

Mini-groupwork 1, solutions, 2015

a)

$$h = \frac{17}{2} \cdot 6378 \text{ km} = 54000 \text{ km}$$

The thermal energy is divided into motion in the three dimensions, two of which only give rise to a gyro motion around the magnetic field lines, with the motion along the magnetic field corresponding to an energy

$$E = \frac{k_B T}{2} = \frac{1.38 \cdot 10^{-23} \cdot 2 \cdot 10^6}{2} = 1.4 \cdot 10^{-17} \text{ J}$$

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 1.4 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 5500 \text{ km/s}$$

Approximating the loop with a quarter-circle, the electron has to travel a length

$$s = \pi h / 2 = 85\,000 \text{ km}$$

Then we get $t = 15 \text{ s}$.

b)

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi} \frac{qB}{m} \Rightarrow$$
$$B = \frac{2\pi f_c m}{q} = \frac{2\pi \cdot 8 \cdot 10^9 \cdot 0.91 \cdot 10^{-30}}{1.6 \cdot 10^{-19}} = 0.29 \text{ T}$$

The perpendicular energy is given by

$$E = \frac{2k_B T}{2} = \frac{2 \cdot 1.38 \cdot 10^{-23} \cdot 2 \cdot 10^6}{2} = 2.8 \cdot 10^{-17} \text{ J}$$

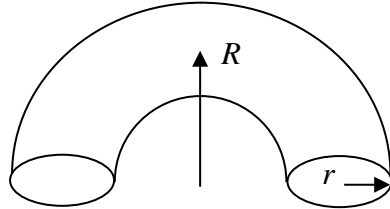
\Rightarrow

$$v_{\perp} = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 2.8 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 7.8 \cdot 10^6 \text{ m/s}$$

$$\rho = \frac{m_e v_{\perp}}{qB} = \frac{0.91 \cdot 10^{-30} \cdot 7.8 \cdot 10^6}{1.6 \cdot 10^{-19} \cdot 0.29} = 1.5 \cdot 10^{-4} \text{ m}$$

c)

Model the flare by a half torus with minor axis r , and major axis R . From the figure I estimate $R = 8 R_E$, and $r = 2 R_E$.



Let this half-torus be filled with a magnetic field of strength $B \sim 0.29$ T (using the value in b)). If the volume of the half-torus is V and the magnetic energy density is p_B , the total energy is

$$\begin{aligned} W = V p_B &= \pi R \pi r^2 \frac{B^2}{2\mu_0} = \pi^2 \cdot 8 \cdot 2^2 R_E^3 \frac{B^2}{2\mu_0} = \pi^2 \cdot 32 \cdot (6378 \cdot 10^3)^3 \frac{(0.29)^2}{2\mu_0} \\ &= 5.5 \cdot 10^{27} \text{ J} \end{aligned}$$

which is enough to produce a solar flare, even if only a fraction of the available magnetic field energy is converted into kinetic energy of the plasma.