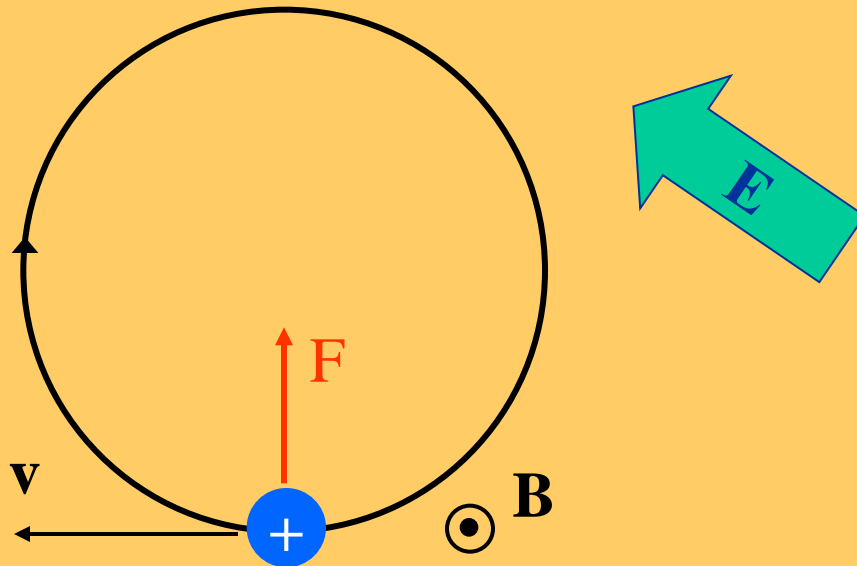


Think about this:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$



What happens if you add an electric field \mathbf{E} ?



Last lecture (3)

- Solar activity
- Solar wind – basic facts
- Solar wind – magnetic structure

Today's lecture (4)

- Ionosphere
 - layers
 - radio wave reflection



Today

Activity	Date	Time	Room	Subject	Litterature
L1	31/8	13-15	V22	Course description, Introduction, The Sun 1, Plasma physics 1	CGF Ch 1, 5, (p 110-113)
L2	3/9	15-17	Q36	The Sun 2, Plasma physics 2	CGF Ch 5 (p 114-121), 6.3
L3	7/9	13-15	Q36	Solar wind, The ionosphere and atmosphere 1 , Plasma physics 3	CGF Ch 6.1, 2.1-2.6, 3.1-3.2, 3.5, LL Ch III, Extra material
T1	10/9	15-17	Q36	Mini-group work 1	
L4	14/9	13-15	E2	The ionosphere 2 , Plasma physics 4	CGF Ch 3.4, 3.7, 3.8
T2	17/9	8-10	Q31	Mini-group work 2	
L5	17/9	15-17	L52	The Earth's magnetosphere 1, Plasma physics 5	CGF 4.1-4.3, LL Ch I, II, IV.A
L6	21/9	13-15	L52	The Earth's magnetosphere 2, Other magnetospheres	CGF Ch 4.6-4.9, LL Ch V.
T3	24/9	16-18	Q36	Mini-group work 3	
L7	28/9	13-15	Q36	Aurora, Measurement methods in space plasmas and data analysis 1	CGF Ch 4.5, 10, LL Ch VI, Extra material
T4	1/10	15-17	V22	Mini-group work 4	
L8	5/10	13-15	M33	Space weather and geomagnetic storms	CGF Ch 4.4, LL Ch IV.B-C, VII.A-C
L9	6/10	8-10	Q36	Interstellar and intergalactic plasma, Cosmic radiation,	CGF Ch 7-9
T5	8/10	15-17	Q34	Mini-group work 5	
L10	12/10	13-15	Q36	Swedish and international space physics research.	
T6	15/10	15-17	Q33	Round-up.	
Written examination	28/10	8-13	Q21, Q26		

L = Lecture, T = Tutorial



Mini groupwork 1

$$h = \frac{17}{2} \cdot 6378 \text{ km}$$

a)

The thermal energy is divided into motion in the three dimensions, two of which only give rise to a gyro motion around the magnetic field lines, with the motion along the magnetic field corresponding to an energy

$$E = \frac{k_B T}{2} = \frac{1.38 \cdot 10^{-23} \cdot 2 \cdot 10^6}{2} = 1.4 \cdot 10^{-17} \text{ J}$$

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 1.4 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 5500 \text{ km/s}$$

Approximating the loop with a quarter-circle, the electron has to travel a length

$$s = \pi h / 2 = 85 \text{ 000 km}$$

Then we get $t = 15 \text{ s}$.

Energy - temperature

Average energy of molecule/atom:

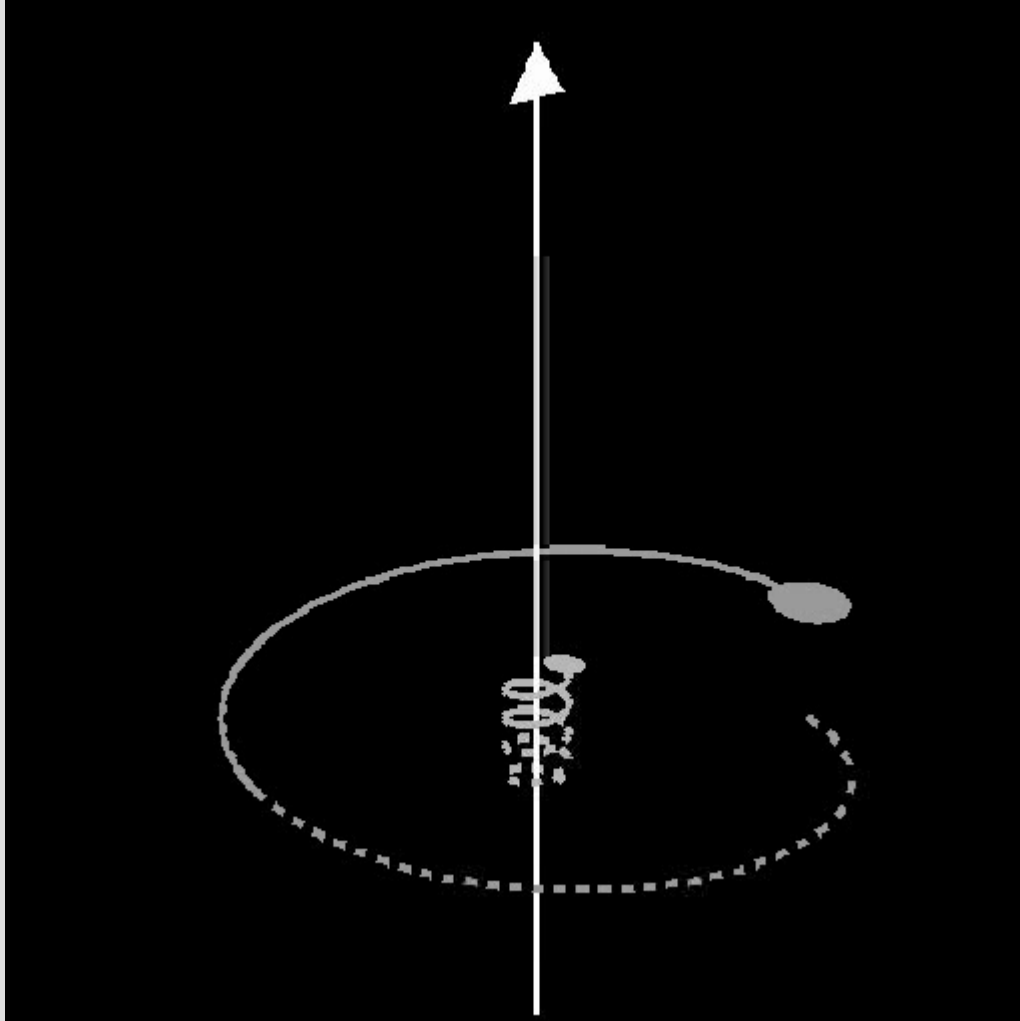
$$E = \frac{3}{2} k_B T \Rightarrow$$

$$T = \frac{2E}{3k_B}$$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J} \Rightarrow$$

$$T = \frac{2E}{3k_B} = \frac{2 \cdot 1.6 \cdot 10^{-19} \text{ J}}{3 \cdot 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}} = 7729 \text{ K}$$

Gyro motion



Equipartition principle

Statistically the kinetic energy is equally distributed along the three dimensions:

$$E_{\parallel} = \frac{1}{2} k_B T$$

$$E_{\perp} = \frac{2}{2} k_B T$$

Mini groupwork 1

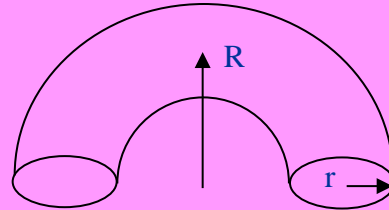
b)
$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi} \frac{qB}{m} \Rightarrow$$
$$B = \frac{2\pi f_c m}{q} = \frac{2\pi \cdot 8 \cdot 10^9 \cdot 0.91 \cdot 10^{-30}}{1.6 \cdot 10^{-19}} = 0.29 \text{ T}$$

The perpendicular energy is given by

$$E = \frac{2k_B T}{2} = \frac{2 \cdot 1.38 \cdot 10^{-23} \cdot 2 \cdot 10^6}{2} = 2.8 \cdot 10^{-17} \text{ J}$$
$$v_{\perp} = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 2.8 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 7.8 \cdot 10^6 \text{ m/s}$$
$$\rho = \frac{m_e v_{\perp}}{qB} = \frac{0.91 \cdot 10^{-30} \cdot 7.8 \cdot 10^6}{1.6 \cdot 10^{-19} \cdot 0.29} = 1.5 \cdot 10^{-4} \text{ m}$$

Mini groupwork 1

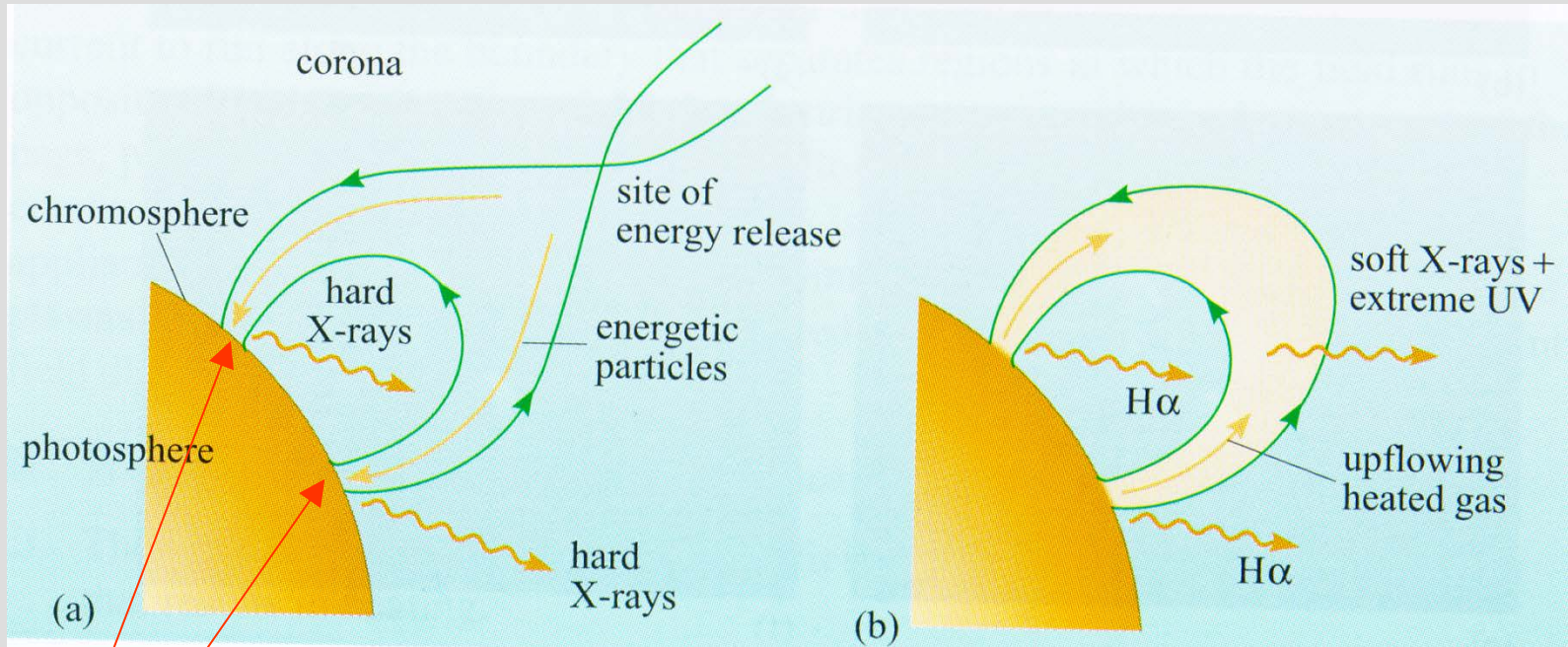
- c) Model the flare by a half torus with minor axis r , and major axis. From the figure, estimate $R = 8 R_E$, and $r = 2 R_E$.



