Principles of Wireless Sensor Networks

https://www.kth.se/social/course/EL2745/

Lecture 9

Dynamic Distributed Estimation

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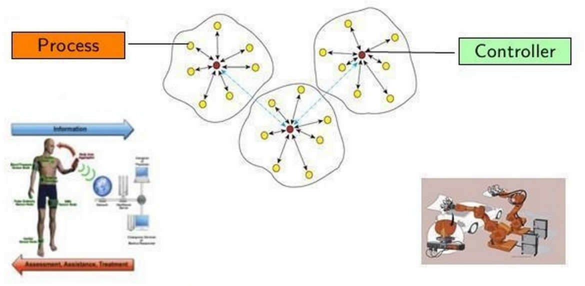
Course content

- Part 1
  - Lec 1: Introduction to WSNs
  - Lec 2: Introduction to Programming WSNs
- Part 2
  - Lec 3: Wireless Channel
  - Lec 4: Physical Layer
  - Lec 5: Medium Access Control Layer
  - Lec 6: Routing
- Part 3
  - Lec 7: Distributed Detection
  - Lec 8: Static Distributed Estimation
  - Lec 9: Dynamic Distributed Estimation
  - Lec 10: Positioning and Localization
  - Lec 11: Time Synchronization
- Part 4
  - Lec 12: Wireless Sensor Network Control Systems 1
  - Lec 13: Wireless Sensor Network Control Systems 2

Previous lecture

- Star and general topology
- Estimation from one sensor
- Distributed estimation in a start topology
- Distributed estimation in a general topology

Today's lecture



- Today we study how to perform dynamic estimation from erroneous or noisy measurements of the sensors
- "Dynamic" means that we take advantage of the time evolution of signals to build the estimators

Today's learning goals

- How to perform estimation of a dynamic signal from one sensor?
- How to perform estimation of a dynamic signal from many sensors?
- How to make a sensor fusion of a dynamic signal by the distributed Kalman filter?

Outline

- Dynamic estimation from one sensor
- Dynamic estimation from many sensors in a star topology
- Dynamic estimation from many sensors by the distributed Kalman filter

Outline

- Dynamic estimation from one sensor
- Dynamic estimation from many sensors in a star topology
  - Dynamic sensor fusion, centralized setup
  - Dynamic sensor fusion, centralized setup (drawbacks)
  - Dynamic sensor fusion, distributed Kalman filtering

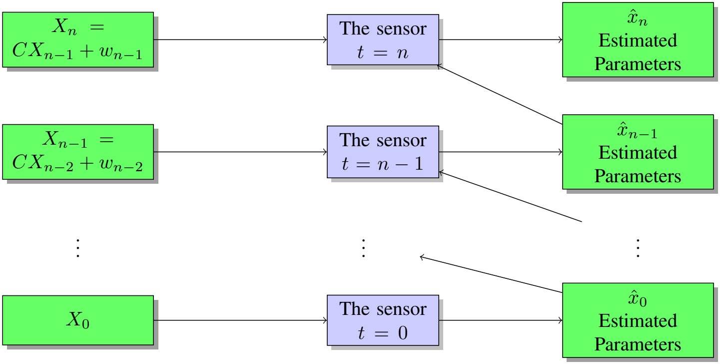


Figure: Illustration of how the fusion of sequential measurement works to combine measurements in one sensor.

