Signal Theory: suggested problems for final exam

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Problem 1(Chapter 1):
Two stochastic variables X and Y have the following means and variances:

$$X: \ m_X, \sigma_X^2$$
$$Y: \ m_Y, \sigma_Y^2.$$

These stochastic variables have the covariance of σ_{XY}. We form new stochastic variables U and W such that $U = X + Y$ and $W = X - aY$.
Find parameter 'a' such that variables U and W become uncorrelated.

Problem 2(Chapter 2):
Let $X(t)$ and $Y(t)$ be two stochastic processes as follows:

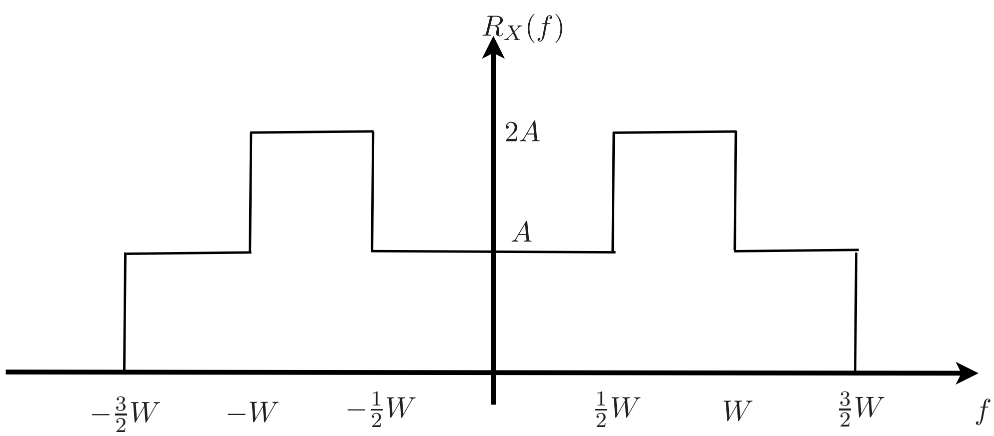
$$X(t) = A_X cos(\omega_0 + \phi_X)$$
$$Y(t) = A_Y cos(\omega_0 + \phi_Y),$$

where, A_X, A_Y, ϕ_X and ϕ_Y are independent stochastic variables. Variables ϕ_X and ϕ_Y are uniformly distributed in $[0, 2\pi]$. We define another stochastic process $Z(t)$ as follows:

$$Z(t) = X(t)Y(t)$$

Is $Z(t)$ a wide-sense stationary process? what is the autocorrelation function of $Z(t)$?

Problem 3(Chapter 4):
A continues process $X(t)$ has a power spectral density shown below:



Determine the corresponding ACF of $X(t)$.

Problem 4(Chapter 5):
Consider a stationary stochastic process $X(t)$ with power spectral density $R_X(f)$, and the stationary stochastic process $Y(t)$ that is defined by

$$\frac{dY(t)}{dt} + 2Y(t) = 3X(t)$$

a) Determine the power spectral density of $Y(t)$ ($R_Y(f)$) as a function of the power spectral density of $X(t)$.
b) Determine $E\{Y^2(t)\}$ if $X(t)$ is a zero mean white noise with ACF $r_X(\tau) = \delta(\tau)$.