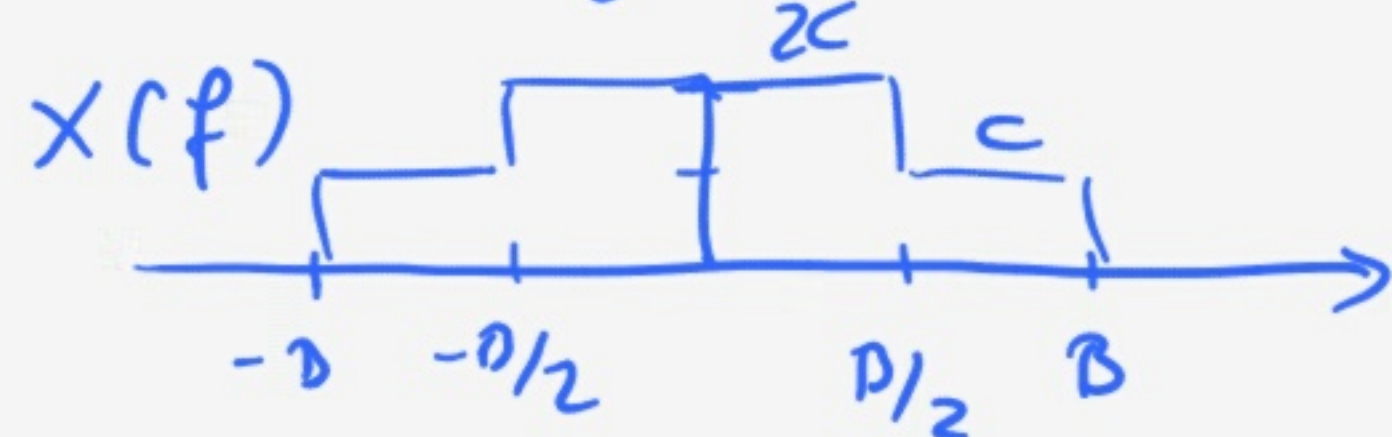
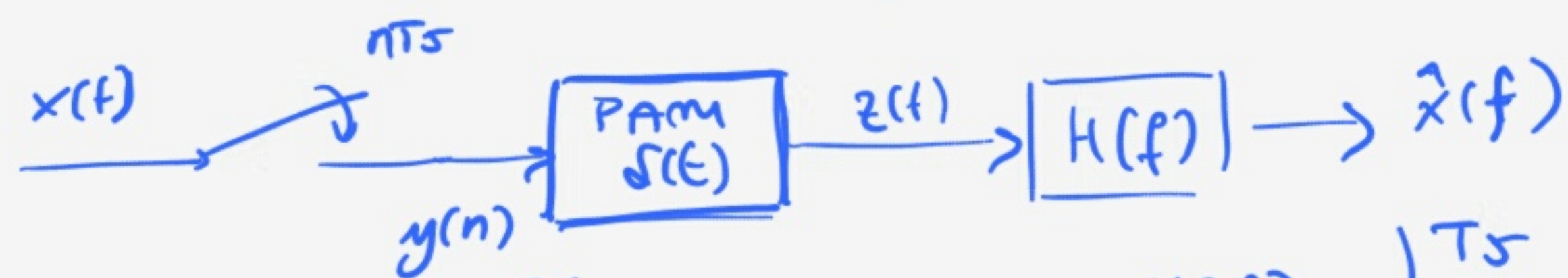


(12.1) $x(t)$ sampled $f_s = \frac{1}{T_s}$ Reconstructed using:



$$H(f) = \begin{cases} T_s & |f| \leq B \\ 0 & \text{otherwise} \end{cases}$$

$$X(f) = C(\text{rect}_B(f) + \text{rect}_{2B}(f))$$

(a) $\hat{X}(f)$ if $T_s = 2/5B$ $f_s = \frac{5B}{2} > 2B$ hence there is no aliasing.

