Theory of PDE SF2739 Homework.

John Andersson johnan@kth.se

This is your homework assignments for the first part of the course. It is to be handed in to me on the 24th of September on the lecture or emailed to me no later than midnight on Sunday the 27th of September. Both questions should be answered.

1. Consider the partial differential equation

$$u_x(x,y) + (u(x,y)+1)u_y(x,y) = 0 \quad \text{in } \mathbb{R}^2 \cap \{x > 0\}$$
$$u(0,y) = \begin{cases} 1 & \text{if } y < 0 \\ 1-y & \text{if } 0 \le y \le 1 \\ 0 & \text{if } y > 1. \end{cases}$$

Write down its solution and sketch the projected characteristics in a coordinate system. Specify any shocks and or rarefacations in the set $\mathbb{R}^2 \cap \{x > 0\}$.

[5 marks]

2. Write a two page summary of the theory of first order partial differential equations and the method of characteristics. Two pages is not much and therefore you will not be able to cover the entire theory. Instead you should pick some aspect (or a few aspects) of the theory that interests you. It could be the phenomena of the PDE deciding its own natural domain of definition, it could be an exposition of the 1st year calculus used in higher mathematics or it could be something entirely different. The important thing is that you carefully read through the text and try to come up with your own interpretation of it or of an aspect of it. Just repeating what I have said during the lectures will probably be about as interesting as the lectures themselves.[1] Part of this exercise is about convincing me that you have read the material and thought it through carefully.

[5 marks]

[1] That is, a hogwash of abject non-sense.