Let this half-torus be filled with a magnetic field of strength $B \sim 0.36$ T (using the value in b)). If the volume of the half-torus is V and the magnetic energy density is p_B , the total energy is

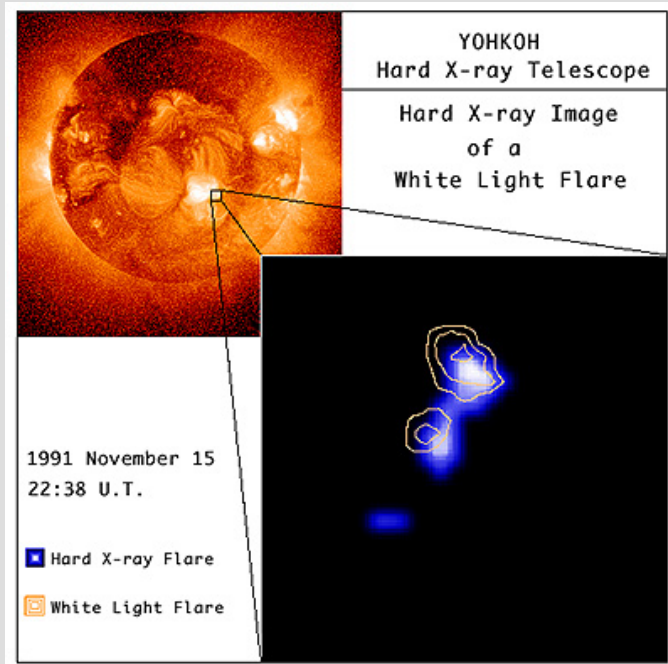
$$\begin{aligned}
 W &= V p_B = \pi R \pi r^2 \frac{B^2}{2\mu_0} = \pi^2 \cdot 8 \cdot 2^2 R_E^3 \frac{B^2}{2\mu_0} \\
 &= \pi^2 \cdot 32 \cdot (6378 \cdot 10^3)^3 \frac{(0.29)^2}{2\mu_0} \\
 &= 5.5 \cdot 10^{27} \text{ J}
 \end{aligned}$$

Solar flare mechanism

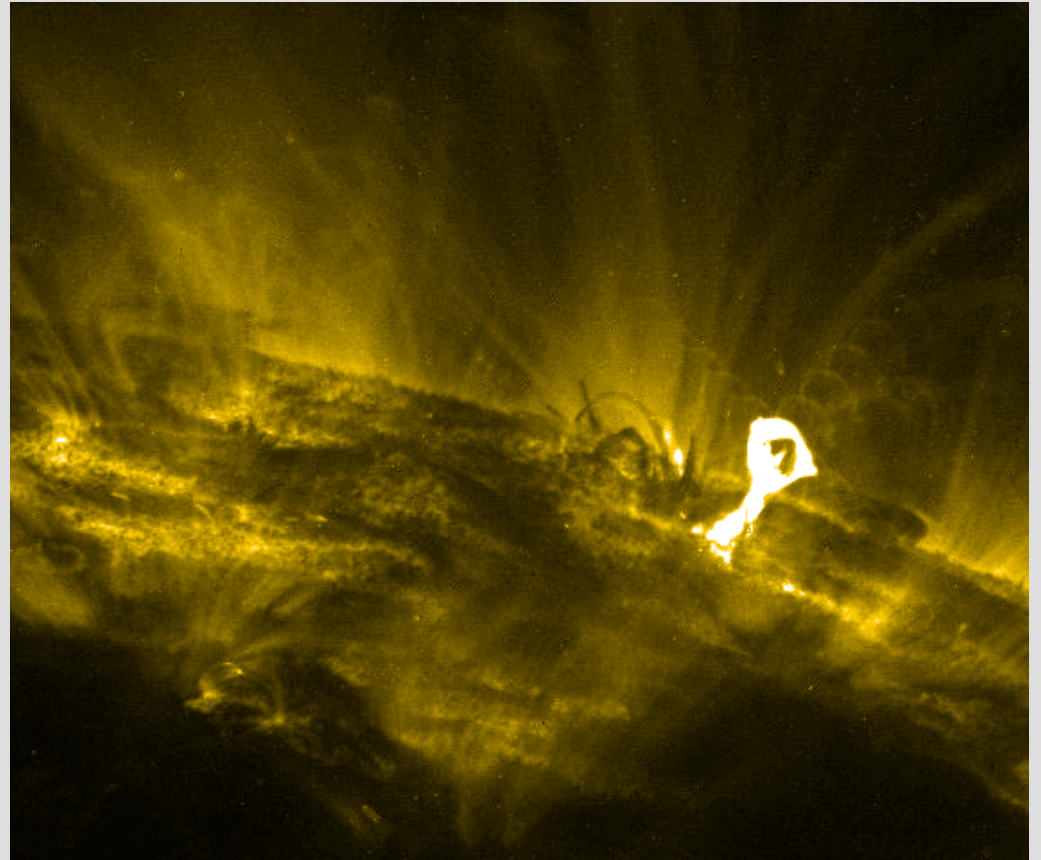


Electrons are accelerated, collide with solar surface (photosphere) and emit bremsstrahlung (X-rays).

Solar flare observations



(a) double signature of x-ray emissions at foot of flare



(b) coronal loop filled with hot gas

Frozen in magnetic flux *PROOF II*

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{v} \times \mathbf{B})}_A + \underbrace{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}}_B$$

Order of magnitude estimate:

$$\frac{A}{B} = \frac{\nabla \times (\mathbf{v} \times \mathbf{B})}{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}} \approx \frac{\frac{v \Delta B}{L}}{\frac{\Delta B}{\mu_0 \sigma L^2}} = v L \mu_0 \sigma \equiv R_m$$

Magnetic Reynolds number R_m :

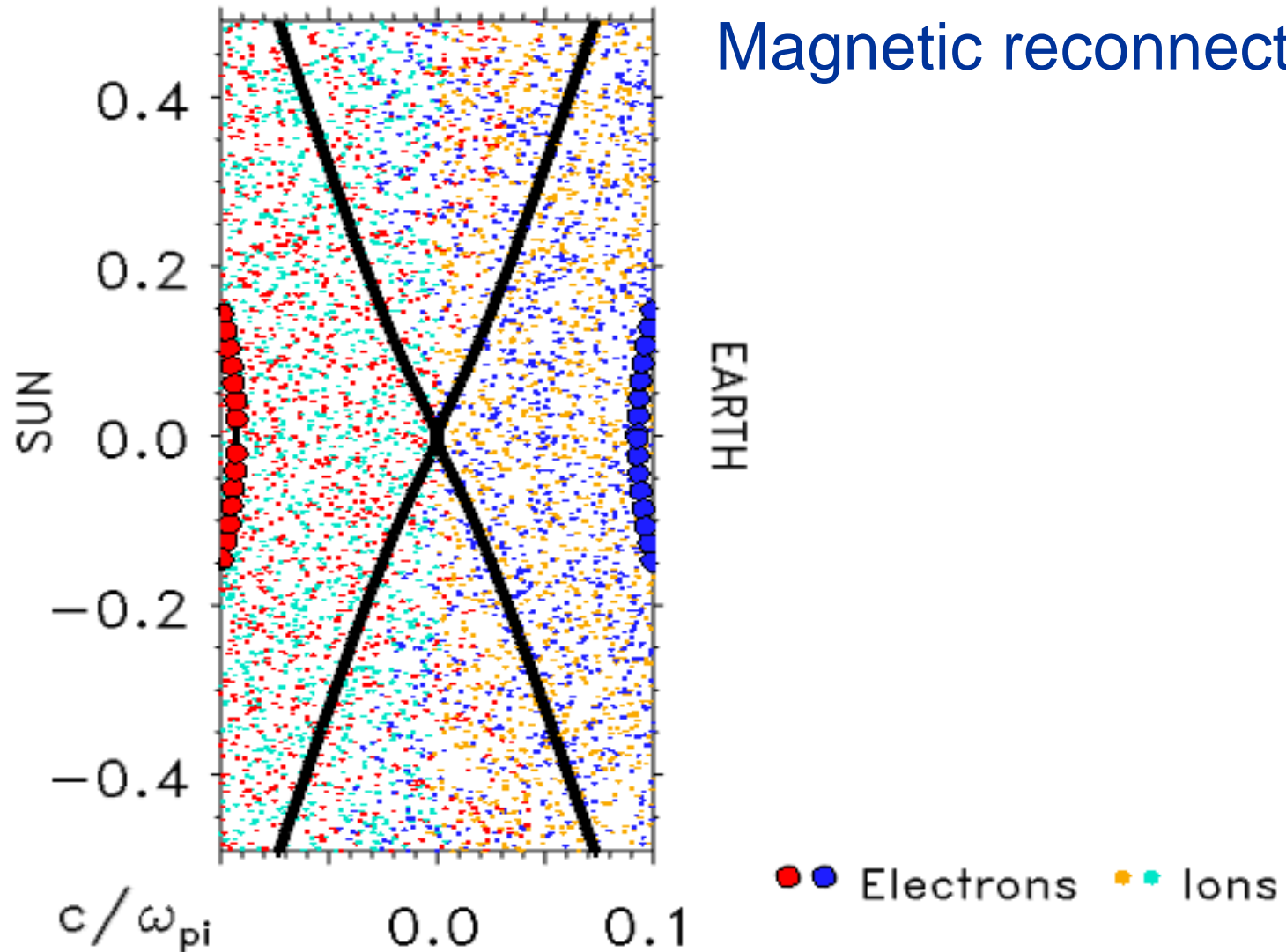
$$R_m \gg 1 \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Frozen-in fields!

$$R_m \ll 1 \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Diffusion equation!

Magnetic reconnection



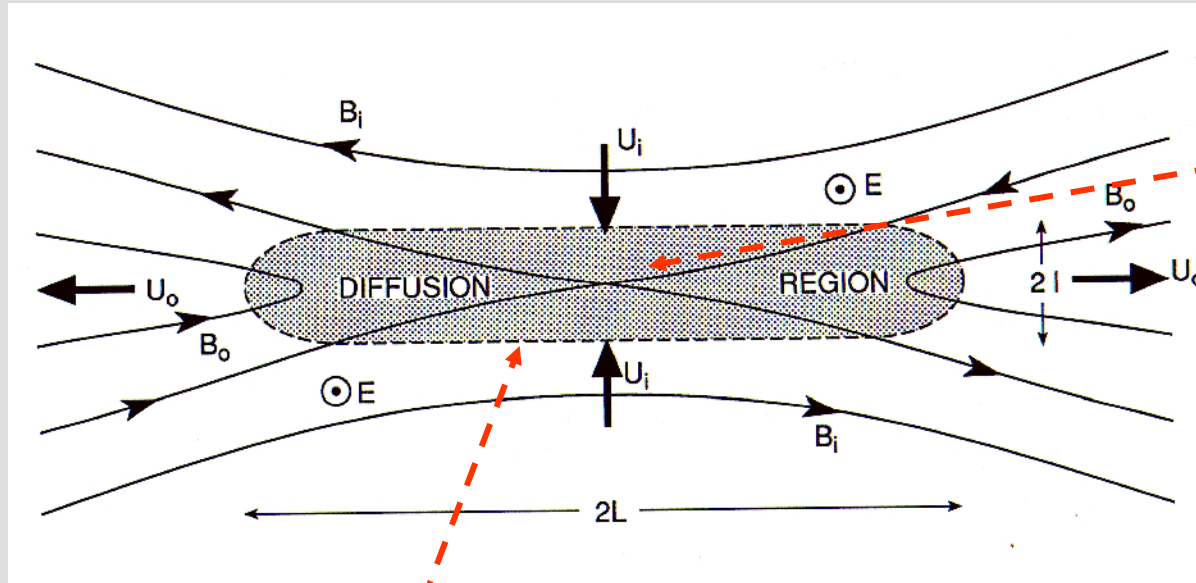
Reconnection

In 'diffusion region':

$$R_m = \mu_0 \sigma l v \sim 1$$

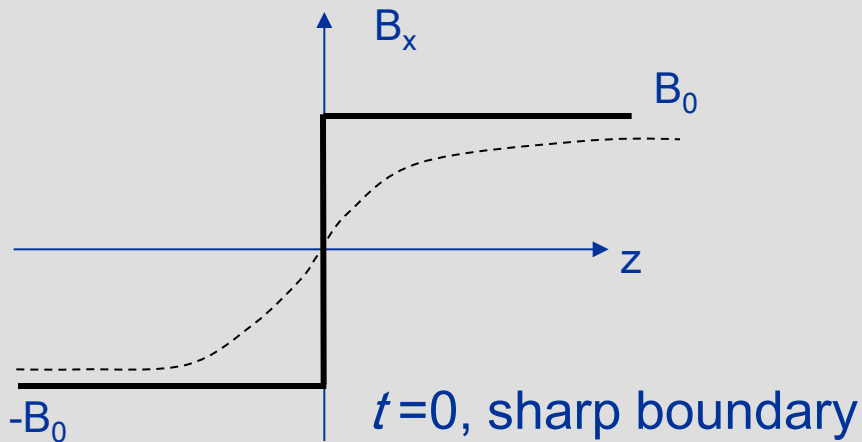
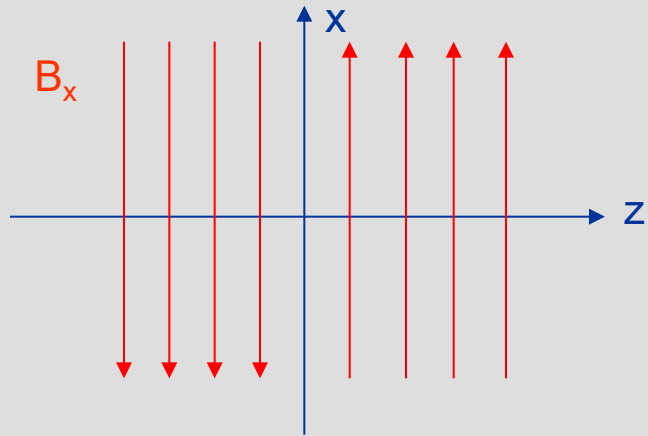
Thus: **condition** for frozen-in magnetic field breaks down.