- We want to combine many dynamic measurements in one sensor

Preliminaries

- Given two random variables $\mathbf{x}$ and $\mathbf{y}$, the MMSE estimator of $\mathbf{x}$ given a realization $\mathbf{y} = y$ is given by the conditional expectation

$$\hat{\mathbf{x}} = \mathsf{E}\{\mathbf{x}|\mathbf{y} = y\}$$

Dynamic estimation from one sensor

Proposition 1

Consider a phenomenon $\mathbf{x}$ *evolving in time (indexed by* n*) according to*

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{w}_n$$

Dynamic estimation from one sensor

Proposition 1

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Every time step a sensor generates measurement of the form

$$\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{v}_n$$

- $\mathbf{w}_n$ *and* $\mathbf{v}_n$ *are zero mean Gaussian noises with* $E\{\mathbf{w}_n\mathbf{w}_n^T\} = \mathbf{Q}$ *and* $E\{\mathbf{v}_n\mathbf{v}_n^T\} = \mathbf{R}$
- $\mathbf{A}$, $\mathbf{C}$ *are known matrices, with* $\mathbf{A}$ *invertible*

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- $\mathbf{A}$, $\mathbf{C}$ *are known matrices, with* $\mathbf{A}$ *invertible*

Then the MMSE estimate $\hat{\mathbf{x}}_n$ *given the past measurements* $(\mathbf{y}_0, \dots, \mathbf{y}_{n-1})$ *is such that*

$$\hat{\mathbf{x}}_{n|n-1} = \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}$$

$$\mathbf{P}_{n|n-1} = \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^T + \mathbf{Q}$$

- $\hat{\mathbf{x}}_{n-1|n-1}$ *is the estimate of* $\mathbf{x}_{n-1}$ *given* $(\mathbf{y}_0, \dots, \mathbf{y}_{n-1})$
- $\mathbf{P}_{n-1|n-1}$ *is the corresponding error covariance matrix*

Proof of proposition 1

Step 1: Show that $\hat{\mathbf{x}}_{n|n-1} = \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}$

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Step 2: Show that $\mathbf{P}_{n|n-1} = \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$

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Step 1: Show that $\hat{\mathbf{x}}_{n|n-1} = \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}$

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Step 2: Show that $\mathbf{P}_{n|n-1} = \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$

$$\begin{aligned}\mathbf{P}_{n|n-1} &= \mathsf{E}\{(\hat{\mathbf{x}}_{n|n-1}-\mathbf{x}_n)(\hat{\mathbf{x}}_{n|n-1}-\mathbf{x}_n)^{\mathsf{T}}\}\\ &= \mathsf{E}\{(\mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}-\mathbf{A}\mathbf{x}_{n-1}-\mathbf{w}_{n-1})(\mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}-\mathbf{A}\mathbf{x}_{n-1}-\mathbf{w}_{n-1})^{\mathsf{T}}\}\end{aligned}$$

Proof of proposition 1

Step 1: Show that $\hat{\mathbf{x}}_{n|n-1} = \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}$

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Step 2: Show that $\mathbf{P}_{n|n-1} = \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$

$$\begin{aligned}\mathbf{P}_{n|n-1} &= \mathsf{E}\{(\hat{\mathbf{x}}_{n|n-1} - \mathbf{x}_n)(\hat{\mathbf{x}}_{n|n-1} - \mathbf{x}_n)^{\mathsf{T}}\} \\ &= \mathsf{E}\{(\mathbf{A}\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{A}\mathbf{x}_{n-1} - \mathbf{w}_{n-1})(\mathbf{A}\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{A}\mathbf{x}_{n-1} - \mathbf{w}_{n-1})^{\mathsf{T}}\} \\ &= \mathsf{E}\{\mathbf{A}(\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{x}_{n-1})(\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{x}_{n-1})^{\mathsf{T}}\mathbf{A}^{\mathsf{T}} + \mathbf{w}_{n-1}\mathbf{w}_{n-1}^{\mathsf{T}}\} \\ &= \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}\end{aligned}$$

where

$$\mathbf{P}_{n-1|n-1} = \mathsf{E}\{(\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{x}_{n-1})(\hat{\mathbf{x}}_{n-1|n-1} - \mathbf{x}_{n-1})^{\mathsf{T}}\}$$