$$Y(v) = \frac{1}{T_s} \sum_m X\left(\frac{v-m}{T_s}\right)$$

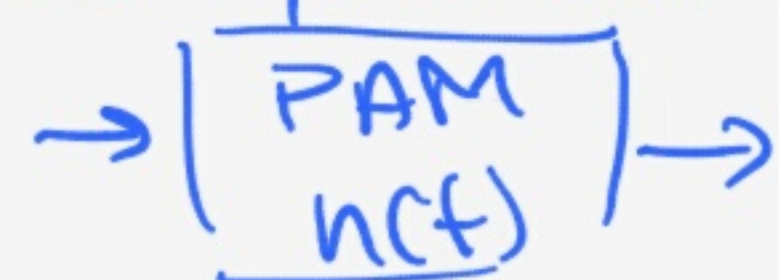
$$Y(v) = \frac{C}{T_s} \sum_m \text{rect}_B\left(\frac{v-m}{T_s}\right) + \text{rect}_{2B}\left(\frac{v-m}{T_s}\right)$$

$$\hat{X}(f) = H(f) Y(f T_s)^*$$

$$Y(f T_s) = \frac{C}{T_s} \sum_m \text{rect}_B\left(\frac{f T_s - m}{T_s}\right) + \text{rect}_{2B}\left(\frac{f T_s - m}{T_s}\right) \quad \text{for } |f| \leq B$$

$$\text{Hence, } \hat{X}(f) = C(\text{rect}_B(f) + \text{rect}_{2B}(f))$$

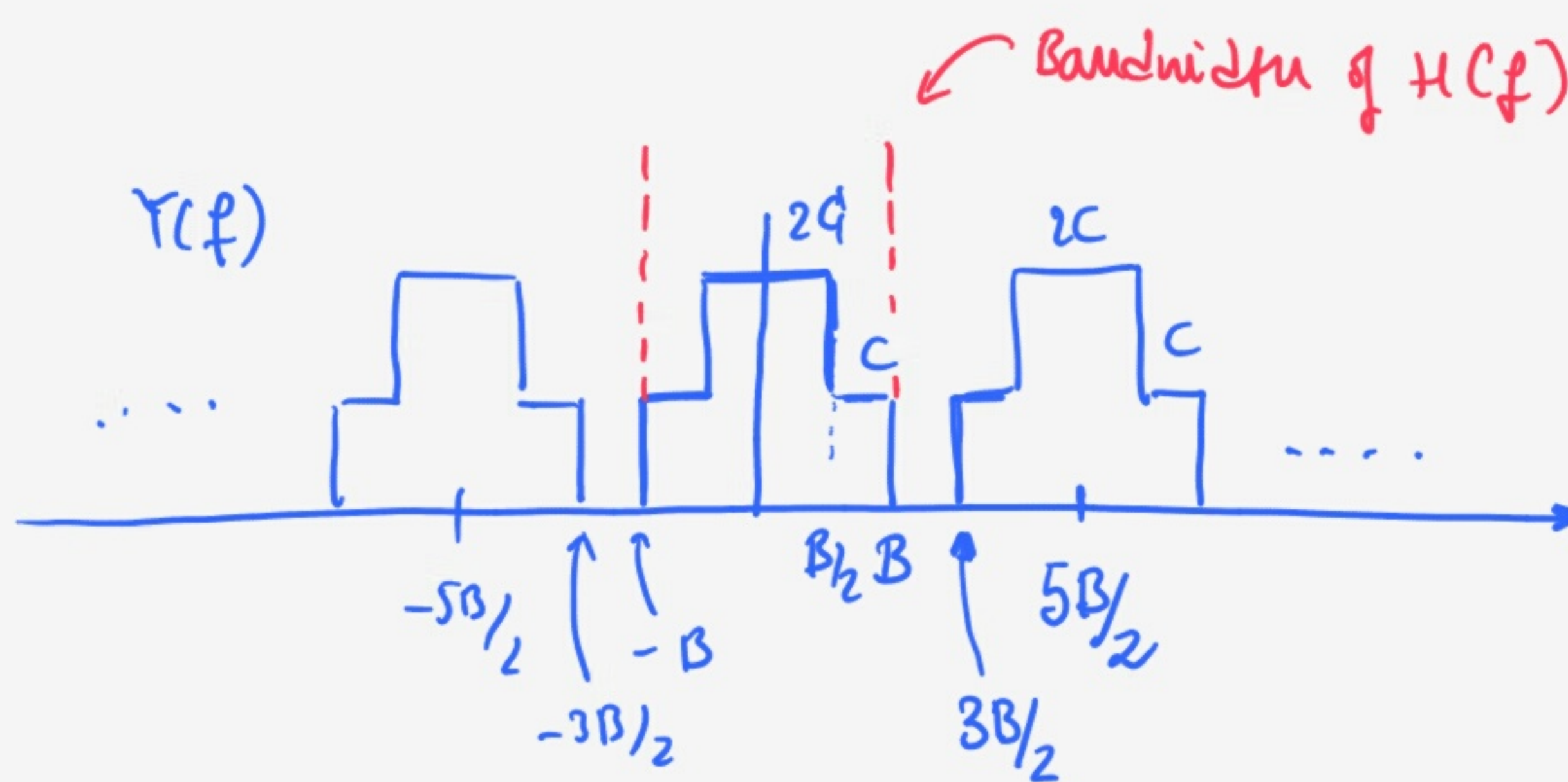
* the two blocks together are equivalent to