A second **condition** is that there are two regions of magnetic field pointing in *opposite* direction:



- Field lines are “cut” and can be re-connected to other field lines
- **Magnetic energy is transformed into kinetic energy ($U_o \gg U_i$)**
- **Plasma from different field lines can mix**

Reconnection in 1D



$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \quad \rightarrow \quad \frac{\partial B_x}{\partial t} = \frac{1}{\mu_0 \sigma} \frac{\partial^2 B_x}{\partial z^2}$$

Diffusion equation! Has solution

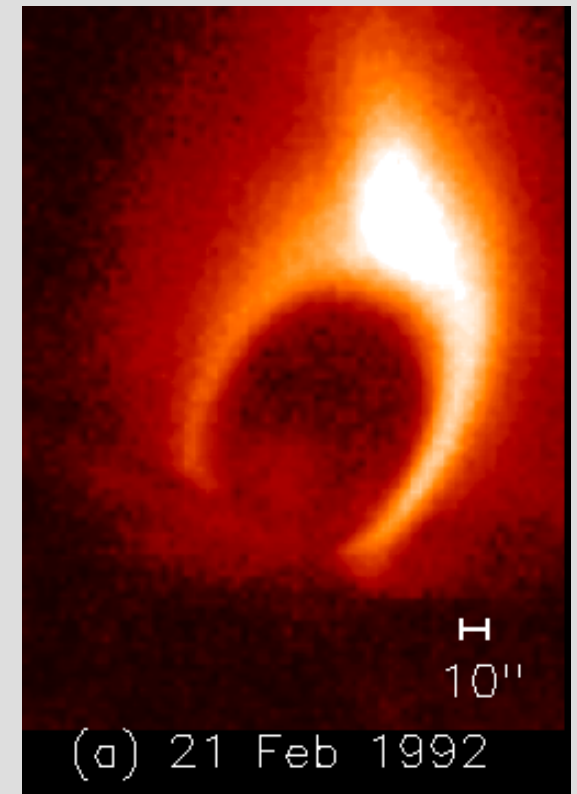
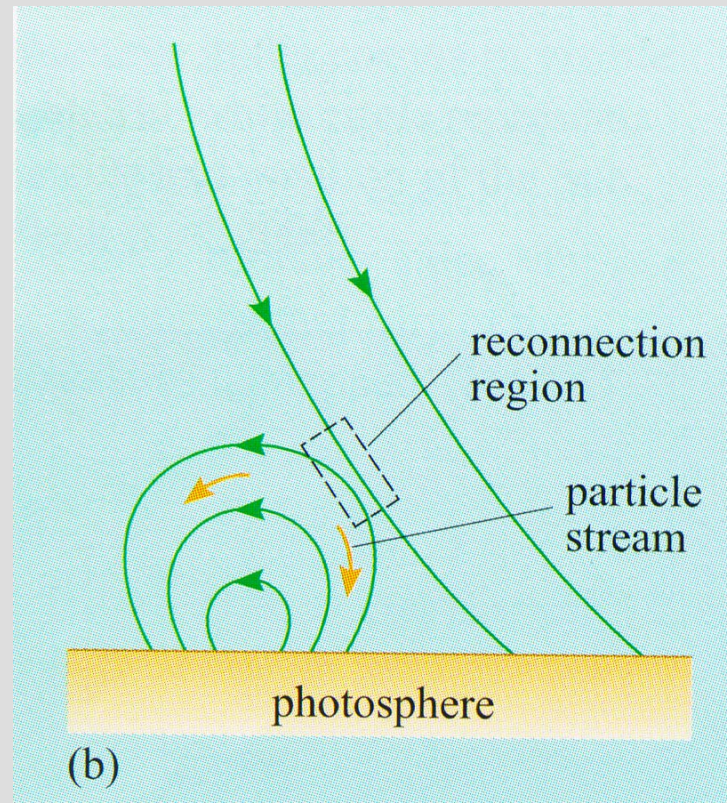
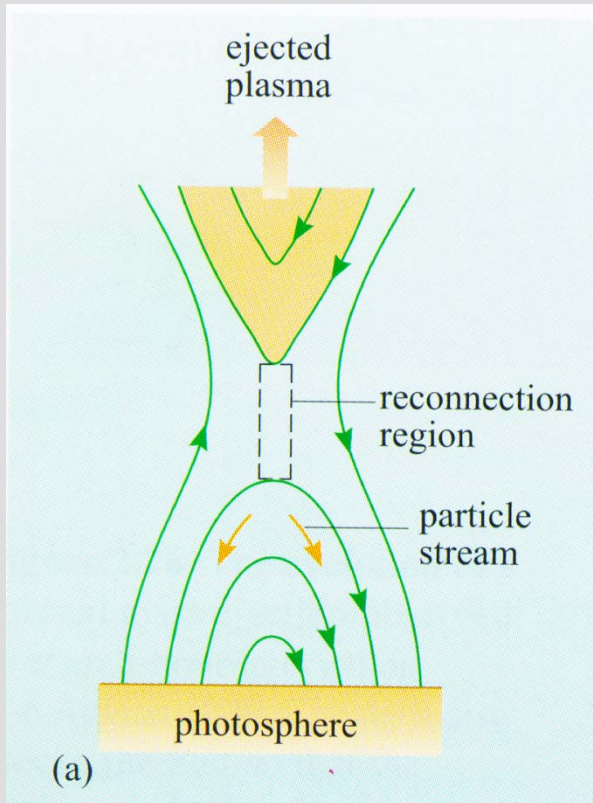
$$B_x(z, t) = B_0 \operatorname{erf} \left(\left[\frac{\mu_0 \sigma}{4t} \right]^{1/2} z \right)$$

The total magnetic energy then decreases with time:

$$W_B = \int_{-\infty}^{\infty} \frac{B^2}{2\mu_0} dz$$

The magnetic energy is converted into heat and kinetic energy in 2D

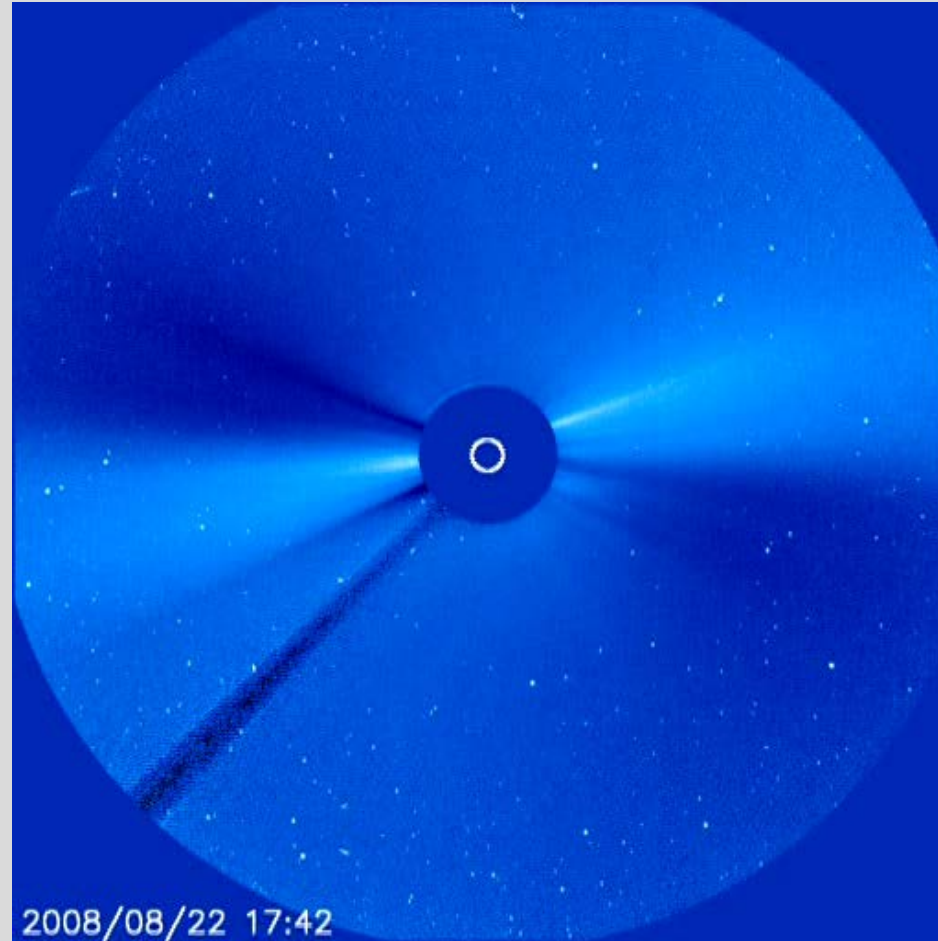
Solar flare *energization mechanism*



Two possible reconnection geometries

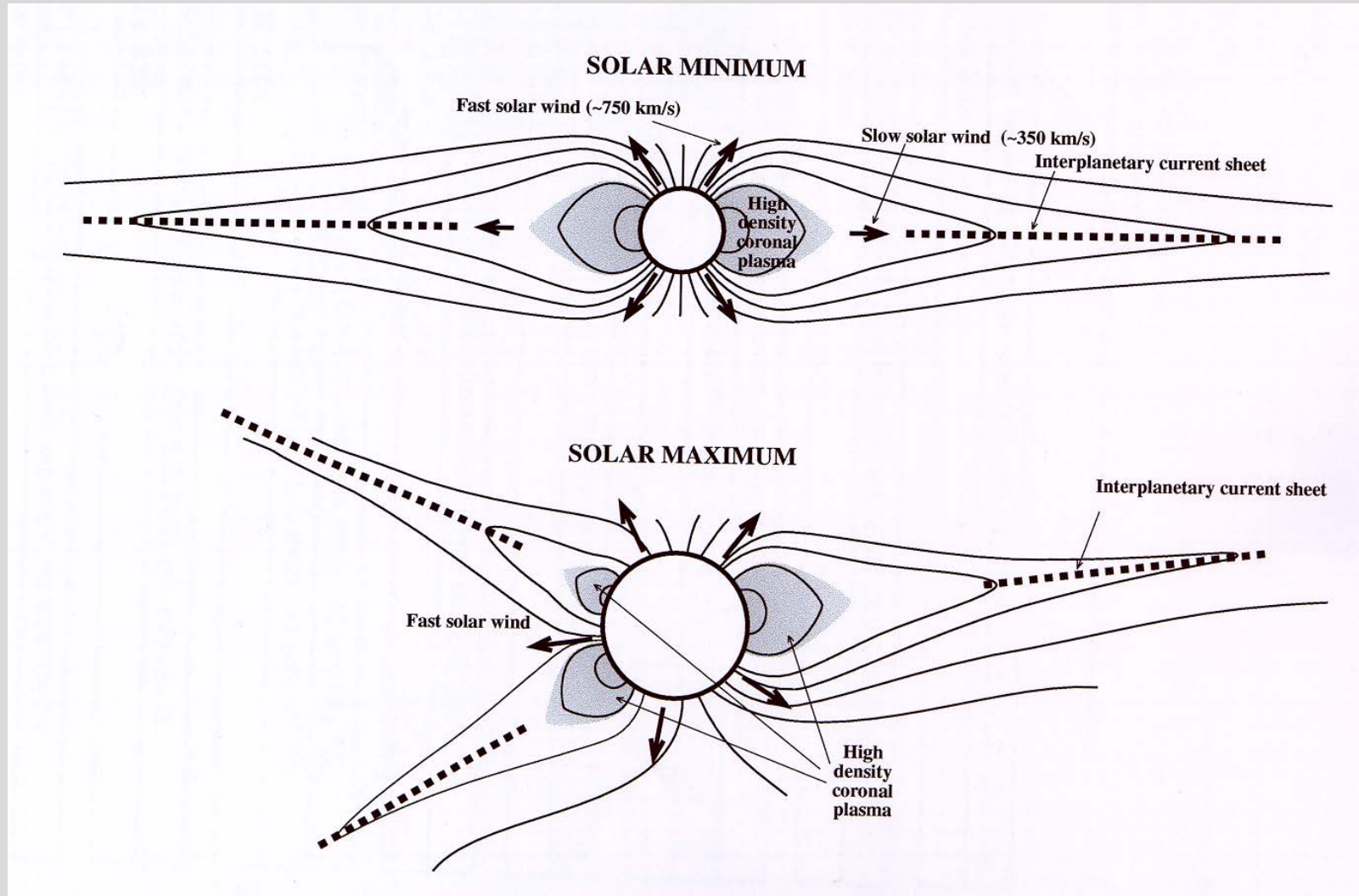
Solar wind

***Corona
continuously
merges into
solar wind***



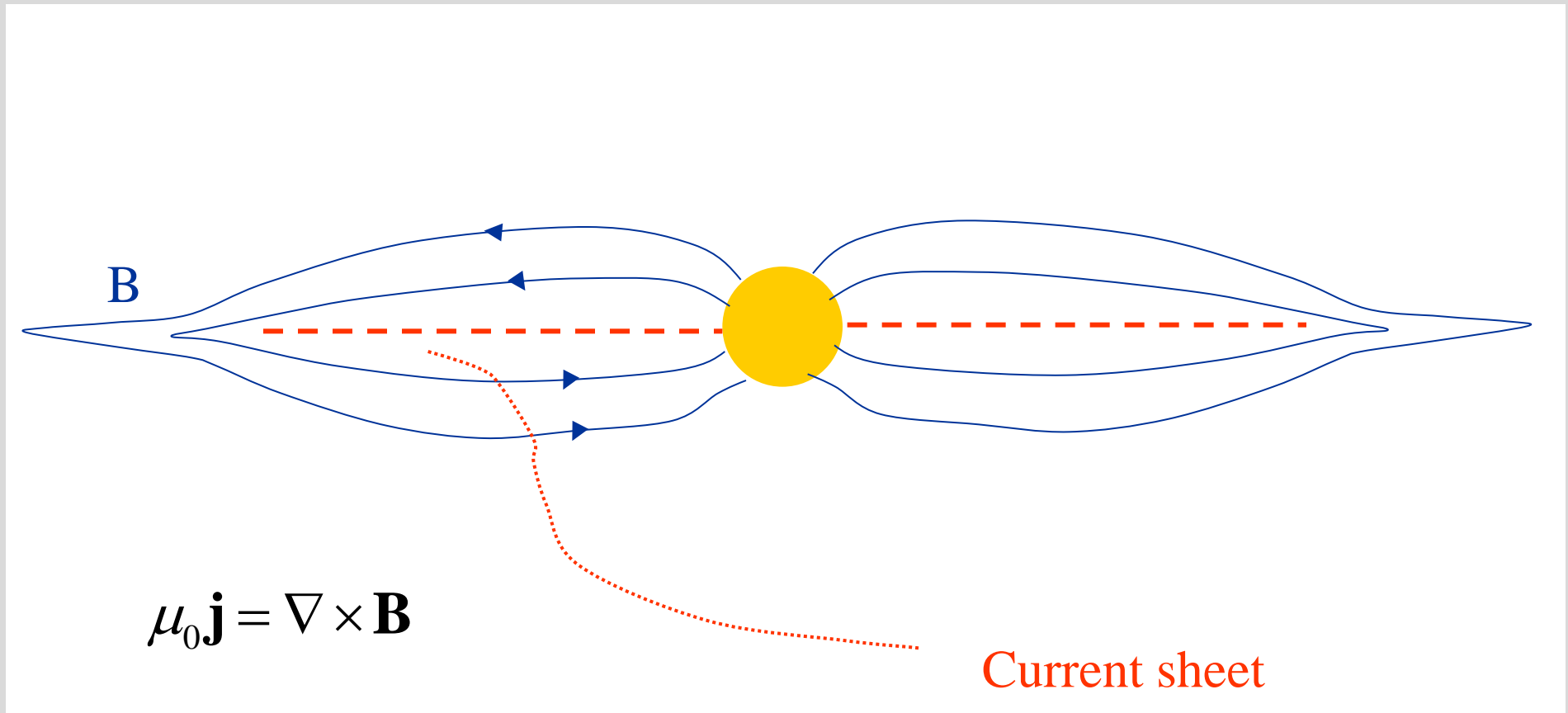
Solar and Heliospheric Observatory (SOHO)
LASCO C3 Coronagraph Movie

