Principles of Wireless Sensor Networks

https://www.kth.se/social/course/EL2745/

Lecture 10

Positioning and Localization

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Course content

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  - Lec 1: Introduction to WSNs
  - Lec 2: Introduction to Programming WSNs
- Part 2
  - Lec 3: Wireless Channel
  - Lec 4: Physical Layer
  - Lec 5: Medium Access Control Layer
  - Lec 6: Routing
- Part 3
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  - Lec 8: Static Distributed Estimation
  - Lec 9: Dynamic Distributed Estimation
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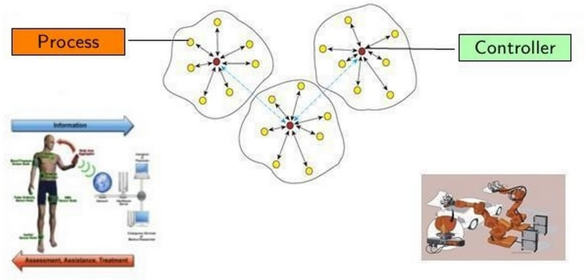
Previous lecture

Application
Presentation
Session
Transport
Routing
MAC
Phy



How to estimate phenomena from noisy measurements?

Today's learning goals

- Which measurements are used for estimating the position of a node?
- How to estimate the position of a node?
- What is the effect of measurement errors?

Outline

- Introduction
- Specific sources of measurements
- Estimation of the position

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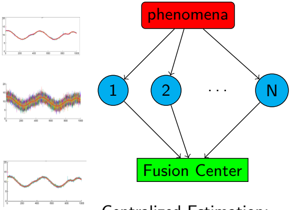
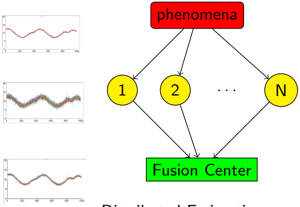
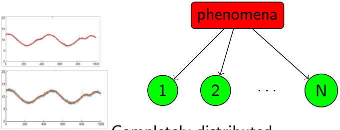
Estimation



Centralized Estimation:
no intelligence on sensors



Distributed Estimation:
some intelligence on sensors



Completely distributed
(or decentralized) Estimation

Positioning and localization

Localization is defined as a technique to estimate the positions of nodes

- It can be categorized into
  - Centralized, where a central node estimates the position of the nodes
  - Distributed, where many nodes help each other to find their own positions

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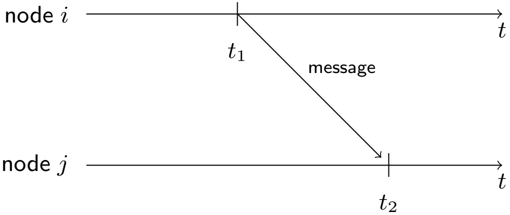
Time of arrival



- Assuming that the nodes are synchronized the distance measurement is

$$d_{ij} \simeq (t_2 - t_1) \cdot v$$

  where v is the propagation speed of the message

- Uncertainty occurs when $t_2 \approx t_1$
- Problems
  - Packet losses
  - MAC delays
  - CPU delay

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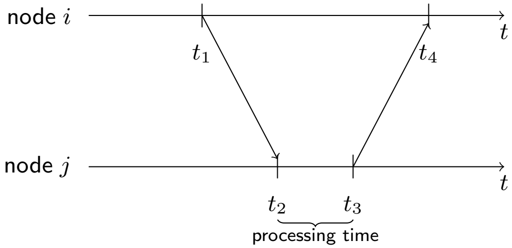
Time difference of arrival



- The distance measurement is

$$d_{ij} \simeq \frac{(t_2 - t_1) + (t_4 - t_3)}{2} \cdot v$$

  where v is the propagation speed of the message

- Problems
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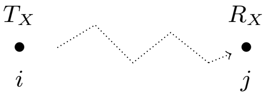
Received signal strength (RSS)