Dynamic estimation from one sensor

Let us consider the compact form

$$\underbrace{\begin{bmatrix} \mathbf{y}_{n-1} \\ \vdots \\ \mathbf{y}_0 \end{bmatrix}}_{\mathbf{z}} = \underbrace{\begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C}\mathbf{A}^{-(n-1)} \end{bmatrix}}_{\mathbf{H}} \mathbf{x}_{n-1} + \underbrace{\begin{bmatrix} \mathbf{v}_{n-1} \\ \vdots \\ \mathbf{v}_0 - \cdots - \mathbf{C}\mathbf{A}^{-1}\mathbf{w}_0 \end{bmatrix}}_{\mathbf{u}}$$

obtained with the following recursion

$$\begin{aligned} \mathbf{y}_{n-1} &= \mathbf{C}\mathbf{x}_{n-1} + \mathbf{v}_{n-1} \\ \mathbf{y}_{n-2} &= \mathbf{C}\mathbf{x}_{n-2} + \mathbf{v}_{n-2} = \mathbf{C}\mathbf{A}^{-1}\mathbf{x}_{n-1} + \left(\mathbf{v}_{n-2} - \mathbf{C}\mathbf{A}^{-1}\mathbf{w}_{n-2}\right) \\ &\cdots \\ \mathbf{y}_0 &= \mathbf{C}\mathbf{A}^{-(n-1)}\mathbf{x}_{n-1} + \left(\mathbf{v}_0 - \mathbf{C}\mathbf{A}^{-(n-1)}\mathbf{w}_{n-2} - \cdots - \mathbf{C}\mathbf{A}^{-1}\mathbf{w}_0\right) \end{aligned}$$

Dynamic estimation from one sensor

Let us consider the compact form

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&\cdots\\
\mathbf{y}_0 &= \mathbf{C}\mathbf{A}^{-(n-1)}\mathbf{x}_{n-1} + \left(\mathbf{v}_0 - \mathbf{C}\mathbf{A}^{-(n-1)}\mathbf{w}_{n-2} - \cdots - \mathbf{C}\mathbf{A}^{-1}\mathbf{w}_0\right)
\end{aligned}$$

Then, it is possible to use Proposition 1 of Lecture 8 to obtain

$$\mathbf{P}^{-1}_{n-1|n-1}\hat{\mathbf{x}}_{n-1|n-1} = \mathbf{H}^{\mathsf{T}}\mathbf{R}_{\mathbf{u}}^{-1}\mathbf{z}$$

$$\mathbf{P}_{n-1|n-1} = \left(\mathbf{R}^{-1}_{\mathbf{x}_{n-1}} + \mathbf{H}^{\mathsf{T}}\mathbf{R}_{\mathbf{u}}^{-1}\mathbf{H}\right)^{-1}$$

Dynamic estimation from one sensor

- From Proposition 1, we know the estimate of $\mathbf{x}_n$ and the error covariance as a function of the past measurements $\mathbf{z} = (\mathbf{y}_0, \ldots, \mathbf{y}_{n-1})$

$$\hat{\mathbf{x}}_{n|n-1} = \mathbf{A}\hat{\mathbf{x}}_{n-1|n-1}$$
$$\mathbf{P}_{n|n-1} = \mathbf{A}\mathbf{P}_{n-1|n-1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

where

$$\hat{\mathbf{x}}_{n-1|n-1} = \mathbf{P}_{n-1|n-1}\mathbf{H}^{\mathsf{T}}\mathbf{R}_{\mathbf{u}}^{-1}\mathbf{z}$$
$$\mathbf{P}_{n-1|n-1} = \left(\mathbf{R}_{\mathbf{x}_{n-1}}^{-1} + \mathbf{H}^{\mathsf{T}}\mathbf{R}_{\mathbf{u}}^{-1}\mathbf{H}\right)^{-1} \quad (1)$$