$$\hat{x}(t) = \sum_n y(n) h(t - n T_s)$$

$$\hat{X}(f) = \sum_n y(n) e^{-j 2 \pi f n T_s} H(f)$$

$$\hat{X}(f) = H(f) Y(f T_s)$$



(b) Find $\hat{X}(f)$ if $T_s = 1/B \Rightarrow$ will have aliasing!

$$Y(v) = \frac{1}{T_s} \sum_m X\left(\frac{v-m}{T_s}\right)$$

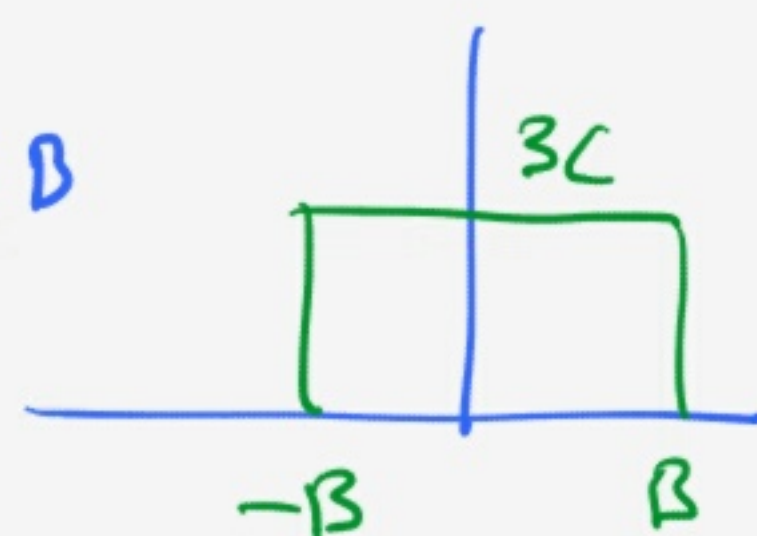
$$X(f) = C(\text{rect}_B(f) + \text{rect}_{2B}(f))$$

$$Y(v) \quad \text{periodic} \quad \leftarrow \frac{3C}{T_s} \quad \text{periodic}$$

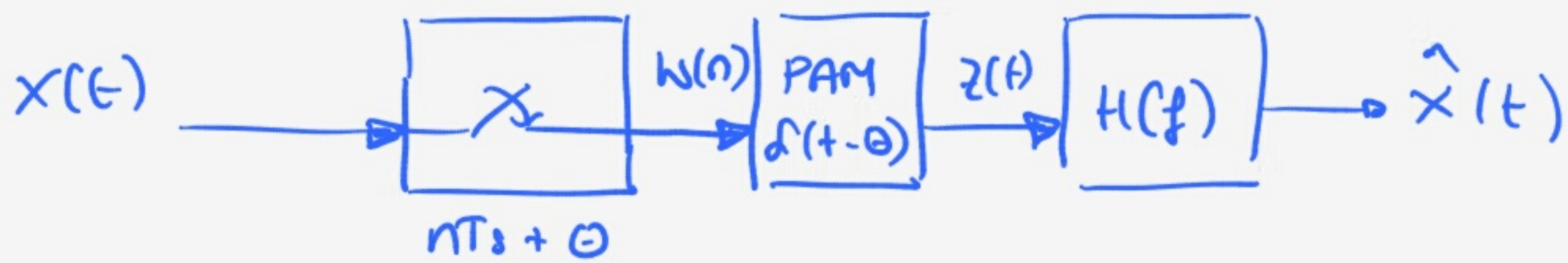


$$Y(v) = \frac{3C}{T_s} \quad \text{Then } \hat{X}(f) = H(f) Y(f T_s) = 3C \quad \text{for } |f| \leq B$$