Solar wind



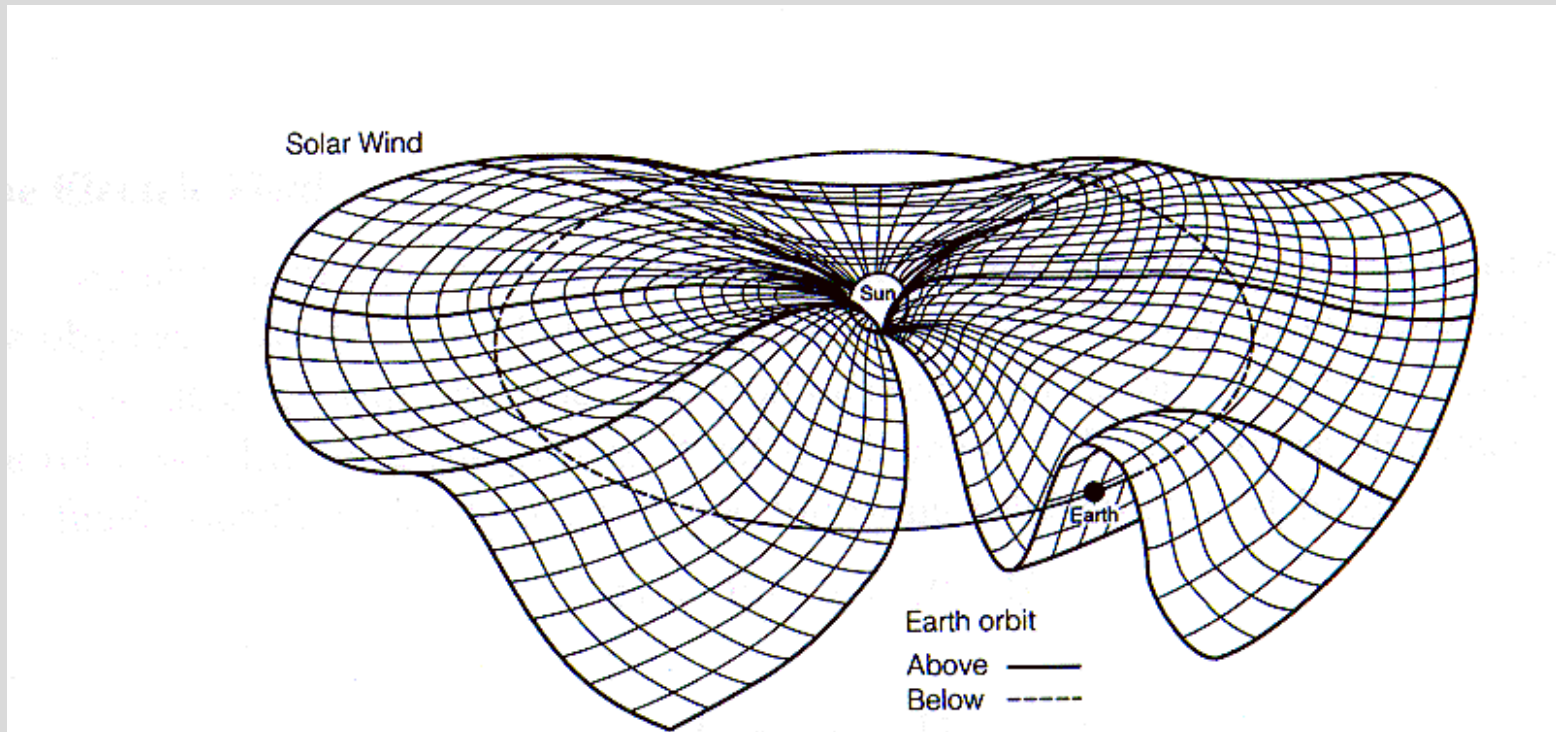
Solar wind

Interplanetary current sheet



Solar wind

Interplanetary current sheet



Later we will see that the N-S component of the interplanetary magnetic field (IMF) is important for the coupling between solar wind and magnetosphere)

Solar wind

Some basic facts

Average values

$$n_p = 8 \text{ cm}^{-3}$$

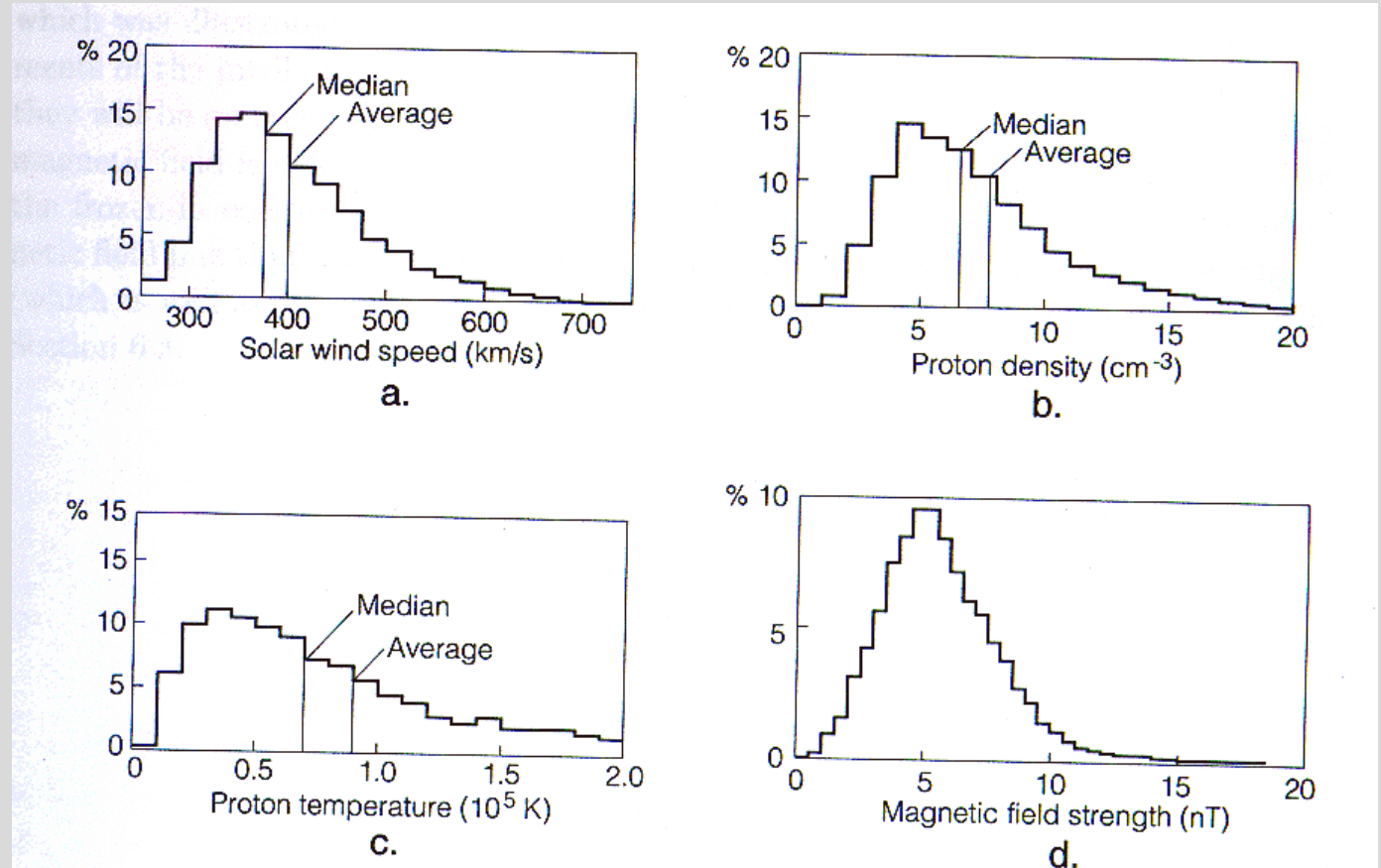
$$v = 320 \text{ km/s}$$

$$T_p = 4 \cdot 10^4 \text{ K}$$

$$T_e = 10^5 \text{ K}$$

$$B = 5 \text{ nT}$$

$$\Phi_K = \rho v^3 / 2 = 0.22 \text{ mW/m}^2$$

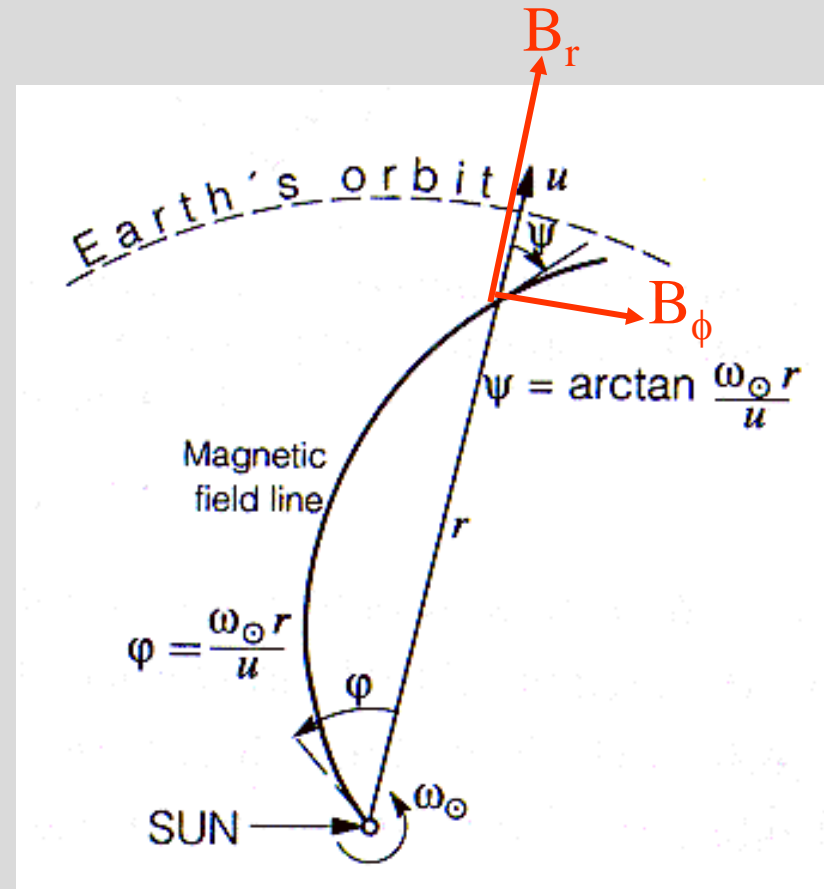


Solar wind

Parker spiral

Archimedean spiral:

$$\frac{B_{\phi}}{B_r} = \tan \psi = \left(\frac{\omega r}{u_{SW}} \right)$$





Classification of plasmas

- **High density plasmas**

- $\lambda \ll \rho$
- *magnetic field not important, collisions dominate, isotropic.*

- **Medium density plasmas**

- $\rho \ll \lambda \ll l_c$
- *magnetic field important, collisions important, anisotropies.*

- **Low density plasmas**

- $l_c \ll \lambda$
- *magnetic field important, anisotropies, uninhibited motion along magnetic field*

ρ : gyro radius

λ : mean free path

l_c : dimension of the plasma



Plasma models/descriptions

- Single particle motion
- Computer simulations of many-particle dynamics
- Generalization of statistical mechanics (kinetic theory)
- Generalization of fluid mechanics:
Magneto-hydrodynamics (MHD)