- The estimate of $\mathbf{x}_n$ given only the current measurement $\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{v}_n$ can be obtained by the standard MMSE (Proposition 1 of Lecture 8)

$$\hat{\mathbf{x}} = \mathbf{M}\mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{y}_n \text{ , where } \mathbf{M} = \left(\mathbf{R}_{\mathbf{x}_n}^{-1} + \mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{C}\right)^{-1}$$

Next step: Derive the MMSE estimate of $\mathbf{x}_n$ given $(\mathbf{y}_0, \ldots, \mathbf{y}_{n-1}, \mathbf{y}_n) = (\mathbf{z}, \mathbf{y}_n)$

Dynamic estimation from one sensor

Result 1

The MMSE estimate of $\mathbf{x}_n$ given $(\mathbf{y}_0, \ldots, \mathbf{y}_{n-1}, \mathbf{y}_n) = (\mathbf{z}, \mathbf{y}_n)$ can be obtained by combining the available estimates at time n i.e, $\hat{\mathbf{x}}_n$ and $\hat{\mathbf{x}}_{n|n-1}$, with a **static sensor fusion** (Proposition 2 of Lecture 8),

$$\begin{aligned}\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} &= \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \mathbf{M}^{-1}\hat{\mathbf{x}} \\ &= \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{y}_n\end{aligned}$$

where

$$\begin{aligned}\mathbf{P}_{n|n}^{-1} &= -\mathbf{R}_{\mathbf{x}_n}^{-1} + \mathbf{P}_{n|n-1}^{-1} + \mathbf{M}^{-1} \\ &= \mathbf{P}_{n|n-1}^{-1} + \mathbf{C}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{C}^{-1}\end{aligned}$$

- The above equations represent the update steps of the so called **Kalman filter**.
- The **Kalman filter** can be seen as a **combination of estimators** that is **optimal** in the **minimum mean squared** sense.

Outline

- Dynamic estimation from one sensor
- Dynamic estimation from many sensors in a star topology
  - Dynamic sensor fusion, centralized setup
  - Dynamic sensor fusion, centralized setup (drawbacks)
  - Dynamic sensor fusion, distributed Kalman filtering

Dynamic sensor fusion

Consider a phenomenon $\mathbf{x}$ evolving in time (indexed by n) according to the law

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{w}_n$$

Dynamic sensor fusion

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Every time step, sensor k generates a measurement of the form

$$\mathbf{y}_{n,k} = \mathbf{C}_k\mathbf{x}_n + \mathbf{v}_{n,k}$$

Dynamic sensor fusion

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- **Multiple sensors** that generate measurements about the random variable that is evolving in time
- **Question:** How to **fuse data** from all the sensors for an estimate of the state $\mathbf{x}_n$ at time step n?

Dynamic sensor fusion

- **Centralized** approach
  1. At every time step n, all the sensors transmit their **measurements** $\mathbf{y}_{n,k}$ to a central node that implements a **Kalman Filter**.
  2. At every time step n, all the sensors perform local estimation and transmit their current **estimates** to a central node that combines them opportunely (**static sensor fusion + Kalman updates**).
- **Decentralized** approach
  1. At every time step n, all the sensors perform local Kalman filtering and transmit the **updates** to a central node that combines them opportunely. (**Distributed Kalman Filtering**)

Centralized setup: transmitting measurements

- At time step n, the central node collects all the measurements $\mathbf{y}_{n,k}$, $k = 1, ..., K$ and then computes the current estimate $\hat{\mathbf{x}}$ and the Kalman updates according to Result 1.
- There are two reasons why this **may not be the preferred** implementation

(1) number of sensors increases $\Rightarrow$ the computational effort required at the central node increases.

(2) the sensors may not be able to transmit measurements at every time step.