0 otherwise



12.4



$$\Theta \sim \mathcal{U}[0, T_s]$$

$$r_x(z) = \sigma_x^2 e^{-|z|/T_s}$$

$$H(f) = \begin{cases} T_s & |f| \leq 1/2T_s \\ 0 & \text{otherwise} \end{cases}$$

$$E[w(n+k)w(n)] = E[X((n+k)T_s + \Theta)X(nT_s + \Theta)] =$$

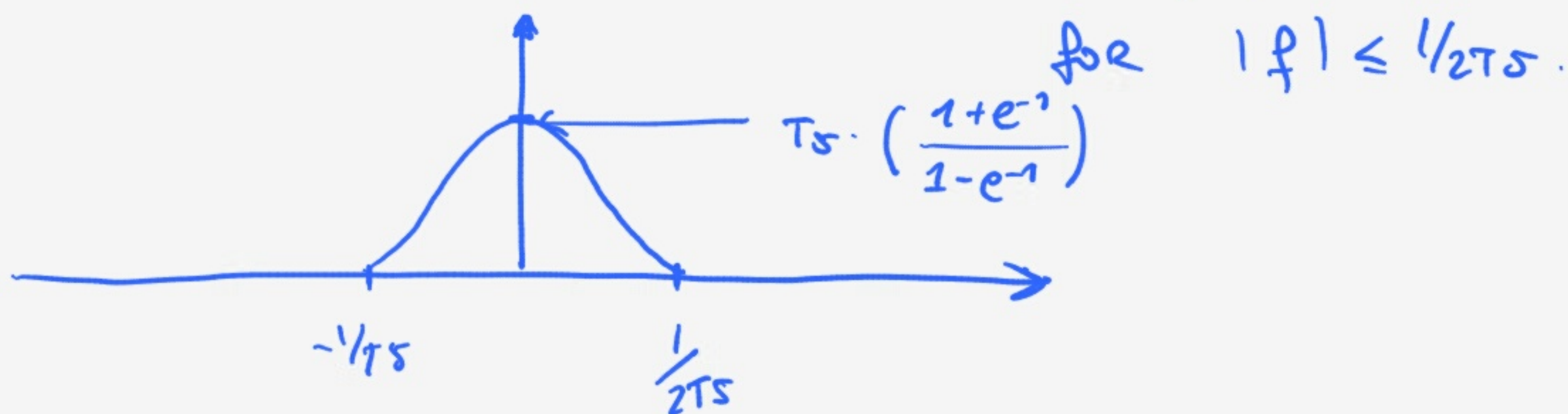
$$= r_x((n+k)T_s + \Theta - nT_s - \Theta) = \text{same value}$$

$$= r_x(kT_s) = \sigma_x^2 e^{-|k|} \quad R_w(\nu) = \frac{1 - e^{-2}}{1 + e^{-2} - 2e^{-1}\cos(2\pi\nu)} \cdot \sigma_x^2$$

$$R_z(f) = \frac{|P(f)|^2}{T_s} R_w(fT_s) = \frac{1}{T_s} R_w(fT_s)$$

then $R_x^*(f) = |H(f)|^2 \frac{1}{T_s} R_w(fT_s)$ $|H(f)|^2 = \begin{cases} T_s^2 & |f| \leq 1/2T_s \\ 0 & \text{otherwise} \end{cases}$