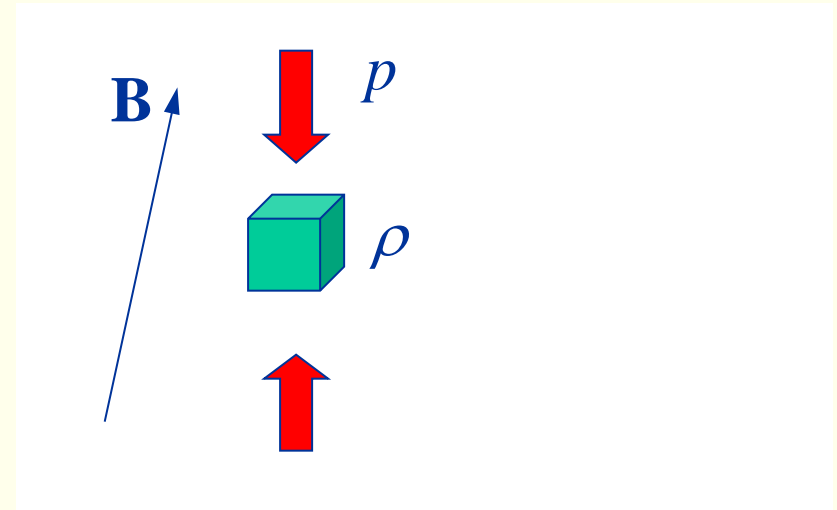
Plasma physics

Magnetohydrodynamics (MHD)

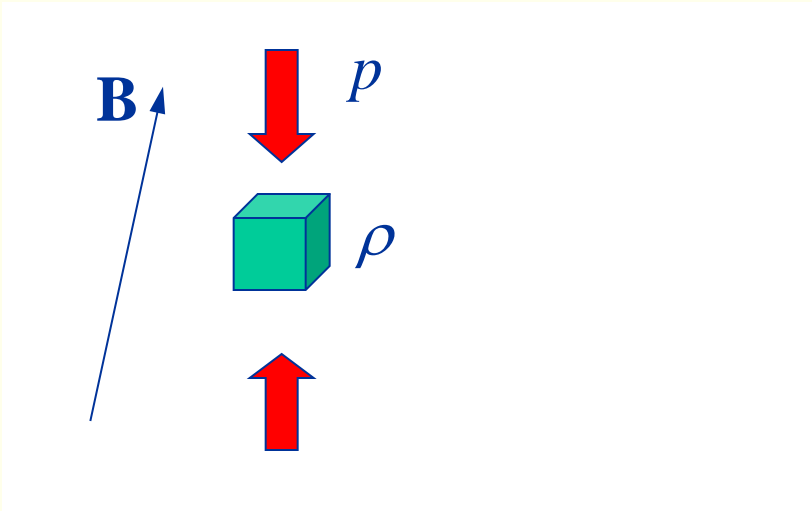
MHD is a combination of

- *fluid-/hydrodynamics* (which is based on Newton's laws of motion)
- *Maxwell's equations* (electrodynamics)

applied on a plasma volume element.



Magnetohydrodynamics (MHD)



For a volume element of plasma:

$$\mathbf{F} = m\mathbf{a} \quad \Rightarrow$$

$$-\nabla p + n_e q \mathbf{v}_e \times \mathbf{B} + \cancel{\rho q \mathbf{E}} = \rho \frac{d\mathbf{v}}{dt} \quad \Rightarrow$$

quasineutrality

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

Magnetohydrodynamics (MHD)

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

This together with two of Maxwell's equations and Ohm's law make up the most common MHD equations:

$$(2) \quad \mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$(3) \quad \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Only consider slow variations

$$(4) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Magnetohydrodynamics (MHD)

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

In equilibrium:

$$0 = -\nabla p + \mathbf{j} \times \mathbf{B} \quad \longleftrightarrow$$

$$-\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} = 0$$

$$-\nabla p - \nabla \left(\frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} = 0$$

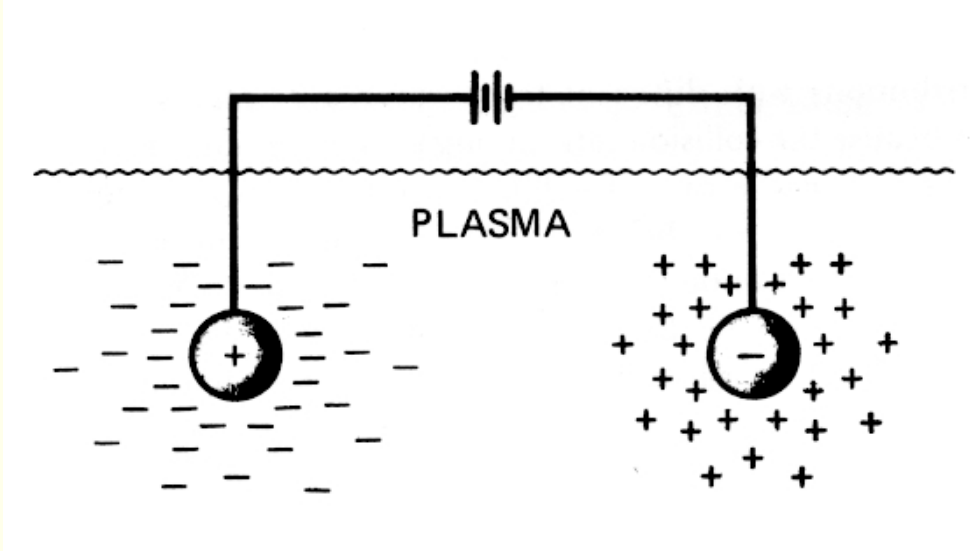
Represents tension in magnetic field

If magnetic tension = 0

$$p + \frac{B^2}{2\mu_0} = \textit{konst}$$

Magnetic pressure

Quasineutrality



$$\Phi = \Phi_0 e^{-x/\lambda_D}$$

Debye length

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_e e^2}}$$

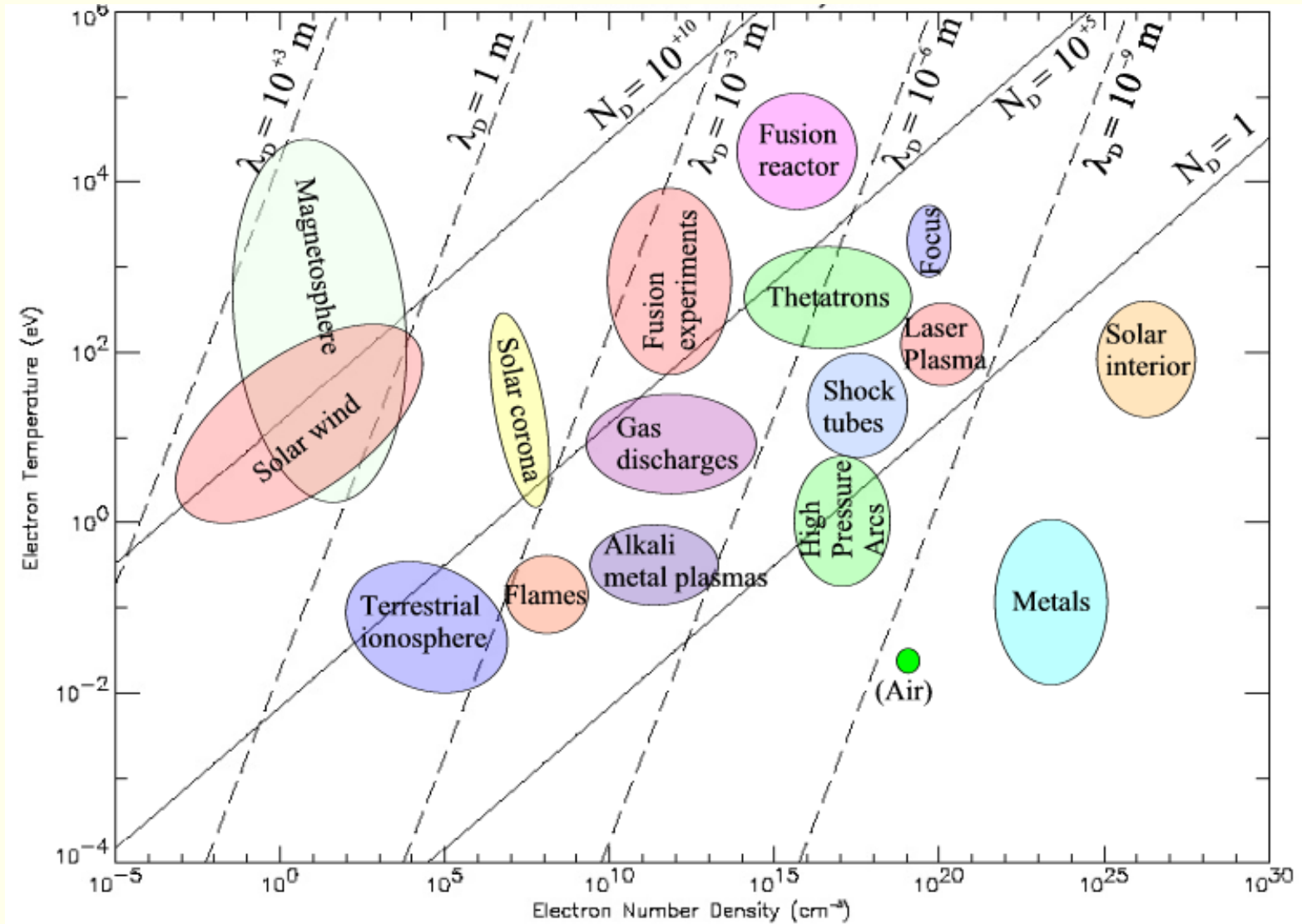
$$\frac{\Delta n}{n} = \frac{(n_e - n_i)}{n_e} < \left(\frac{\lambda_D}{l_c} \right)^2$$

$$l_C \gg \lambda_D \Rightarrow$$

Plasma close to neutral:

$$n_e \approx n_i$$

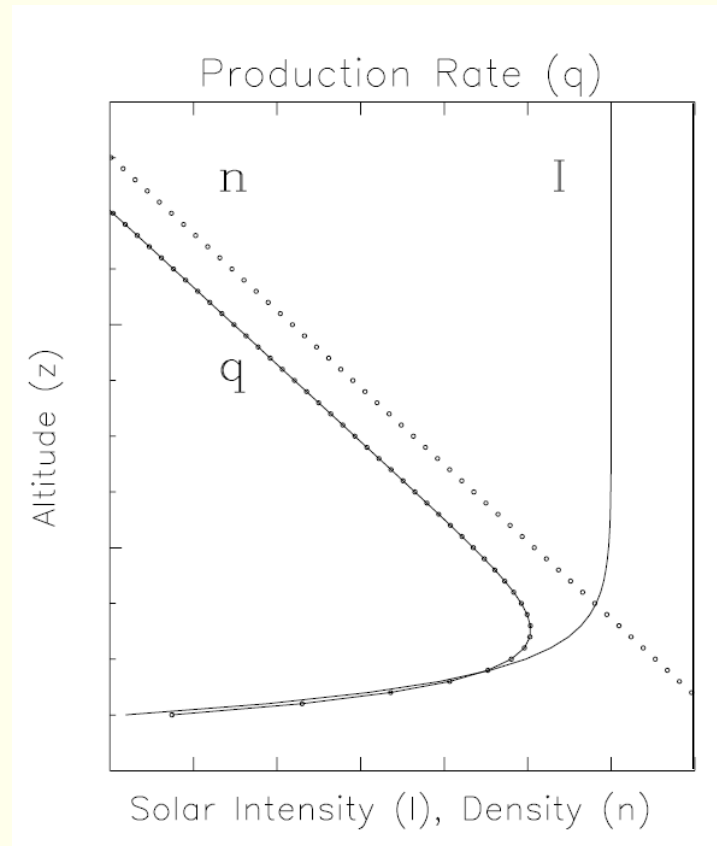
Debye lengths

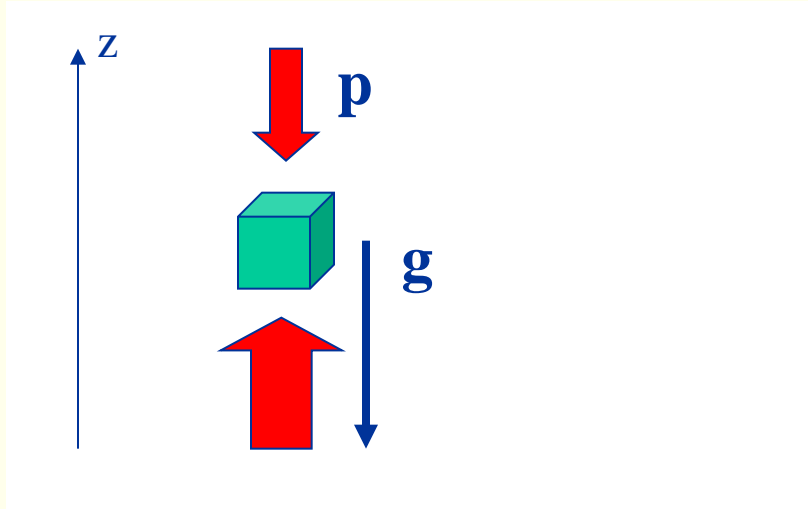




The ionosphere

Basic principle for creation of ionospheric layer





Atmospheric scale height

$$-\frac{dp}{dz} = g\rho_m \quad \text{hydrostatic equilibrium for a volume element}$$

$$p = nk_B T = \frac{\rho k_B T}{m} \quad \text{ideal gas law}$$

$$-\frac{k_B T}{m} \frac{d\rho_m}{dz} = g\rho_m \quad \text{if } T \text{ is constant}$$

$$\rho_m = \text{const} \cdot e^{-z/(k_B T / gm)} = \text{const} \cdot e^{-z/H}$$

Scale height

$$H = k_B T / gm$$

Scale height

$$H = k_B T / gm$$

What is the approximate scale height in the atmosphere right here, right now?