Centralized setup: transmitting local estimates

Potential method 1: At each time step n, all sensors transmit the current estimate $\hat{\mathbf{x}}_n$

The overall linear system is given by

$$\underbrace{\begin{bmatrix} \mathbf{y}_{n,k} \\ \mathbf{y}_{n-1,k} \\ \vdots \\ \mathbf{y}_{0,k} \end{bmatrix}}_{\mathbf{y}_k} = \underbrace{\begin{bmatrix} \mathbf{C}_k \\ \mathbf{C}_k\mathbf{A}^{-1} \\ \vdots \\ \mathbf{C}_k\mathbf{A}^{-n} \end{bmatrix}}_{\mathbf{H}_k} \mathbf{x}_n + \underbrace{\begin{bmatrix} \mathbf{v}_{n,k} \\ \mathbf{v}_{n-1,k} - \mathbf{C}_k\mathbf{A}^{-1}\mathbf{w}_{n-1} \\ \vdots \\ \mathbf{v}_{0,k} - \cdots - \mathbf{C}_k\mathbf{A}^{-1}\mathbf{w}_0 \end{bmatrix}}_{\mathbf{v}_k}$$

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- Process noise $\mathbf{w}_n$ appears in the noise $\Rightarrow$ the **measurement noises** $\mathbf{v}_k$ are **not independent** as desired
- $\mathbf{v}_k$ are not independent $\Rightarrow$ the **noise is correlated**.
- Therefore, we cannot use **Proposition 2 in Lecture 8** to combine local estimates.

Centralized setup: transmitting local estimates

Potential method 2: The current estimate $\hat{\mathbf{x}}_n$ can be obtained by combining the initial estimate $\mathbf{x}_0$ and estimates of the noises $\mathbf{w}_i$, $i = 0, ..., n-1$.

The overall linear system is given by

$$\underbrace{\begin{bmatrix} \mathbf{y}_{n,k} \\ \mathbf{y}_{n-1,k} \\ \vdots \\ \mathbf{y}_{0,k} \end{bmatrix}}_{\mathbf{y}_k} = \underbrace{\begin{bmatrix} \mathbf{C}_k\mathbf{A}^n & \mathbf{C}_k\mathbf{A}^{n-1} & \cdots & \mathbf{C}_k \\ \mathbf{C}_k\mathbf{A}^{n-1} & \cdots & \mathbf{C}_k & 0 \\ \mathbf{C}_k\mathbf{A}^{n-2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ \mathbf{C}_k & 0 & \cdots & 0 \end{bmatrix}}_{\mathbf{H}_k} \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{w}_0 \\ \vdots \\ \mathbf{w}_{n-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{v}_{n,k} \\ \mathbf{v}_{n-1,k} \\ \vdots \\ \mathbf{v}_{0,k} \end{bmatrix}}_{\mathbf{v}_k}$$

Centralized setup: transmitting local estimates

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- The **measurement noises** $\mathbf{v}_k$ are **independent** as desired
- **Proposition 2 in Lecture 8 does apply** for combining local estimates.
- **Vectors** transmitted from sensors are **increasing in dimension** as the time step n increases

Centralized setup - drawbacks

- Practically, it is not feasible to combine local estimates from method 2 to obtain the global estimate (i.e., lots of communication overhead)
- If there is no process noise, then the method 1 is applicable
- However, in general it is not possible

Distributed Kalman filtering

- **Recall:** dynamic estimation from one sensor (Result 1)

Distributed Kalman filtering

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- Random variable evolution: $\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{w}_n$
- Measurements: $\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{v}_n$, where $\mathsf{E}\{\mathbf{v}_n\mathbf{v}_n^\mathsf{T}\} = \mathbf{R}$

Distributed Kalman filtering

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- Measurements: $\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{v}_n$, where $\mathsf{E}\{\mathbf{v}_n\mathbf{v}_n^\mathsf{T}\} = \mathbf{R}$
- We have

$$\begin{bmatrix}\mathbf{y}_n \\ \mathbf{z}\end{bmatrix} = \begin{bmatrix}\mathbf{C} \\ \mathbf{H}\end{bmatrix}\mathbf{x}_n + \begin{bmatrix}\mathbf{v}_n \\ \mathbf{u}\end{bmatrix}$$

$$\hat{\mathbf{x}} = \mathbf{M}\mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{y}_n \text{ , where } \mathbf{M} = \left(\mathbf{R}_{\mathbf{x}_n}^{-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{C}\right)^{-1}$$

