

## Homework to Seminar 4

### (1) MATHEMATICAL INDUCTION

Mathematics is similar to science when it comes to research activities (observations, experiments, models, hypotheses, theories) but the concept of proof is much more stringent in maths. A simple example:

- OBSERVATIONS:  $1 + 3 = 4, 1 + 3 + 5 = 9, 1 + 3 + 5 + 7 = 16$
- EXPERIMENT: for `n in range(10): sum(range(1, 2*n+1, 2))`  
0, 1, 4, 9, 16, 25, 36, 49, 64, 81
- MODEL:  $1 + 3 + 5 + \dots + (2n - 1) = n^2$  for  $0 < n < 10$
- HYPOTHESIS:  $1 + 3 + 5 + \dots + (2n - 1) = n^2$  for  $0 < n < \infty$

In science, the hypothesis, strengthened by the observations and the experiment (and also by the simple and plausible model) would then be temporarily accepted until falsified by a new observation. In mathematics, it would be called a conjecture until a proof has been presented.

One proof method is mathematical induction, which takes scientific induction a step further. The format is the following.

- HYPOTHESIS:  $P(n)$  is true for all  $n > 0$
- BASE CASE:  $P(1)$  is true
- INDUCTION STEP:  $P(k) \Rightarrow P(k + 1)$  for any  $k$

**Task:** Try to write down a correct induction proof for our hypothesis.

### (2) DATABASE OF SEQUENCES

If the observations do not immediately suggest a model, you may consult the On-Line Encyclopedia of Integer Sequences. Let us say that you want a similar formula for the sum of squares  $1 + 4 + 9 + \dots + n^2$ . Make a small number of observations like  $1 = 1, 1 + 4 = 5$  etc and search for the sequence  $1, 5, \dots$  in the database.

**Task:** What is the model you find?

**Task:** Did you get more interesting information?

### (3) BENFORD'S LAW

"Thirty percent of all numbers start with the digit 1." This statement is supposed to be true for collections of numbers with widely varying orders of magnitude, like populations of cities or byte sizes of files.

**Task:** Test the law by generating many powers of 2 and printing the first digit statistics.

**Task:** To demonstrate your shell script skills you may also want to check the law on all byte sizes in your computer. (Not compulsory!)

**Task:** Do you consider the law a hypothesis, a conjecture or a mathematical theorem?

(4) **OVERPOPULATION IS CAUSED BY MEN**

”Men have more children than women! Men are fertile longer and men often remarry with a younger wife and get more children. In Cameroon, the average man has eight children compared with the average woman’s four children.”

This was stated in Dagens Nyheter, which cited a U.N. report. But is it even possible that men have more children and more marriages than women?

**Task:** How would you use the computer to test the possibility?

**Task:** Can you give an argument to falsify the statement?