(0° C = 273 K)

Blue

1 km

Yellow

30 km

Green

9 km

Red

100 km



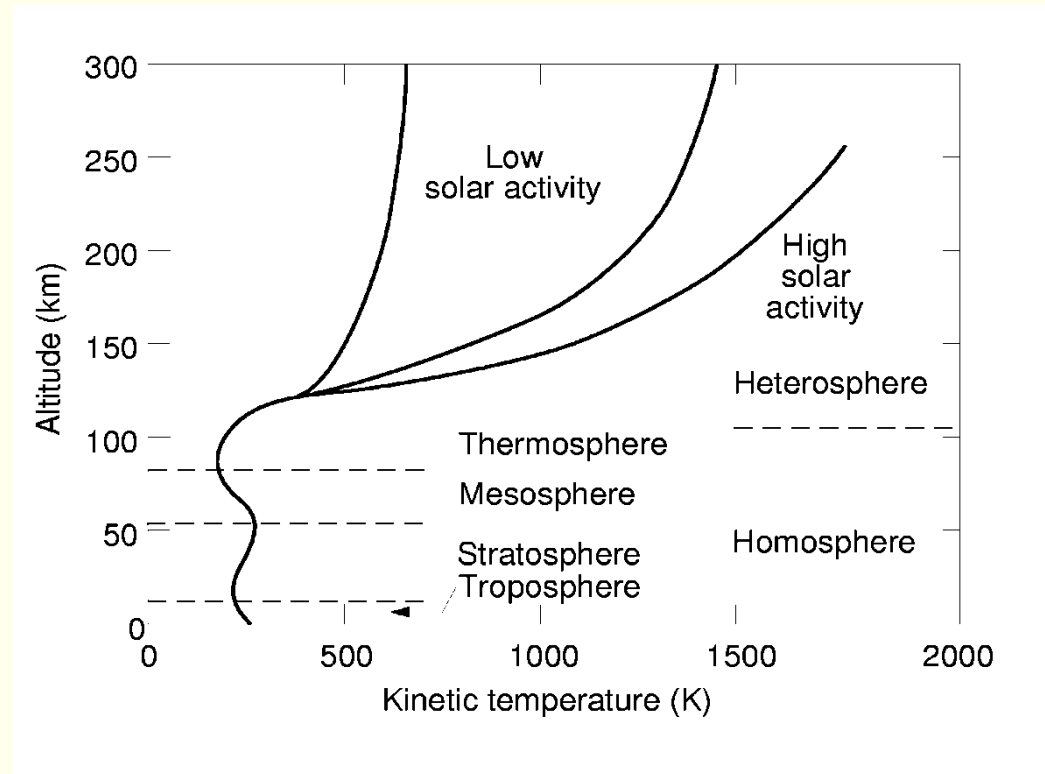
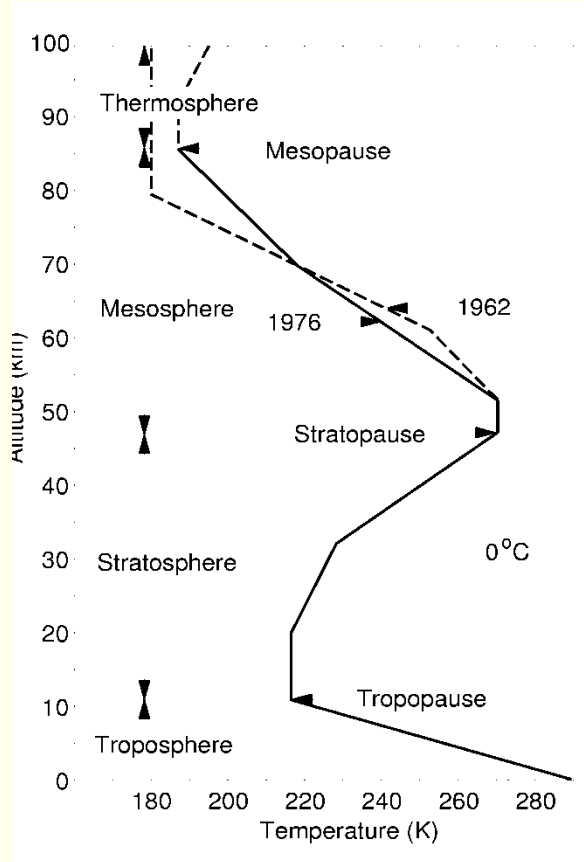
$$H = k_B T / gm = (1.38 \cdot 10^{-23} \cdot 290) / (9.81 \cdot 14 \cdot 2 \cdot 1.67 \cdot 10^{-27}) =$$
$$= 8724 \text{ m} \approx 9 \text{ km}$$

Green



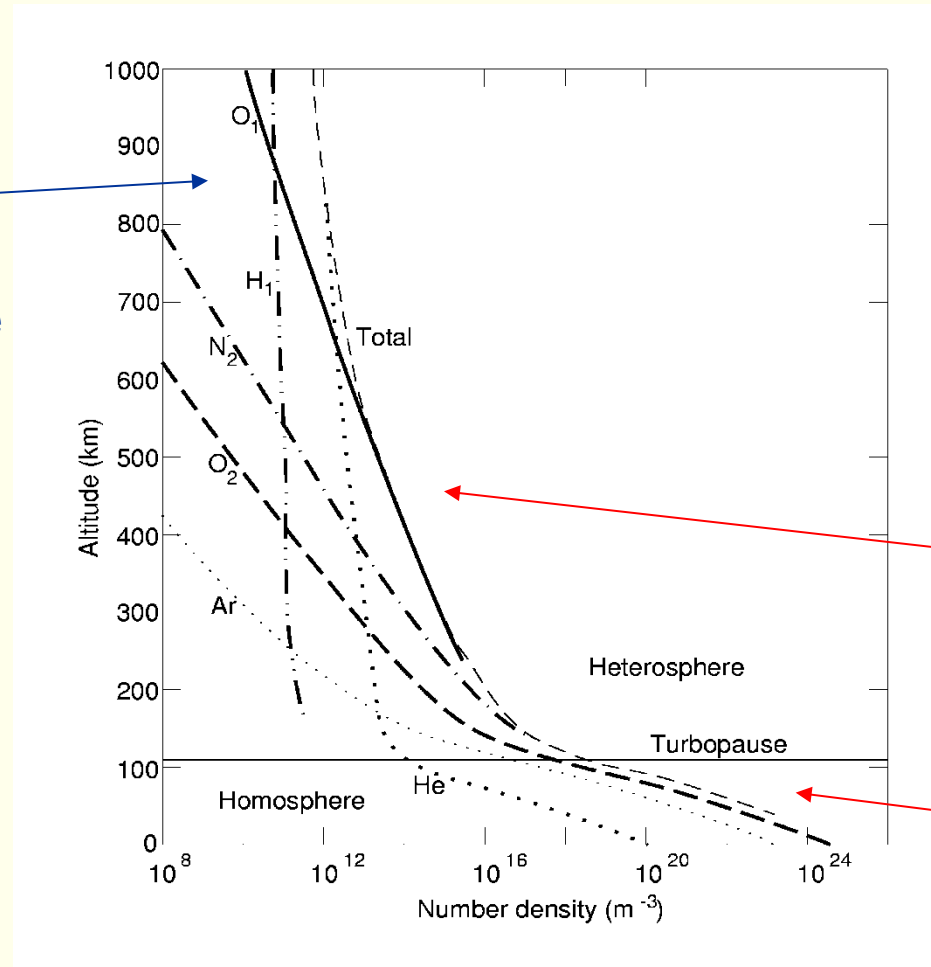
What did we neglect
when we derived the
scale height?

Temperature profile



Atmospheric composition

Longer scale height due to higher temperature



Separate scale heights for different components

Turbulent mixing – one scale height



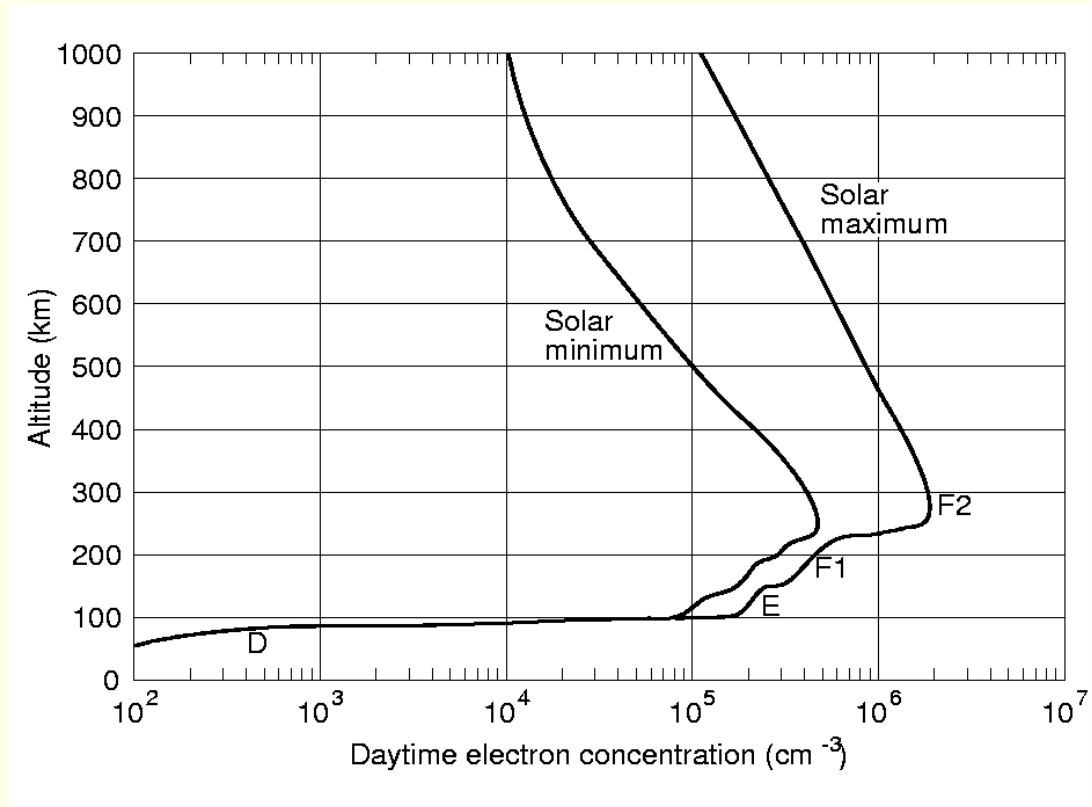
Ionosphere

- The ionized, electrically conducting part of the upper atmosphere
- The ionosphere is a **plasma**

History

- Stewart, 1882: Explained variations in the geomagnetic field
- Kenelly & Heavyside, 1902: explained Marconis transatlantic radio communication experiments
- Appleton & Barnett: experimental proof

Altitude distribution of electron density (n_e)



Continuity equation = conservation of ?

$$\frac{\partial n_e}{\partial t} = q - r - \nabla \cdot (n_e \mathbf{v}_e)$$

Ionization ($\text{m}^{-3}\text{s}^{-1}$)

Recombination ($\text{m}^{-3}\text{s}^{-1}$)

Flow ($\text{m}^{-3}\text{s}^{-1}$)



Ionization and recombination

Continuity equation

$$\frac{dn_e}{dt} = q - r$$

$$q = a_i I n_n$$

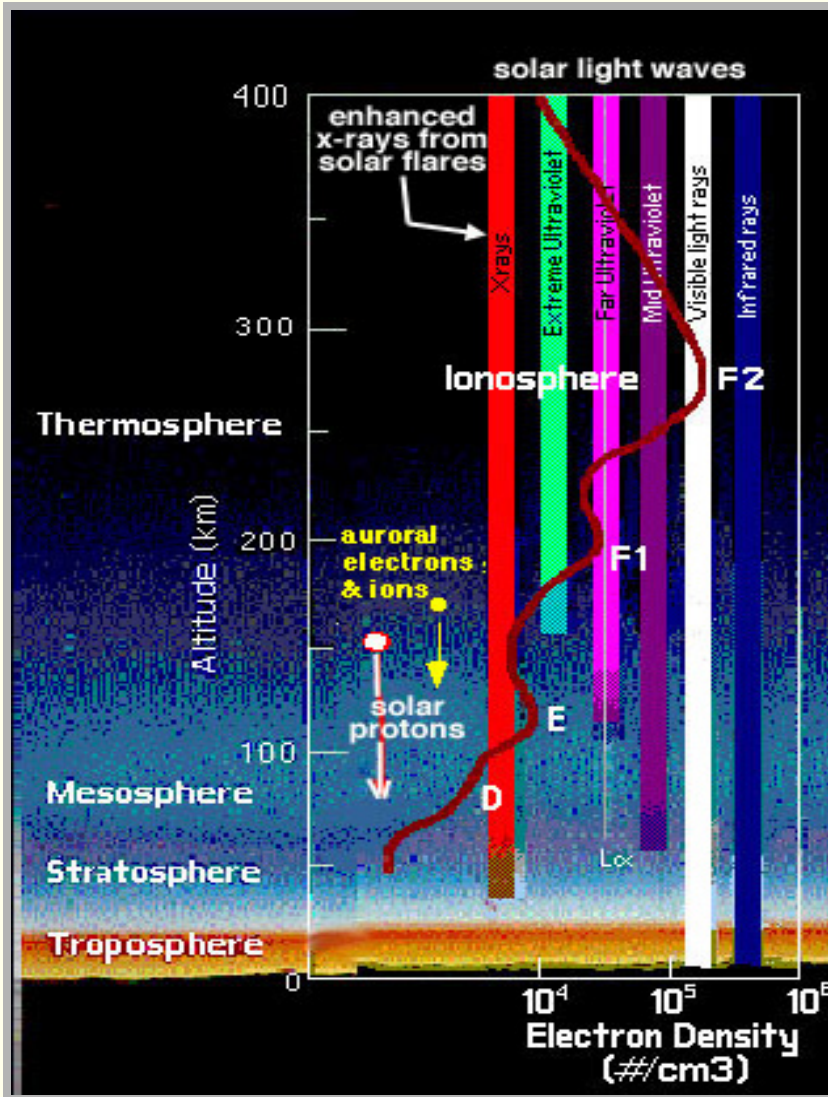
Ionization ($\text{m}^{-3}\text{s}^{-1}$)

Recombination ($\text{m}^{-3}\text{s}^{-1}$)

$$r = a_r n_e n_i = a_r n_e^2$$

Example: $e + \text{O}_2^+ \rightarrow \text{O} + \text{O}$ (dissociative recombination)

UV and X-ray radiation



$$\frac{dI}{dz} = I n_n a_a$$



Derive Chapman layer

Electron density in Chapman layer

$$n_e = \left\{ \frac{a_i}{a_r} I_0 n_0 e^{-\left(H a_a n_0 e^{-z/H} + z/H \right)} \right\}^{1/2}$$



What does it look like in reality?

