Principles of Wireless Sensor Networks

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Lecture 11

Time Synchronization

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Previous lecture

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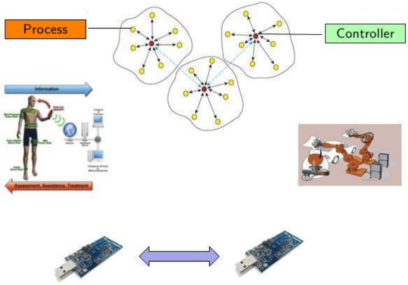

How to estimate the position of fixed and mobile nodes?

Today's learning goals

- Which measurements are used for synchronizing the nodes?
- How to synchronize pair of nodes?
- How to synchronize a network of nodes?

Outline

- Basics of time synchronization
- Synchronization protocols

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- Basics of time synchronization
  - Hardware clock - Software clock
  - Message exchanges
- Synchronization protocols
  - Time synchronization protocol
    - Estimation based on LS
    - Estimation based on MMSE
  - Distributed clock synchronization

Basics of time synchronization

Time synchronization is defined as the procedure for at least two nodes to have a common reference clock

A typical node possesses an oscillator of a specified frequency (**hardware clock**) and a counter register, which is incremented after a certain number of oscillator pulses.

The nodes software has access only to the counter register (**software clock**).

The time distance between two increments (ticks) determines the achievable **time resolution** Consider two nodes: node i, node j. Then,

- **Clock offset** is defined as the difference between time at node i and time at node j
- **Clock rate** is the frequency at which clock progresses
- **Clock skew** represents the difference in the frequency of two clocks

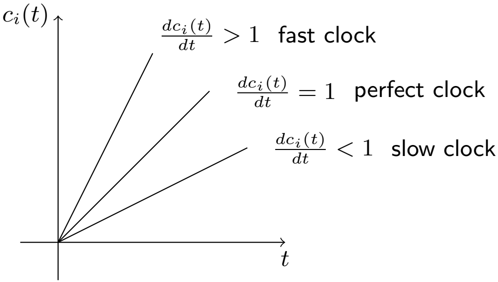
Basics of time synchronization

- Denote the nominal hardware clock of node i as $H_i(t)$.
- The software time measured at node i at time t is

$$c_i(t) = \rho_i(t) H_i(t) + \phi_i(t)$$

where $\phi_i(t)$ is called **phase shift** and $\rho_i(t)$ is called **drift rate**.

- Ideally, $H_i(t) = t$, $\rho_i(t) = 1$, and $\phi_i(t) = 0$.



Basics of time synchronization

Clock rate: $\frac{dc_i(t)}{dt}$

Defining ρ_i as the drift rate at node i,

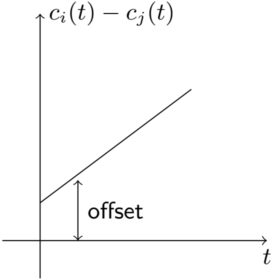
$$1 - \rho_i(t) \leq \frac{dc_i(t)}{dt} \leq 1 + \rho_i(t)$$

Two synchronized nodes, node i and node j, before being resynchronized can drift of at maximum $2\rho_{\max}$, that is

$$\frac{dc_i(t)}{dt} - \frac{dc_j(t)}{dt} \leq 2\rho_{\max}$$

where $2\rho_{\max} \cdot \tau_{\text{synch}} < \delta_{\max}$ and $\delta_{\max}$ a precision parameter that is equal to the maximum offset between two clocks

Basics of time synchronization



Offset is worsening as time goes by!

How can we model the offset between node i and node j ?

$$c_i\,(t) - c_j\,(t) = t_0 + \Delta f\,(t) \cdot t + \Delta\tau(t)$$

constant offset

time dependent frequency offset

jitter due to noise

Frequency offset $\Delta f\,(t)$ is due to different rates among the clocks and environmental effects (e.g. temperature, humidity)

Basics of time synchronization

How do we synchronize the clocks?

Problem: Non-determinism of communication delay

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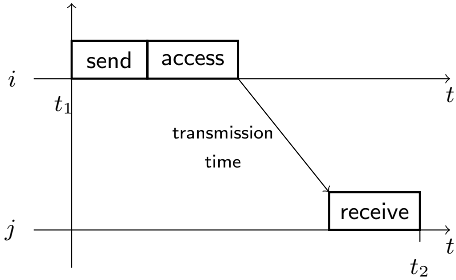
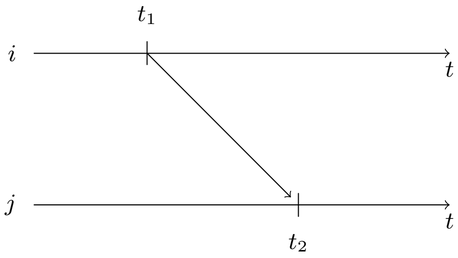
Message exchanges

