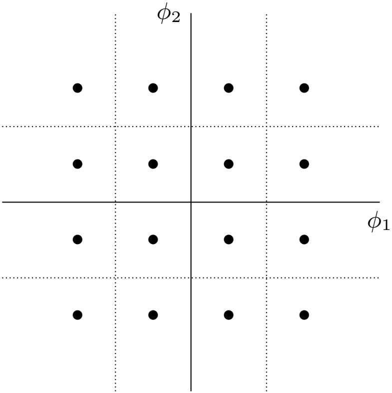
Probability of error in 4-QAM

Minimum distance between two symbols $d_{\min} = \sqrt{2E}$

2 neighbors at minimum distance

$$\mathrm{P}_e \simeq 2Q\left(\sqrt{\frac{E}{5N_0}}\right) = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

A more complex modulation: 16-QAM



More points can be added, this increases the transmit bit rate, but increases also the probability of error

Comparison of probabilities or error

$$\mathrm{P}_{e,\mathrm{BPSK}} = Q\left(\sqrt{\frac{2E}{N_0}}\right) \qquad \mathrm{SNR} = \frac{E}{N_0}$$

$$\mathrm{P}_{e,\text{4-PAM}} \simeq 1.5Q\left(\sqrt{\frac{2E}{5N_0}}\right)$$

$$\mathrm{P}_{e,\text{4-QAM}} \simeq 2Q\left(\sqrt{\frac{E}{N_0}}\right)$$

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Communication over wireless channel

- So far, we have seen only channels where the transmitted signal is received corrupted by AWGN, but in real channel

$$r\left(t\right) = \sqrt{A}s\left(t\right) + n_0\left(t\right)$$

- The power of the transmitted signals is attenuated by the wireless channel

- How the bit probability of error is affected by fading channels?

Probability of error over fading channels

- Consider a AWGN + Rayleigh channel with path loss, Rayleigh fast fading, and fixed shadow fading

$$P_r = P_t \underbrace{G_t(\theta_t, \psi_t)\, G_r(\theta_r, \psi_r)\, \frac{\lambda^2}{(4\pi r)^2}\, \overline{\mathrm{PL}} \cdot y}_{C} \cdot z$$

$$\triangleq P_t C \cdot z$$

Probability of error over fading channels

- The receiver sees the transmit power $P_t = E/T_s$ received with an attenuation $C \cdot z$
- We can reuse the probability for AWGN channels AS if there WERE no fading and the transmit power WERE

$$P_t = \frac{E}{T_s} C z$$

- Thus, the probability of error with fading has similar expression of simple AWGN channels, but with the SNR of a fading channel

$$\mathrm{SNR} = \frac{E}{N_0} C z$$

$$\mathrm{P}_{e,\mathrm{BPSK}} = Q\left(\sqrt{\frac{2E}{N_0}}\right) \longrightarrow \mathrm{P}_{e,\mathrm{BPSK}}(z) = Q\left(\sqrt{\frac{2ECz}{N_0}}\right)$$

AWGN no fading — AWGN + Rayleigh fading

Probability of error over fading channels

- The probability so derived over fading channel is instantaneous
  - Depends on the given realization of the fading channel z
- What is the average probability of error, where the average is taken over the distribution of the fading?
- Just take the expectation of $\mathrm{P}_{e,\mathrm{BPSK}}(z)$ over the distribution of z
- Thus (remember from Exercise session that the square of a Rayleigh random variable is an exponential random variable)

$$p(z) = \frac{1}{\gamma^*} e^{-\frac{z}{\gamma^*}} \qquad \gamma^* \triangleq \mathbb{E}\{\mathrm{SNR}\} = \frac{E}{N_0} C \qquad \mathbb{E}\{z\} = 1$$

$$\overline{\mathrm{P}}_{e,\mathrm{BPSK}} = \int_0^\infty \mathrm{P}_{e,\mathrm{BPSK}}(z)\, p(z)\, dz = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma^*}{1+\gamma^*}}\right] \simeq \frac{1}{4\gamma^*}$$

Error probability comparison over fading channels

- Simple AWGN channel:

$$\mathrm{P}_{e,\mathrm{BPSK}} = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

  - Linear increase of SNR results in exponential decrease in the error probability

- AWGN + Rayleigh channel:

$$\overline{\mathrm{P}}_{e,\mathrm{BPSK}} \simeq \frac{1}{4\gamma^*} \qquad \gamma^* \triangleq \mathbb{E}\{\mathrm{SNR}\} = \frac{E}{N_0}C$$

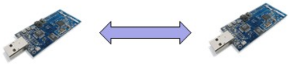
  - Linear increase of the SNR gives only a linear decrease of error probability
  - 40dB higher SNR than the simple AWGN channel to have same probability $\mathrm{P}_e = 10^{-6}$

Packet error probability

- Bits are grouped in units denoted physical layer data frames, “physical layer messages”
- What is the probability that the message is erroneously received?
- Suppose that the message is formed by L bits and BPSK is used, then the probability that the packet is in error is bounded by

$$\mathrm{P}_m \leq 1 - \left(1 - \mathrm{P}_{e,\mathrm{BPSK}}\right)^L$$

Conclusions



- We studied how bits are modulated and transmitted over the wireless channel
- The probability of successful reception of messages was characterized for AWGN and for AWGN + fading channels

Next lecture

- But how a node has the right to transmit a message?

- Medium access control
  - When a node gets the right to transmit?
  - What is the mechanism to get such a right?