Ideal propagation

$$P_r = P_t G_t G_r \overline{\mathrm{PL}} \frac{\lambda^2}{(4\pi d_{ij})^2} \Rightarrow d_{ij} \simeq \frac{\lambda}{4\pi} \sqrt{\frac{P_t G_t G_r \overline{\mathrm{PL}}}{P_r}}$$

Problems: multi-path, noise, etc, affect the propagation characteristics
$\Rightarrow$ Channel estimation needed

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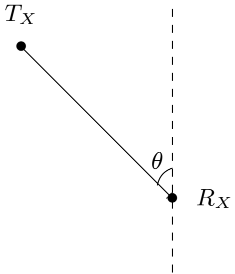
Angle of arrival

Example: Sound propagation by multiple microphones, amplitude and phase

- Odometry sensor



Problems: Errors in measurements of θ (e.g., due to magnetic fields of earth), estimation accuracy decreases with the distance.

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Estimation of the position

We now examine the specific techniques that are used to estimate the sensors' position based on the measurements that were previously presented

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Angle of arrival + velocity

$$\widetilde{\theta}(k) = \theta(k) + n_\theta(k)$$

- $\widetilde{\theta}(k)$ the angle measurement at discrete time k given by, e.g., odometry sensor
- $\theta(k)$ the real angle value
- $n_\theta(k)$ the additive noise

$$\widetilde{v}(k) = v(k) + n_v(k)$$

- $\widetilde{v}(k)$ the velocity measurement at discrete time k given by, e.g., accelerometer
- $v(k)$ the real velocity value
- $n_v(k)$ the additive noise

Angle of arrival + velocity

Node moving
with velocity v $\bullet\!\longrightarrow$ v

Let $X_r(k) = \begin{bmatrix} x(k) \\ y(k) \end{bmatrix}$ be the true position of the node that we want to estimate

Then

$$\widehat{x}(k+1) = \widehat{x}(k) + \widetilde{v}(k) \cdot T \cdot \cos\widetilde{\theta}(k)$$
$$\widehat{y}(k+1) = \widehat{y}(k) + \widetilde{v}(k) \cdot T \cdot \sin\widetilde{\theta}(k)$$

where T is the sampling time

A problem is that the resulting estimator is **biased**

$$\mathbb{E}\left\{\widehat{x}(k+1)\right\} = \mathbb{E}\left\{\widehat{x}(k)\right\} + \mathbb{E}\left\{\widetilde{v}(k)\right\} \cdot T \cdot \mathbb{E}\left\{\cos\widetilde{\theta}(k)\right\} =$$
$$= \mathbb{E}\left\{\widehat{x}(k)\right\} + v(k) \cdot T \cdot \cos\theta(k) \cdot e^{-\frac{\sigma_\theta^2}{2}} \neq x(k+1)$$

We need to estimate the bias as well.

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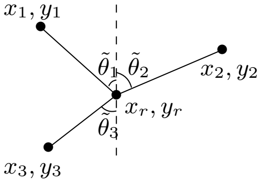
Triangulation

- Anchors: nodes with fixed position used for determining the unknown position of a node



Let $X_r = \begin{bmatrix} x_r \\ y_r \end{bmatrix}$ be the true position of the node that we want to estimate

The angle measurements are

$$\widetilde{\theta}_i = \theta_i \left(X_r \right) + n_i \qquad i = 1, 2, 3$$

where $\theta_i \left(X_r \right) = \arctan \dfrac{x_r - x_i}{y_r - y_i}$

Triangulation

Setting

- $\underline{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$ as the vector of noises with $\mathbb{E}\left\{n \cdot n^T\right\} = R = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$
- $\theta\left(X_r\right) = \begin{bmatrix} \theta_1\left(X_r\right) \\ \theta_2\left(X_r\right) \\ \theta_3\left(X_r\right) \end{bmatrix}$ as the vector of true node position and
- $Y = \begin{bmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \\ \widetilde{\theta}_3 \end{bmatrix}$ as the vector of angle measurements