Consider two nodes: node i and node j.
A message is sent to synchronize i with j



Message exchanges

One way message exchange



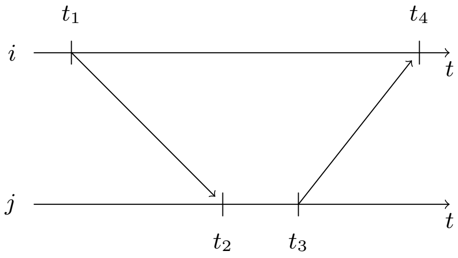
$$c_i\,(t_1) = t_1 + n_1$$

$$c_j\,(t_2) = t_2 = t_1 + \underbrace{D}_{\text{propagation delay}} + \underbrace{\delta}_{\text{offset}} + n_2$$

Node j makes an estimate of the offset and adjusts its clock if $D \simeq 0$ (negligible) and $n_1 \approx n_2$

Message exchanges

Two way message exchange (in case D is non negligible)



$t_2 = t_1 + D + \delta + n_2$

$t_4 = t_3 + D - \delta + n_4$

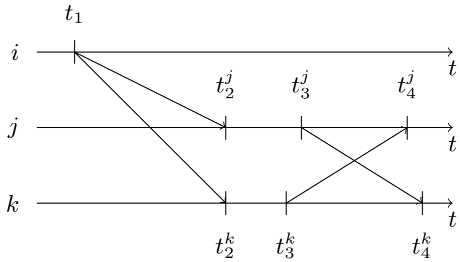
t_1, t_2, t_3, t_4 measured $\Rightarrow$ solve for D and δ , assuming that $n_2 \approx n_4$

Therefore,

$$D = \frac{((t_2 - t_1) + (t_4 - t_3))}{2} \qquad \delta = \frac{((t_2 - t_1) - (t_4 - t_3))}{2}$$

Message exchanges

Receiver-receiver synchronization



$$t_4^k = t_3^j + \delta_{jk} + n_4^k$$
$$t_3^j = t_2^j + \Delta_j = t_1 + \delta_{ij} + \Delta_j + n_3^j$$
$$t_4^j = t_3^k + \delta_{kj} + n_4^j$$
$$t_3^k = t_2^k + \Delta_k = t_1 + \delta_{ik} + \Delta_k + n_3^k$$

where Δ_j and Δ_k are known and δ_{ik}, δ_{kj}, δ_{jk}, δ_{kj} unknown

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Time synchronization protocol

Consider two nodes, node i and node j, with different drifts and offsets

Let us define $c_i(t)$ as the clock of node i and $c_j(t)$ as the clock of node j

Consider the following clock synchronization model

$$c_j(t) = a_0 + a_1 \cdot c_i(t) + n_{ij}$$

For synchronization, measurements are required in order to determine a_0 and a_1

Time synchronization protocol

$$c_j(t) = a_0 + a_1 \cdot c_i(t) + n_{ij}$$

STEP 1

$c_i(t_0) \triangleq x_0$ (measured)

$c_j(t_0) = a_0 + a_1 \cdot c_i(t_0) + n_{ij,0} = a_0 + a_1 \cdot x_0 + n_{ij,0} \triangleq y_0 + n_{ij,0}$ (measured)

STEP 2

$c_i(t_1) \triangleq x_1$

$c_j(t_1) = a_0 + a_1 \cdot x_1 + n_{ij,1} \triangleq y_1 + n_{ij,1}$

$\vdots$

STEP n

$c_i(t_{n-1}) \triangleq x_{n-1}$

$c_j(t_{n-1}) = a_0 + a_1 \cdot x_{n-1} + n_{ij,n-1} \triangleq y_{n-1} + n_{ij,n-1}$

Time synchronization protocol

Putting all the measurements together, we end up in the following system of equations

$$A \cdot X + N = Y$$

where

$$A = \begin{bmatrix} x_0 & 1 \\ x_1 & 1 \\ \vdots & \vdots \\ x_{n-1} & 1 \end{bmatrix} \qquad X = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix} \qquad N = \begin{bmatrix} n_{ij,0} \\ n_{ij,1} \\ \vdots \\ n_{ij,n-1} \end{bmatrix} \qquad Y = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{bmatrix}$$

Estimation based on LS

Least Square Estimator

$$\widehat{X} = L \cdot Y$$

where

$$L = \left(A^T \cdot A\right)^{-1} A^T$$

$$A^T \cdot A = \begin{bmatrix} \sum_{i=0}^{n-1} {x_i}^2 & \sum_{i=0}^{n-1} x_i \\ \sum_{i=0}^{n-1} x_i & n \end{bmatrix} \qquad A^T \cdot Y = \begin{bmatrix} \sum_{i=0}^{n-1} x_i y_i \\ \sum_{i=0}^{n-1} y_i \end{bmatrix}$$