$$= T_s R_w(fT_s)$$

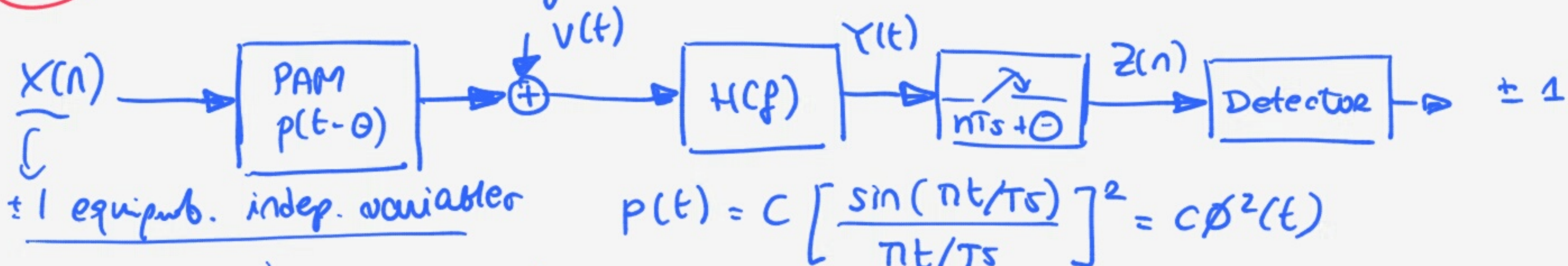


Compare to $R_x(f) = \sigma_x^2 \frac{2/T_s}{(\frac{1}{T_s})^2 + (2\pi f)^2} = \sigma_x^2 \frac{2T_s}{1 + (2\pi fT_s)^2}$] not band-limited $\Rightarrow$ aliasing effect.

write first in board

12.17

$x(n)$ reconstructed using PAM.



$$R_v(f) = R_0 \quad H(f) = \begin{cases} \frac{1}{1 - |f|T_s} & |f| \leq \frac{1}{2T_s} \\ 0 & \text{otherwise} \end{cases}$$

SNR at $z(n)$? $\rightarrow r_x(k) = d(k)$

$$\frac{1}{T_s} |P(f)|^2 R_x(fT_s) = \frac{1}{T_s} |P(f)|^2$$

spectrum before anti-aliasing filter:

$$\frac{1}{T_s} |P(f)|^2 + R_0$$

$$R_y(f) = |H(f)|^2 \left(\frac{1}{T_s} |P(f)|^2 + R_0 \right)$$

$$\begin{cases} \text{for } |f| \leq \frac{1}{2T_s} \\ 0 \text{ otherwise} \end{cases}$$

$$C \cdot \mathcal{F} \left\{ \frac{\sin(\pi t/T_s)}{\pi t/T_s} \right\} = T_s \cdot \text{Rect}_{1/2T_s}(f)$$

$$\text{Rect}_{1/2T_s} * \text{Rect}_{1/2T_s}(f) =$$

$$\Rightarrow P(f) = \begin{cases} T_s(1 - fT_s) & 0 \leq f \leq 1/2T_s \\ T_s(1 + fT_s) & -1/2T_s \leq f \leq 0 \end{cases}$$

$$P(f) = \begin{cases} c^2 T_s (1 - |f| T_s) & |f| \leq \frac{1}{T_s} \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{Then } R_y(f) = |H(f)|^2 \left(\frac{1}{T_s} |P(f)|^2 + R_0 \right) = \left| \frac{1}{1 - |f| T_s} \right|^2 \left(\frac{c^2 T_s^2}{T_s} (1 - |f| T_s)^2 + R_0 \right)$$

$$= \underbrace{c^2 T_s}_{R_{ys}(f)} + \underbrace{\frac{R_0}{|1 - |f| T_s|^2}}_{R_{yn}(f)} \quad \text{for } |f| \leq \frac{1}{2T_s} \quad \Rightarrow \text{no aliasing}$$

0 elsewhere.

$$R_{zs}(v) = \frac{1}{T_s} \sum_m R_{ys}\left(\frac{v-m}{T_s}\right) = c^2. \quad \text{Then } P_S = \int_{-1/2}^{1/2} c^2 dv = c^2(v) \Big|_{-1/2}^{1/2} = c^2$$

$$P_N = \int_{-1/2}^{1/2} \frac{R_0}{T_s} \sum_m \frac{1}{|1 - |v-m||^2} dv = \int_{-1/2}^{1/2} \frac{R_0}{T_s} \frac{1}{(1 - |v|)^2} dv =$$

$$= 2 \int_0^{1/2} \frac{R_0}{T_s} \frac{1}{(1-v)^2} dv = \frac{2R_0}{T_s} \left[\frac{1}{1-v} \right]_0^{1/2} = \frac{2R_0}{T_s} \quad \text{SNR} = \frac{c^2}{2R_0/T_s}$$