Principles of Wireless Sensor Networks

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Lecture 12

Wireless Sensor Network Control Systems 1

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- Part 2
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  - Lec 4: Physical Layer
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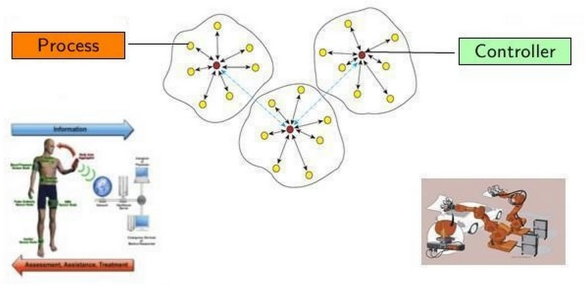
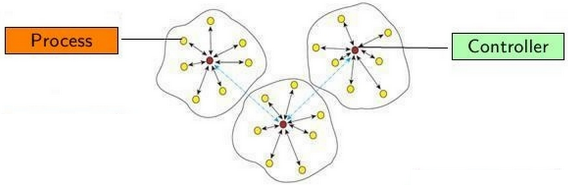
Previous lecture



How to synchronize nodes?

Today's learning goals



- How the process state dynamics over time are mathematically modeled?
- How such state dynamics can be controlled by closing the loop process→controller→process?
- How to discretize the continuous time model of the dynamics?
- What is the concept of state stability of closed loop control systems?

Outline

- Wireless Sensor Network Control Systems (WSNCS)
- State space description of a control system
- Stability and asymptotic stability of a control system

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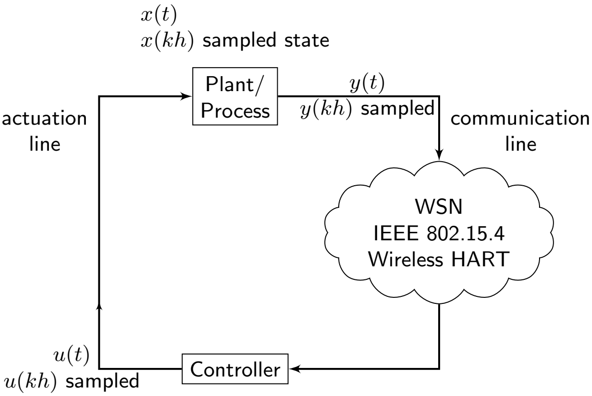
Wireless Sensor Network Control Systems (WSNCS)

Closed-loop system



Wireless Sensor Network Control Systems

k: discrete time

h: sampling interval

- $u(kh)$: control decision
- $x(kh)$: state of the process/plant
- $y(kh)$: output of the state (measured by sensors)

- The **GOAL** of the controller is to bring the state $x(kh)$ in a desired region by taking measurements $y(kh)$ and a control decision $u(kh)$
- Delay and packet loss probability affect the way the measurements $y(t)$ are received in the controller

This lecture gives the basic control theory background for WSNCS. The effect of the network on the controller is studied next lecture

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Continuous time description

Let $x(t)$ be the state of the system (temperature, position,...)
We assume that the physical process is described by the time-invariant state space model

Linear model

$$\frac{dx(t)}{dt} \triangleq \dot{x}(t) = Ax(t) + Bu(t) \qquad \text{state model} \tag{1}$$

$$y(t) = Cx(t) + Du(t) \qquad \text{measurement model}$$

where A, B, C, D are assumed to be known matrices

Continuous time description

Assuming that $x\,(kh)$ is known, then the solution of the differential equation (1) is

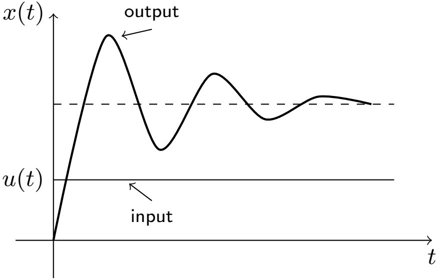
$$x\,(t) = e^{A(t-kh)} \cdot x\,(kh) + \int_{kh}^{t} e^{A(t-\tau)} Bu\,(t)\, d\tau \qquad t > kh \qquad (2)$$

The control decision $u\,(t)$ can be properly chosen to bring the system state $x\,(t)$ in a desired region.

Example: "The step response"

Suppose $x(0) = 0$ and $x(t) \in \mathbb{R}$

The step response is defined as the evolution of the state $x(t)$, i.e., the solution of (1), for an input $u(t) = \begin{cases} 0 & t \leq 0 \\ 1 & t > 0 \end{cases}$



The state may evolve to a stabilized condition after possible oscillations

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Discretization of state space model

Assume $u(t)$ constant in the interval $kh \leq t \leq kh + h$

Then, (2) becomes

$$x(t) = e^{A(t-kh)} \cdot x(kh) + \int_{kh}^{t} e^{A(t-\tau)} d\tau B u(t) =$$

$$= e^{A(t-kh)} \cdot x(kh) + \int_{0}^{t-kh} e^{A\tau} d\tau B u(kh) = \phi_t x(kh) + \Gamma_t u(kh)$$