Estimation based on MMSE

MMSE Estimator

The MMSE estimator of X given that $Y = y$ is

$$P^{-1}\hat{X} = AR_N^{-1}y$$

with error covariance

$$P^{-1} = R_X^{-1} + A^T R_N^{-1} A$$

where R_N the covariance of zero mean Gaussian noise

In this specific case, $\hat{X} = PAR_N^{-1}y$ where

$$P^{-1} = \begin{bmatrix} \sum_{k=0}^{n-1} \frac{x_k^2}{\sigma^2_{n_{ij,k}}} & \sum_{k=0}^{n-1} \frac{x_k}{\sigma^2_{n_{ij,k}}} \\ \sum_{k=0}^{n-1} \frac{x_k}{\sigma^2_{n_{ij,k}}} & \sum_{k=0}^{n-1} \frac{1}{\sigma^2_{n_{ij,k}}} \end{bmatrix}$$

$$R_N^{-1} = \begin{bmatrix} \frac{1}{\sigma^2_{n_{ij,0}}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma^2_{n_{ij,1}}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\sigma^2_{n_{ij,n-1}}} \end{bmatrix}$$

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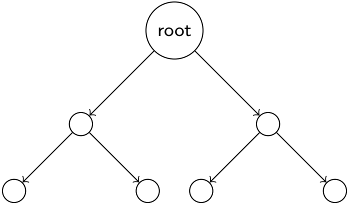
Distributed clock synchronization

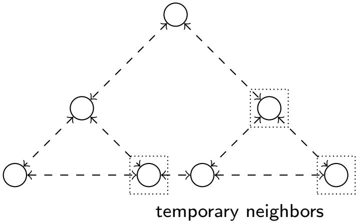
Previous part of lecture



- Nodes synchronize with the root
- Synchronization tends to deteriorate as we move away from the root

Distributed clock synchronization

Now, we would like to have a real common synchronization across the network



No root for synchronization - only P2P

Distributed clock synchronization

Consider the following hardware clock model at node i:

$$H_i(t) = \int_{t_0}^{t} h_i(\tau)\, d\tau + \phi_0(t_0)$$

where $h_i(\tau)$ is the hardware clock rate and $\phi_0(t_0)$ is the hardware clock offset at time t_0.

Assume that the clock rate has a bounded drift ρ, i.e.,

$$1 - \rho_{\max} \leq h_i(t) \leq 1 + \rho_{\max}$$

Distributed clock synchronization

The software clock of node i at time t can be expressed as

$$c_i(t) = \int_{t_0}^{t} h_i(\tau)\, l_i(\tau)\, d\tau + \theta_i(t_0)$$

where $l_i(\tau)$ the relative logical clock rate which can be properly tuned to achieve synchronization and $\theta_i(t_0)$ a clock offset.

We can then define the absolute logical clock rate of node i at time t as

$$x_i(t) \triangleq h_i(t) \cdot l_i(t)$$

Then, nodes are synchronized if the $x_i(t)$ converge to the same value. How to do so?

Distributed clock synchronization

Define the following update rules for all nodes

$$x_i\left(k+1\right) \triangleq \frac{\sum\limits_{j \in N_i(k)} x_j\left(k\right) + x_i\left(k\right)}{\left|N_i\right| + 1}$$

where

- $N_i\left(k\right)$ the set of neighbors of node i at time k
- $\left|N_i\right|$ the cardinality of neighbor nodes

Distributed clock synchronization

By setting $X(k) = \begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix}$

we obtain the compact form

$$X(k+1) = A(k) \cdot X(k)$$

where

$$A(k) = [a_{i,j}(k)] = \begin{cases} \frac{1}{|N_i|+1} & \text{if } i,j \text{ are connected,} \\ 0 & \text{otherwise.} \end{cases}$$

and $A(k) \cdot \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ (raw stochastic matrix)

Note that the matrix dimensions of $A(k)$ depend on the topology and change every time the neighbors of node i change.

Distributed clock synchronization

Proposition

Suppose the communication graph is strongly connected, then all the logical clock rates converge to a steady state value, i.e.,

$$\lim_{k \to \infty} X(k) = x_{ss} \cdot \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Practical implementation

1. Each node periodically broadcasts a synchronization beacon (to all neighbors) containing its current measured software time $c_i(t)$ and the relative logical clock rate $l_i(t)$.
2. The information received by neighbors is used to estimate the absolute logical clock rate $x_j(t)$ of those neighbors.
3. Each node updates its own relative clock rate $l_i(t)$ as

$$l_i\left(k+1\right) \triangleq \frac{\sum_{j \in N_i(k)} \frac{x_j(k)}{h_i(t)} + l_i\left(k\right)}{|N_i| + 1}$$

4. The procedure is repeated.

Summary

- We have studied the basic of synchronization for sensor networks
- Synchronizing the nodes consists in applying estimation techniques

Next lecture

- The fourth and last part of the course starts: control over WSNs