we can estimate X_r from

$$Y = \theta\left(X_r\right) + \underline{n}$$

where $\theta\left(X_r\right)$ is a non-linear function

Triangulation

Error covariance (cf. slide 19 in Lecture 8)

$$C\left(\widehat{X}_r\right) = \left[\theta\left(\widehat{X}_r\right) - Y\right]^T \cdot R^{-1} \cdot \left[\theta\left(\widehat{X}_r\right) - Y\right] = \sum_{i=1}^{3} \frac{\left(\theta_i\left(\widehat{X}_r\right) - \widetilde{\theta}_i\right)^2}{\sigma_i^2}$$

Goal: Choose $\widehat{X}_r$ that keeps oscillations of $\theta\left(\widehat{X}_r\right)$ around Y

Therefore

$$\min_{\widehat{X}_r} \; C\left(\widehat{X}_r\right) \equiv \left\{ \begin{array}{c} \dfrac{dC\left(\widehat{X}_r\right)}{dx_r} = 0 \\ \dfrac{dC\left(\widehat{X}_r\right)}{dy_r} = 0 \end{array} \right\}$$

Triangulation

We can use iterative methods for solving the above system of non-linear equations, e.g.,

Newton-Gauss method

Consider the non-linear equation

$$g\left(x\right) = 0 \qquad g : \mathbb{R}^N \to \mathbb{R}^N \qquad x \subset \mathbb{R}^N$$

Iterative method $\qquad x_{t+1} = x_t - \delta_t \cdot g\left(x_t\right)$

where $\qquad \delta_t = \nabla^{-1} g\left(x_t\right)$

$$\lim_{t\to\infty} x_t \to x^* : g\left(x^*\right) = 0$$

∇ is the matrix of all first-order partial derivatives (Jacobian)

In this specific case, we use the iterative method to solve

$$g\left(x\right) = \nabla_x C\left(\widehat{X}_r\right) = 0$$

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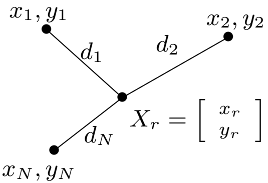
Trilateration



Let $X_r = \begin{bmatrix} x_r \\ y_r \end{bmatrix}$ be the true position of the node that we want to estimate

The distance measurements are

$$\widetilde{d}_i = d_i + n_i \qquad i = 1, .., N$$

From the trigonometry

$$\begin{aligned} (x_1 - x_r)^2 + (y_1 - y_r)^2 &= \widetilde{d}_1^2 \\ &\vdots \\ (x_N - x_r)^2 + (y_N - y_r)^2 &= \widetilde{d}_N^2 \end{aligned}$$

Trilateration

After substracting these N equations we arrive at the following system of equations

$$A \cdot X_r = Y$$

where $$A \in \mathbb{R}^{(N-1)\times 2}, \quad X_r \in \mathbb{R}^2, \quad Y \in \mathbb{R}^{(N-1)\times 1}$$

$$A = 2 \cdot \begin{bmatrix} (x_N - x_1) & (y_N - y_1) \\ \vdots & \vdots \\ (x_N - x_{N-1}) & (y_N - y_{N-1}) \end{bmatrix}$$

$$Y = \begin{bmatrix} \widetilde{d}_1^2 - \widetilde{d}_N^2 - x_1^2 - y_1^2 + x_N^2 + y_N^2 \\ \vdots \\ \widetilde{d}_{N-1}^2 - \widetilde{d}_N^2 - x_{N-1}^2 - y_{N-1}^2 + x_N^2 + y_N^2 \end{bmatrix}$$

Trilateration

From the previous lecture we considered a measurements model $Y = HX + n$, but we now have

$$Y = AX_r$$

A is not a square matrix and there is no explicit noise (all is included in Y).
X_r is constant

We can apply the Linear Minimum Mean Squared Estimator (LMMSE) with

$$\widehat{X}_r = L \cdot Y$$

where

$$L = \left(A^T A\right)^{-1} A^T$$

Trilateration

We can redefine $\widehat{X}_r$ as a **least square** solution.