- Temperature not constant
- Many different wavelengths in solar radiation
- Several different molecules and atoms in neutral atmosphere. Composition also depends on altitude.

"E-region" - simple model calculation

O₂ dominating species, 10 nm X-ray radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

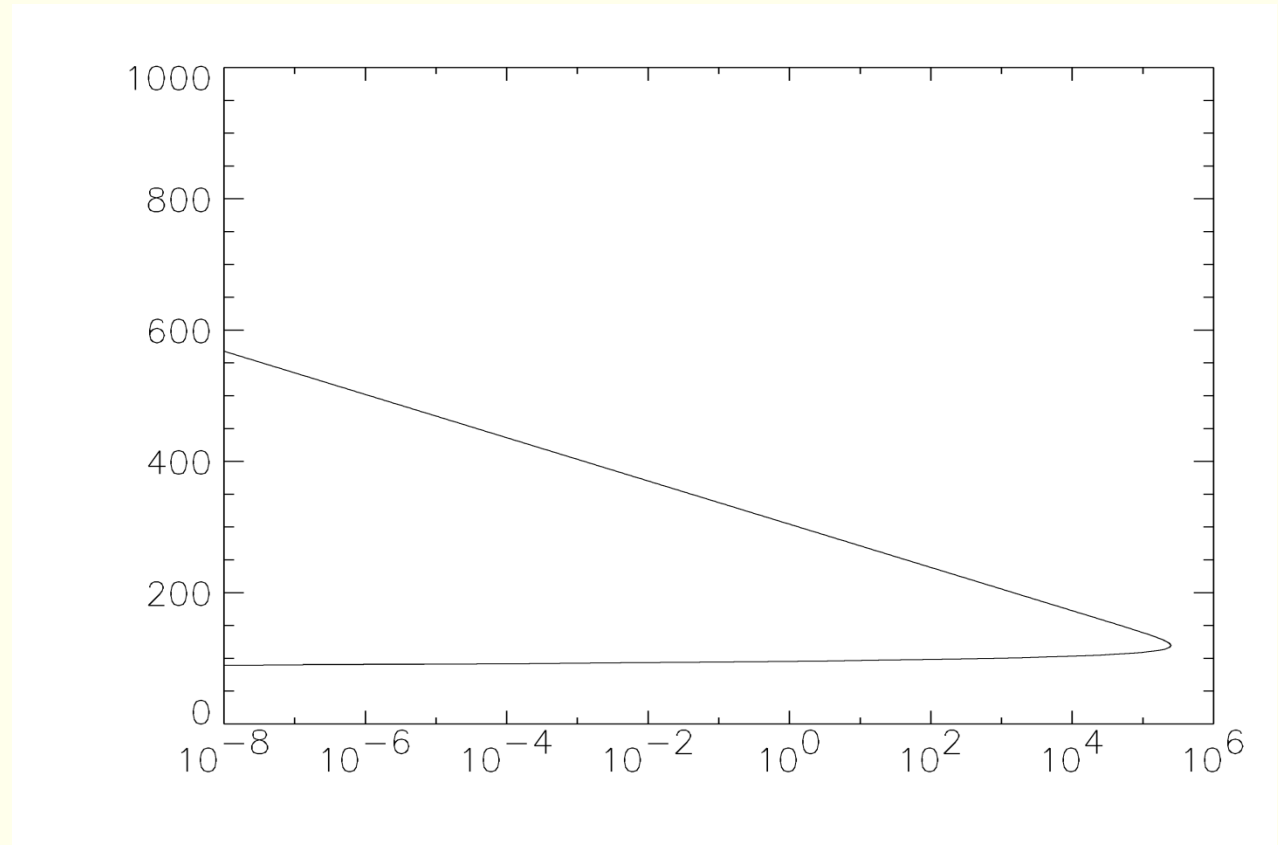
$$a_r = 3.0 \times 10^{-14}$$

$$T = 270$$

$$m = 16 \cdot 2 \cdot \text{amu}$$

$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 3.6 \times 10^{13} \text{ photons/m}^2/\text{s}$$



N₂⁺ produced, but rapidly lost through charge exchange: $N_2^+ + O_2 \rightarrow N_2 + O_2^+$

"F1-region" - simple model calculation

O₂ dominating species, 30 nm UV radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

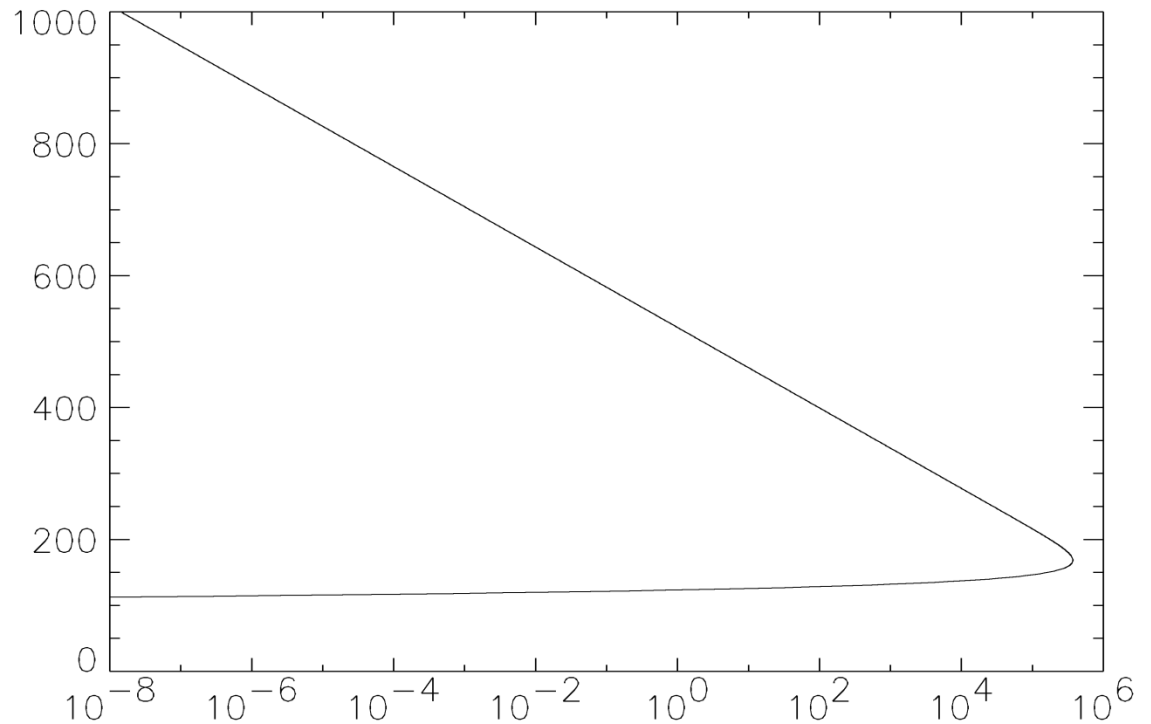
$$a_r = 3.0 \times 10^{-14}$$

$$T = 500$$

$$m = 16 \cdot 2 \cdot \text{amu}$$

$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 1.5 \times 10^{14} \text{ photons/m}^2/\text{s}$$



"F2-region" - simple model calculation

O dominating species, 30 nm UV radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

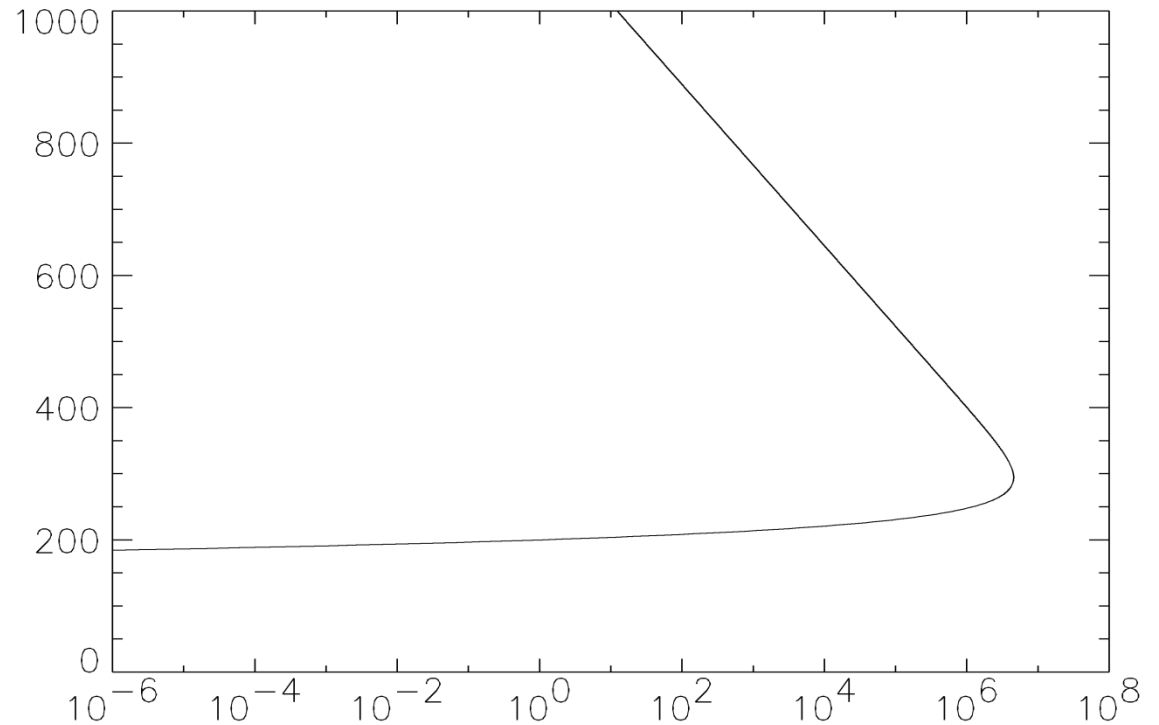
$$a_r = 1.0 \times 10^{-16}$$

$$T = 500$$

$$m = 16 \text{ amu}$$

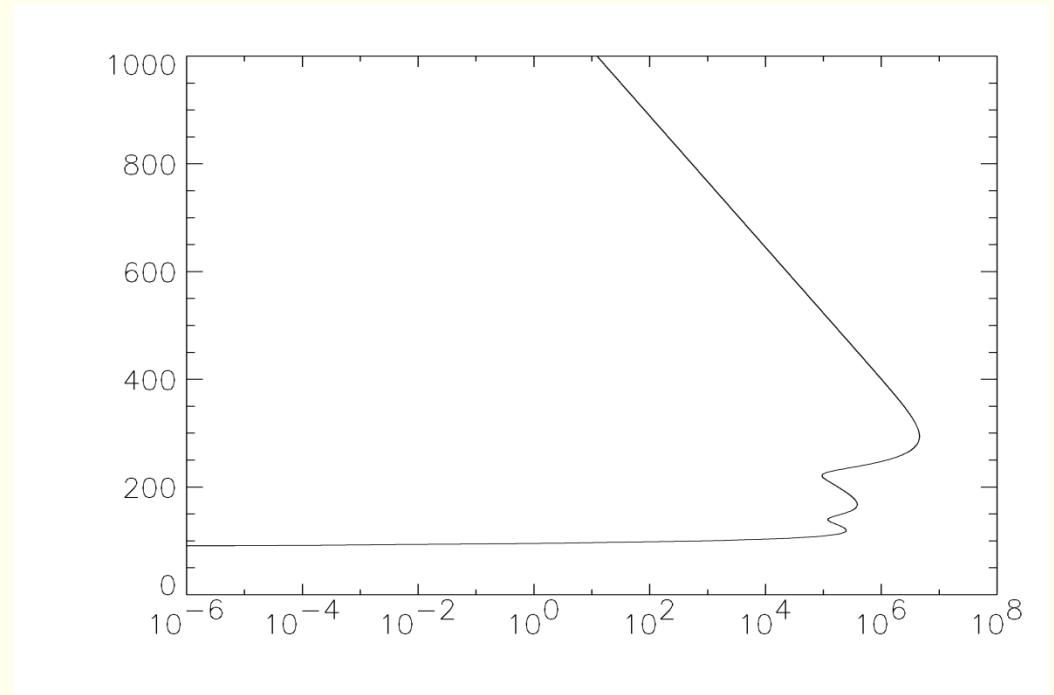
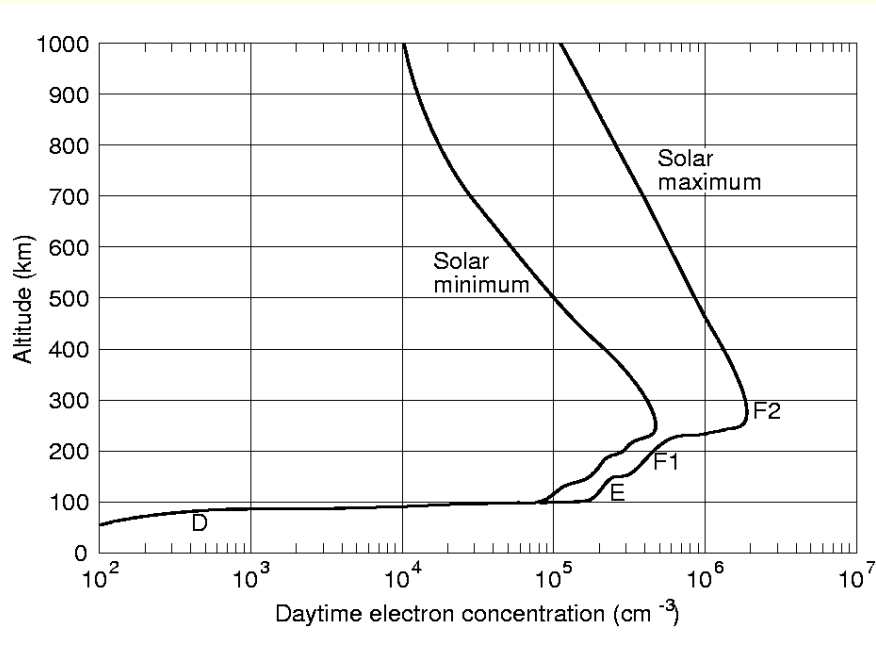
$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 1.5 \times 10^{14} \text{ photons/m}^2/\text{s}$$