Let $t = kh + h$

$$x(kh + h) = \phi x(kh) + \Gamma u(kh) \qquad (3)$$

where $\phi = e^{Ah}$ and $\Gamma = \int_{0}^{h} e^{A\tau} d\tau B$

Discretization of state space model

Recursively from (3),

$$x\left(kh+2h\right)=\phi x\left(kh+h\right)+\Gamma u\left(kh+h\right)$$

Therefore, the solution of (3), given $x\left(0\right)$ and $u\left(kh\right)\ \ \forall k$, is

$$x\left(kh\right)=\phi^{k}x\left(0\right)+\sum_{j=0}^{k-1}\phi^{k-1-j}\Gamma u\left(jh\right)$$

Note that the matrix exponential e^{Ah} can be expressed equivalently as a power series

$$e^{Ah}=I+Ah+\frac{A^{2}h^{2}}{2}+\dots$$

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Non-linear model of the state

Observation

Control decision is chosen as a function of the state

$$u(t) = f(x(t))$$

Therefore, consider a state that evolves according to a non-linear law

$$\dot{x}(t) = a(x(t))$$
$$y(t) = c(x(t))$$

where a and c are be non-linear functions in general

What is the solution of that system?

Non-linear model of the state

Non-linear differential equation

$$x\left(t+kh\right)=x\left(t\right)+\int\limits_{t}^{t+kh}a\left(x\left(\tau\right)\right)d\tau$$

- In general, the integral difficult to solve.
- However, by deriving opportune upper bounds of $a(x(t))$, it is possible to show important properties of the system (e.g., stability).

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Stability

Let us consider the discrete-time differential equation

$$x\left(kh+h\right)=g\left(x\left(kh\right)\right) \tag{4}$$

where g can be linear or non-linear

Definition

A specific solution of (4), $x^*\left(kh\right)$, is called stable,
if $\forall\varepsilon>0 \quad \exists\delta\left(\varepsilon\right):$ $\forall$ other solution $x\left(kh\right)$

$$\left\|x\left(0\right)-x^*\left(0\right)\right\|\leq\delta\Rightarrow\left\|x\left(kh\right)-x^*\left(kh\right)\right\|\leq\varepsilon \quad \forall k$$

Asymptotic stability

We consider the same equation as on the previous slide:

$$x\left(kh+h\right)=g\left(x\left(kh\right)\right) \tag{5}$$

where g can be linear or non-linear.

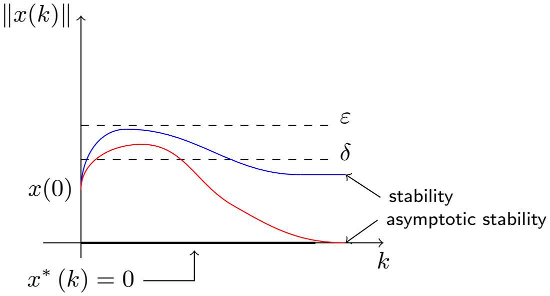
Definition

A specific solution $x^*(k)$ of (5) is called **asymptotically stable** if it is stable and if there is a $\delta > 0$ such that for every other solution $x(k)$ it holds that:

$$\left\|x\left(0\right)-x^*\left(0\right)\right\|\leq\delta\Rightarrow\left\|x\left(k\right)-x^*\left(k\right)\right\|\rightarrow 0 \quad as\ \ k\rightarrow\infty$$

Example 1

Assume that $x^*(kh) = 0$ is a solution of (5). The figure shows the typical behaviour of other solutions in case $x^*(kh)$ is stable or asymptotically stable.



The linear case

Consider a linear difference equation

$$x\,(kh+h) = A \cdot x\,(kh) \tag{6}$$

where A known matrix $\in \mathbb{R}^{n \times n}$

Definition

A linear difference equation of the form (6) is (asymptotically) stable if the constant solution $x^*(k) = 0$ is (asymptotically) stable.

How do we choose matrix A in order to have

1. stability?
2. asymptotic stability?

The linear case

The answer is given by the following theorem:

Theorem (Stability of linear difference equations)

Let $\rho(A) = \max\{|\lambda|, \lambda$ *is an eigenvalue of* $A\}$.

(i) $x\,(kh + h) = A \cdot x\,(kh)$ *is stable if and only* $\rho(A) \leq 1$.

(ii) $x\,(kh + h) = A \cdot x\,(kh)$ *is asymptotically stable if and only* $\rho(A) < 1$.

The linear case: intuition

1. $\|x(0) - 0\| \leq \delta \Rightarrow \|x(kh)\| \leq \varepsilon \quad ?$

   $\|x(kh)\| = \|A \cdot x((k-1)h)\| = \ldots = \left\|A^k \cdot x(0)\right\| \leq \left\|A^k\right\| \cdot \|x(0)\| \leq \|A\|^k \cdot \delta$

   To achieve stability, choose matrix A that does not grow with k

   This is when the maximum absolute eigenvalue of A, $\rho(A) \leq 1$

2. $\lim\limits_{k \to \infty} \|x(kh)\| \leq \lim\limits_{k \to \infty} \|A\|^k \cdot \delta \to 0$

   In this case, $\rho(A) < 1$

- In scalar case, A is constant and $\rho(A) = A$
- If the eigenvalues are larger than $1 \ \Rightarrow$ instability

Summary

- We have seen the basic aspects of control systems
  - Mathematical description of the state evolution
  - Discretization
  - Stability

Next lecture

- WSNCS, robustness to packet delays and losses