This is equivalent to define the following cost function

$$C(X_r) = (AX_r - Y)^T (AX_r - Y)$$

and search for X_r that minimizes $C(X_r)$

$$\frac{dC(X_r)}{dX_r} = 2A^T\left(A\hat{X}_r - Y\right) = 0 \Rightarrow A^T A\hat{X}_r = A^T Y \Rightarrow \boxed{\hat{X}_r = \left(A^T A\right)^{-1} A^T Y}$$

Trilateration

Note that $\widehat{X}_r$ is a random variable due to Y being noisy
Taking the expected values,

$$\mathbb{E}\left\{\hat{X}_r\right\} = \left(A^T A\right)^{-1} A^T \mathbb{E}\left\{Y\right\}$$

$$\mathbb{E}\left\{Y\right\} = \begin{bmatrix} \mathbb{E}\left\{\widetilde{d}_1^2\right\} - \mathbb{E}\left\{\widetilde{d}_N^2\right\} - x_1^2 - y_1^2 + x_N^2 + y_N^2 \\ \vdots \\ \mathbb{E}\left\{\widetilde{d}_{N-1}^2\right\} - \mathbb{E}\left\{\widetilde{d}_N^2\right\} - x_{N-1}^2 - y_{N-1}^2 + x_N^2 + y_N^2 \end{bmatrix}$$

$$\tilde{d}_i = d_i + n_i \Rightarrow \tilde{d}_i^2 = d_i^2 + n_i^2 + 2d_i n_i \Rightarrow E\left\{\tilde{d}_i^2\right\} = d_i^2 + E\left\{n_i^2\right\} = d_i^2 + \sigma_i^2$$

where $\sigma_i^2 = \sigma_0^2 e^{k_\sigma d_i}$

The result is that $\widehat{X}_r$ is a biased estimator since

$$\mathbb{E}\left\{\hat{X}_r\right\} = \left(A^T A\right)^{-1} A^T \mathbb{E}\left\{Y\right\} \neq X_r$$

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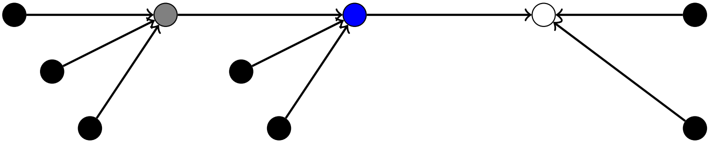
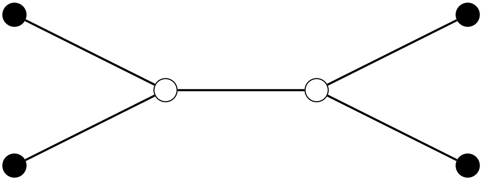
Iterative and collaborative multilateration

- In lateration techniques, at least three anchor nodes are required for estimating position
- Iterative and collaborative multilateration are extension of lateration that do not require three neighbouring anchors
- In iterative multilateration
  - Any node can become anchor after estimating its position and send anchor messages on the network
  - In this way, all nodes can estimate their positions after several iterations
- For example, in the figure below
  - Gray node estimates its position by three black anchors
  - Blue node estimates by gray and two black anchor nodes
  - White node is localized by blue and two black anchor nodes



Iterative and collaborative multilateration

- It is possible that nodes cannot have three anchors even after several iterations
- In that case collaborative multilateration is used in which
  - A graph of participating nodes is constructed
  - Participating nodes are the ones that are either anchors or have at least three participating neighbours
  - This gives set of over-constrained quadratic equations relating distance among nodes and their neighbours
  - These equation are solved to estimate positions



Summary

- We have studied the basic of localization for sensor networks
- Localizing the nodes consists in applying estimation techniques

Next lecture

- Application of estimation and detection to synchronization