$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{y}_n$$

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Distributed Kalman filtering

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$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{y}_n$$

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{C}$$

- The requirements from individual sensors are derived by the equations above

Distributed Kalman filtering

Proposition 2

Consider a random variable $\mathbf{x}_n$ *evolving in time as* $\mathbf{x}_n = \mathbf{A}\mathbf{x}_{n-1} + \mathbf{w}_{n-1}$ *being observed by* K *sensors in every time step* n*. Suppose they generate measurements of the form* $\mathbf{y}_{n,k} = \mathbf{C}_k\mathbf{x}_n + \mathbf{v}_{n,k}$*. Then the global error covariance matrix and the estimate are given in terms of the local covariances and estimates by*

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1}\right)$$

$$\mathbf{P}^{-1}_{n|n}\hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1}\hat{\mathbf{x}}_{n,k|n-1}\right)$$

Distributed Kalman filtering

Proof: Note that overall linear system is given by

$$\begin{bmatrix} \mathbf{y}_{n,1} \\ \vdots \\ \mathbf{y}_{n,K} \\ \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_K \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_K \\ \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} \mathbf{v}_{n,1} \\ \vdots \\ \mathbf{v}_{n,K} \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_K \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{y}_n \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{C} \\ \mathbf{H} \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} \mathbf{v}_n \\ \mathbf{u} \end{bmatrix}$$

Lets now simplify $\mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{y}_n$

$$\begin{aligned}
\mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{y}_n &= \begin{bmatrix} \mathbf{C}_1^\mathsf{T} \cdots \mathbf{C}_K^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathbf{R}_1^{-1} & 0 & \cdots & 0 \\ 0 & \mathbf{R}_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{R}_K^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{n,1} \\ \vdots \\ \mathbf{y}_{n,K} \end{bmatrix} \\
&= \textstyle\sum_{k=1}^K \mathbf{C}_k^\mathsf{T}\mathbf{R}_k^{-1}\mathbf{y}_{n,k} \\
&= \textstyle\sum_{k=1}^K \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1}\right) \\
\mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{C} &= \textstyle\sum_{k=1}^K \mathbf{C}_k^\mathsf{T}\mathbf{R}_k^{-1}\mathbf{C}_k \\
&= \textstyle\sum_{k=1}^K \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1}\right)
\end{aligned}$$

Distributed Kalman filtering

Recap:

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

Based on the result above → **two architectures** for dynamic sensor fusion

- **Method 1: more** computation at the fusion center, **less** communication overhead
- **Method 2: less** computation at the fusion center, **more** communication overhead

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

fusion center

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

fusion center

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

fusion center

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$
$$\mathbf{P}^{-1}_{n|n} \hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2 → fusion center

$\mathbf{P}^{-1}_{0,1|0}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}^{-1}_{0,2|0}, \hat{\mathbf{x}}_{0,2|0}$ ⟼

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$ $\longmapsto$

fusion center

$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$
$$\mathbf{P}_{1,2|0} = \mathbf{A}\mathbf{P}_{0,2|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$
$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$
$$\mathbf{P}_{1,2|0} = \mathbf{A}\mathbf{P}_{0,2|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,2|0} = \mathbf{A}\hat{\mathbf{x}}_{0,2|0}$$

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K}\left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1}\right)$$

$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K}\left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1}\right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{0,1|0}^{-1}, \hat{\mathbf{x}}_{0,1|0}$

$\mathbf{P}_{0,2|0}^{-1}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$
$$\mathbf{P}_{1,2|0} = \mathbf{A}\mathbf{P}_{0,2|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,2|0} = \mathbf{A}\hat{\mathbf{x}}_{0,2|0}$$
$$\mathbf{P}_{1|0} = \mathbf{A}\mathbf{P}_{0|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

Distributed Kalman filtering (method 1)

Say $n = 0$....what will happen?