Measurements

"E" + "F1" + "F2"



Ionospheric layers

Layer	D	E	F ₁	F ₂
Altitude (km)	60-85	85-140	140-200	200 - ca 1500
Nighttime electron density (cm ⁻³)	<10 ²	2 · 10 ³	—	2 - 5 · 10 ⁵
Daytime electron density (cm ⁻³)	10 ³	1 - 2 · 10 ⁵	2 - 5 · 10 ⁵	0.5 - 2 · 10 ⁶
Ion species	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺ O ⁺	O ⁺ He ⁺ H ⁺
Cause of ionization	Lyman α (1215 Å) + cosmic radiation	Lyman β (1025 Å) X-rays	UV	UV

NO⁺ created by chemical reaction $N_2^+ + O \rightarrow NO^+ + N$



Propation of radio waves in the ionosphere

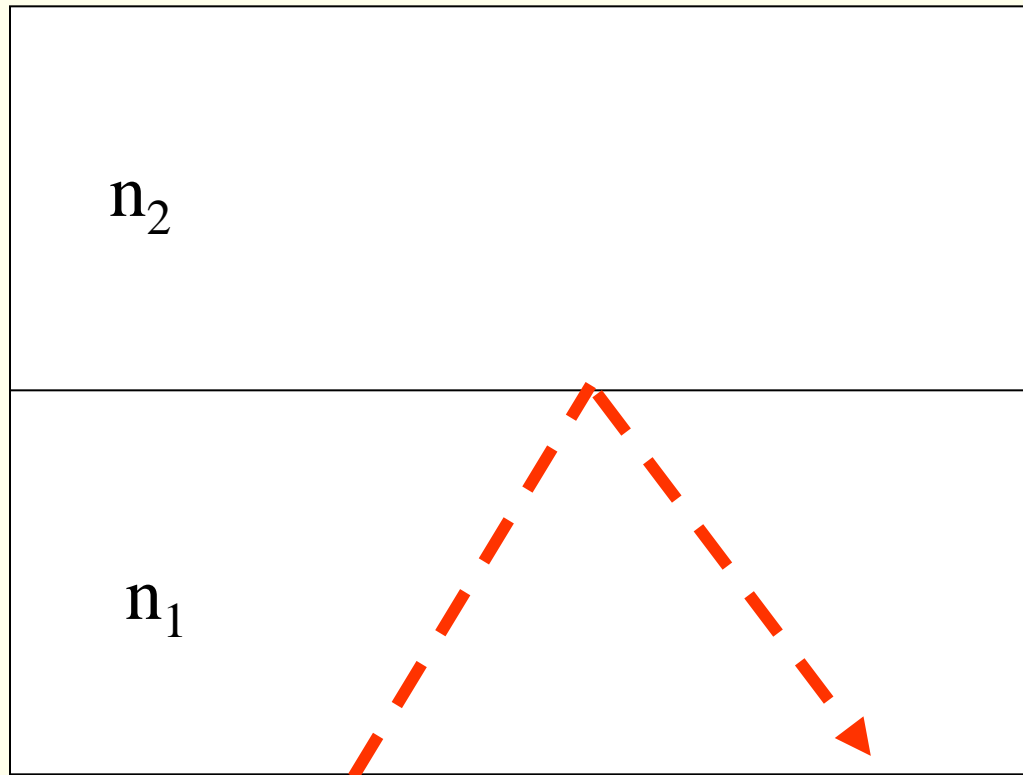
1. Absorption/damping

Takes place in the D-region due to high collision frequency. (Collisions with neutral atoms.)

2. Reflection

takes place in the F-region due to large gradients in the refraction index.

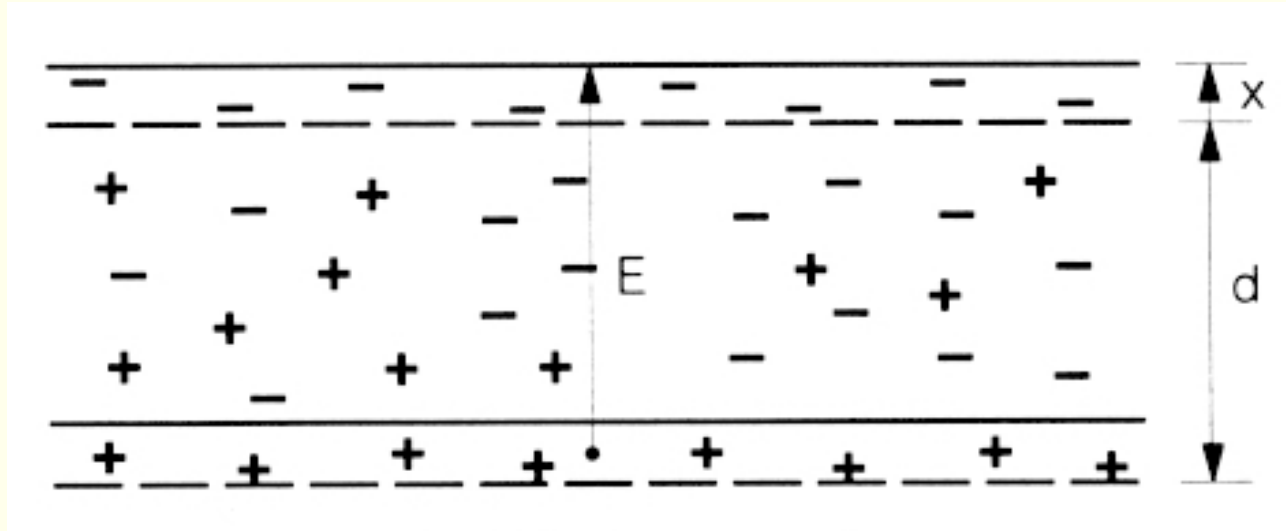
Reflection of radio waves



Total reflection at a sharp boundary (or large gradient) if

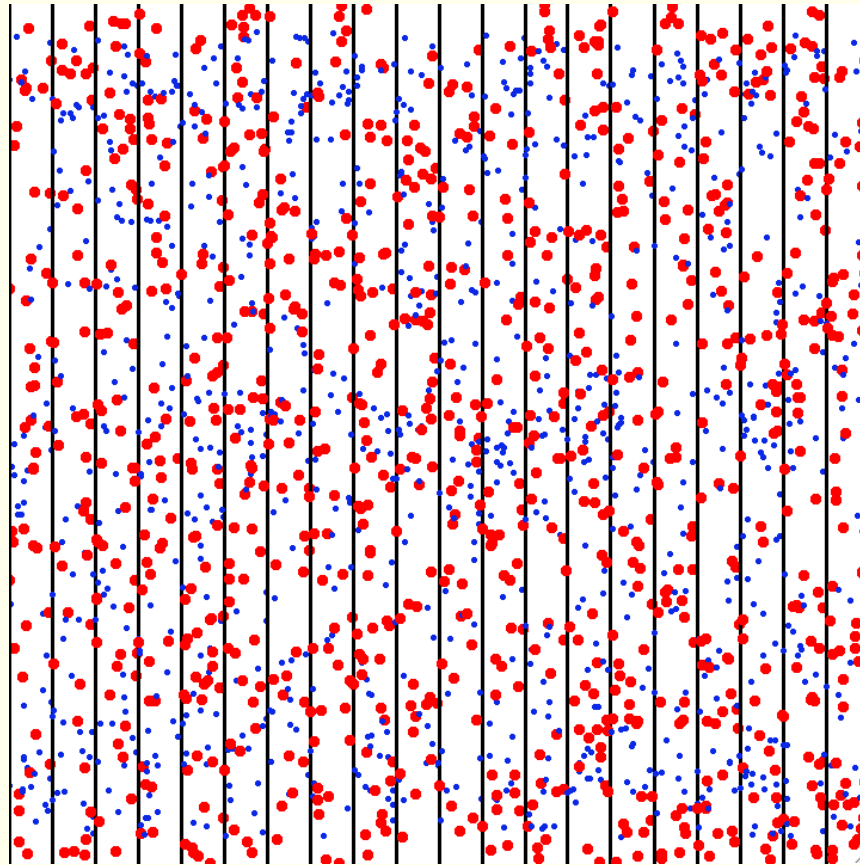
$$n_2 < n_1$$

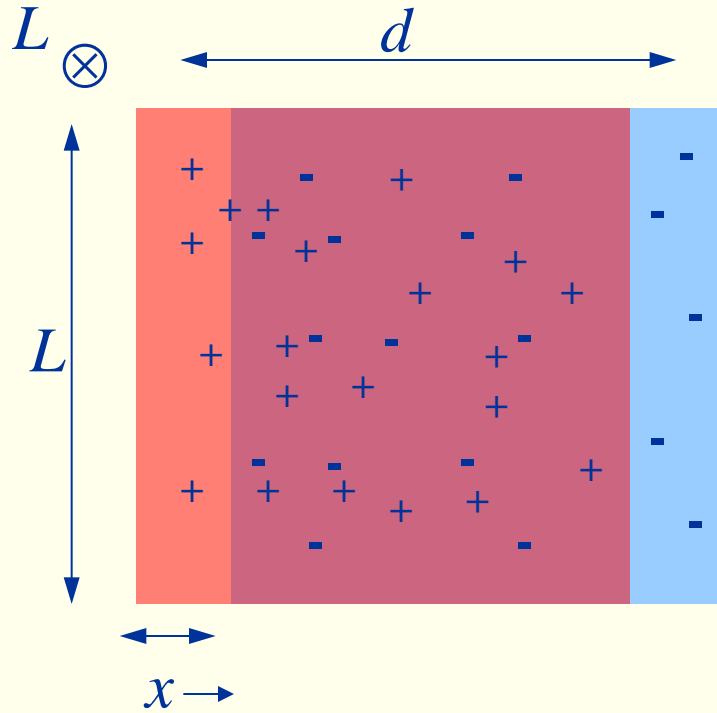
Plasma frequency



Charge imbalance creates an electric field which tends to even out the imbalance.

Plasma oscillations parallel to B





Newton's law on an individual electron inside the slab:

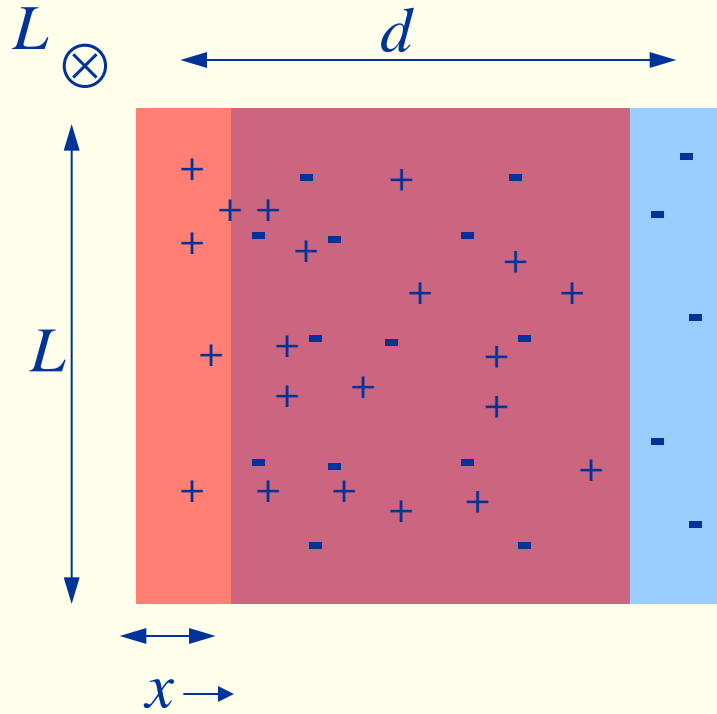
$$F = m_e a$$

$$F = -eE$$

$$E = \frac{\sigma}{\epsilon_0}$$

Surface charge density

$$\sigma = -en_e x$$

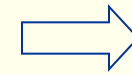


$$F = m_e a$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$F = -eE$$

$$\sigma = en_e x$$



$$-\frac{n_e e^2 x}{\epsilon_0 m_e} = \frac{d^2 x}{dt^2}$$

$$x = \sin(\omega_{pe} t)$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

What is the plasma frequency f_{pe} at the daytime E-region, close to solar minimum? (see Fälthammar p 28)

$$f_{pe} = \frac{\omega_{pe}}{2\pi} \equiv \frac{1}{2\pi} \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

Blue

7 kHz

Yellow

400 MHz

Green

3 MHz

Red

2 GHz

$$f = \frac{\omega_{pe}}{2\pi} \equiv \frac{1}{2\pi} \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} = \frac{1}{2\pi} \sqrt{\frac{(1.6 \cdot 10^{-19})^2}{8.854 \cdot 10^{-12} \cdot 0.91 \cdot 10^{-30}}} \sqrt{n_e} =$$

$$8.97 \sqrt{n_e} = 8.97 \sqrt{10^5 \cdot 10^6} = 2.8 \cdot 10^6 \text{ Hz} = 2.8 \text{ MHz}$$

Green