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$

$$\mathbf{P}^{-1}_{n|n} \hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}^{-1}_{0,1|0}, \hat{\mathbf{x}}_{0,1|0}$

$\mathbf{P}^{-1}_{0,2|0}, \hat{\mathbf{x}}_{0,2|0}$

$\longmapsto$

fusion center

$$\mathbf{P}_{1,1|0} = \mathbf{A}\mathbf{P}_{0,1|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,1|0} = \mathbf{A}\hat{\mathbf{x}}_{0,1|0}$$
$$\mathbf{P}_{1,2|0} = \mathbf{A}\mathbf{P}_{0,2|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1,2|0} = \mathbf{A}\hat{\mathbf{x}}_{0,2|0}$$
$$\mathbf{P}_{1|0} = \mathbf{A}\mathbf{P}_{0|0}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{1|0} = \mathbf{A}\hat{\mathbf{x}}_{0|0}$$

Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$

$$\mathbf{P}^{-1}_{n|n} \hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

- The fusion center now have knowledge about the terms in red for the previous step

Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$
$$\mathbf{P}^{-1}_{n|n}\hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

fusion center

$\mathbf{P}^{-1}_{1,1|1}, \hat{\mathbf{x}}_{1,1|1}$

$\mathbf{P}^{-1}_{1,2|1}, \hat{\mathbf{x}}_{1,2|1}$ $\longmapsto$

Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K}\left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1}\right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K}\left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1}\right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$
$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$ $\longmapsto$

fusion center

$\mathbf{P}_{2,1|1} = \mathbf{A}\mathbf{P}_{1,1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$

Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$

$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$

$\longmapsto$

fusion center

$$\mathbf{P}_{2,1|1} = \mathbf{A}\mathbf{P}_{1,1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{2,1|1} = \mathbf{A}\hat{\mathbf{x}}_{1,1|1}$$

Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$

$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$

$\longmapsto$

fusion center

$$\mathbf{P}_{2,1|1} = \mathbf{A}\mathbf{P}_{1,1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

$$\hat{\mathbf{x}}_{2,1|1} = \mathbf{A}\hat{\mathbf{x}}_{1,1|1}$$

$$\mathbf{P}_{2,2|1} = \mathbf{A}\mathbf{P}_{1,2|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

Distributed Kalman filtering (method 1)

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$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$
$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$

$\longmapsto$

fusion center

$$\mathbf{P}_{2,1|1} = \mathbf{A}\mathbf{P}_{1,1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{2,1|1} = \mathbf{A}\hat{\mathbf{x}}_{1,1|1}$$
$$\mathbf{P}_{2,2|1} = \mathbf{A}\mathbf{P}_{1,2|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{2,2|1} = \mathbf{A}\hat{\mathbf{x}}_{1,2|1}$$

Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

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$$\mathbf{P}^{-1}_{n|n}\hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}^{-1}_{1,1|1}, \hat{\mathbf{x}}_{1,1|1}$

$\mathbf{P}^{-1}_{1,2|1}, \hat{\mathbf{x}}_{1,2|1}$

$\longmapsto$

fusion center

$$\mathbf{P}_{2,1|1} = \mathbf{A}\mathbf{P}_{1,1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{2,1|1} = \mathbf{A}\hat{\mathbf{x}}_{1,1|1}$$
$$\mathbf{P}_{2,2|1} = \mathbf{A}\mathbf{P}_{1,2|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{2,2|1} = \mathbf{A}\hat{\mathbf{x}}_{1,2|1}$$
$$\mathbf{P}_{2|1} = \mathbf{A}\mathbf{P}_{1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$

Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$

$$\mathbf{P}_{n|n}^{-1} \hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}_{1,1|1}^{-1}, \hat{\mathbf{x}}_{1,1|1}$
$\mathbf{P}_{1,2|1}^{-1}, \hat{\mathbf{x}}_{1,2|1}$ $\longmapsto$

fusion center

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$$\hat{\mathbf{x}}_{2|1} = \mathbf{A}\hat{\mathbf{x}}_{1|1}$$

Distributed Kalman filtering (method 1)

Say $n = 1$....what will happen?

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$
$$\mathbf{P}^{-1}_{n|n} \hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

$\mathbf{y}_{0,1}$	$\mathbf{y}_{1,1}$	$\mathbf{y}_{2,1}$	$\mathbf{y}_{3,1}$

sensor 1 measurements

$\mathbf{x}_0$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$

what we want to estimate

$\mathbf{y}_{0,2}$	$\mathbf{y}_{1,2}$	$\mathbf{y}_{2,2}$	$\mathbf{y}_{3,2}$

sensor 2 measurements

sensor 1 / sensor 2

$\mathbf{P}^{-1}_{1,1|1}, \hat{\mathbf{x}}_{1,1|1}$
$\mathbf{P}^{-1}_{1,2|1}, \hat{\mathbf{x}}_{1,2|1}$

$\longmapsto$

fusion center

$$\mathbf{P}_{2,1|1} = \mathbf{A}\mathbf{P}_{1,1|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
$$\hat{\mathbf{x}}_{2,1|1} = \mathbf{A}\hat{\mathbf{x}}_{1,1|1}$$
$$\mathbf{P}_{2,2|1} = \mathbf{A}\mathbf{P}_{1,2|1}\mathbf{A}^{\mathsf{T}} + \mathbf{Q}$$
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$$\hat{\mathbf{x}}_{2|1} = \mathbf{A}\hat{\mathbf{x}}_{1|1}$$

Distributed Kalman filtering (method 2)

$$\mathbf{P}_{n|n}^{-1} = \mathbf{P}_{n|n-1}^{-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1} - \mathbf{P}_{n,k|n-1}^{-1} \right)$$
$$\mathbf{P}_{n|n}^{-1}\hat{\mathbf{x}}_{n|n} = \mathbf{P}_{n|n-1}^{-1}\hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}_{n,k|n}^{-1}\hat{\mathbf{x}}_{n,k|n} - \mathbf{P}_{n,k|n-1}^{-1}\hat{\mathbf{x}}_{n,k|n-1} \right)$$

Distributed Kalman filtering (method 2)

$$\mathbf{P}^{-1}_{n|n} = \mathbf{P}^{-1}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \right)$$
$$\mathbf{P}^{-1}_{n|n} \hat{\mathbf{x}}_{n|n} = \mathbf{P}^{-1}_{n|n-1} \hat{\mathbf{x}}_{n|n-1} + \sum_{k=1}^{K} \left(\mathbf{P}^{-1}_{n,k|n} \hat{\mathbf{x}}_{n,k|n} - \mathbf{P}^{-1}_{n,k|n-1} \hat{\mathbf{x}}_{n,k|n-1} \right)$$

key idea:

- The term $\mathbf{P}^{-1}_{n|n-1} \hat{\mathbf{x}}_{n|n-1}$ can be written in terms of contributions from individual sensors
- The term $\mathbf{P}^{-1}_{n|n-1}$ can be written in terms of contributions from individual sensors
- Allows the fusion center to form the estimate by summing the results sent from the sensors
- Try it or look it up!

Summary

Today we have studied:

- Dynamic estimation from one sensor
- Dynamic estimation from many sensors
- Dynamic sensor fusion, distributed Kalman filtering

Next Lecture

- Application of Lecture 8 and 9 to Positioning and Localization in WSNs