Principles of Wireless Sensor Networks

https://www.kth.se/social/course/EL2745/

Lecture 13

Wireless Sensor Network Control Systems 2

Piergiuseppe Di Marco
Ericsson Research
e-mail:pidm@kth.se
http://pidm.droppages.com/

Royal Institute of Technology
Stockholm, Sweden

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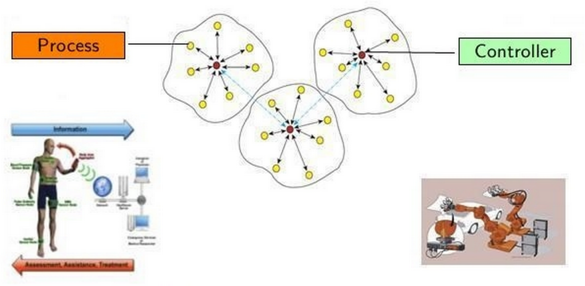
Course content

- Part 1
  - Lec 1: Introduction to WSNs
  - Lec 2: Introduction to Programming WSNs
- Part 2
  - Lec 3: Wireless Channel
  - Lec 4: Physical Layer
  - Lec 5: Medium Access Control Layer
  - Lec 6: Routing
- Part 3
  - Lec 7: Distributed Detection
  - Lec 8: Static Distributed Estimation
  - Lec 9: Dynamic Distributed Estimation
  - Lec 10: Positioning and Localization
  - Lec 11: Time Synchronization
- Part 4
  - Lec 12: Wireless Sensor Network Control Systems 1
  - Lec 13: Wireless Sensor Network Control Systems 2

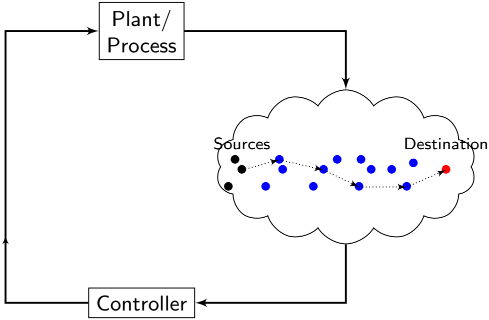
Previous lecture

Application
Presentation
Session
Transport
Routing
MAC
Phy



How to model mathematically a closed loop control system?

Today's learning goals



- How stability is affected by delays introduced by the WSN?
- How stability is affected by packet losses introduced by the WSNs?
- How to design WSNCS?

Outline

- Overview: WSNCS
- WSNCS with constant network delay
- WSNCS with random network delay
- WSNCS with asynchronous events
- WSNCS with packet losses
- Design of WSNCS

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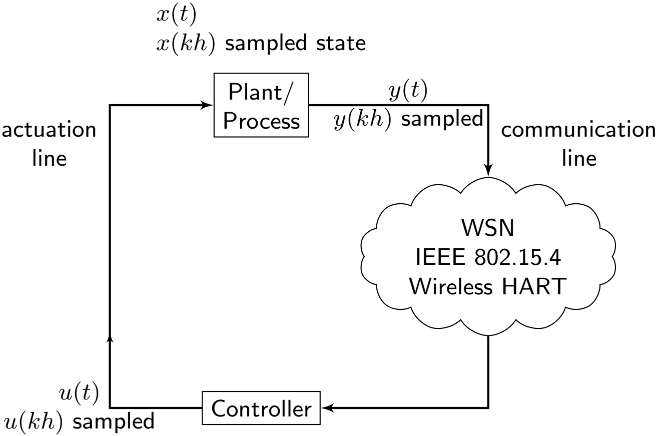
Overview: WSNCS



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WSNCS with constant network delay

Consider a linear model

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

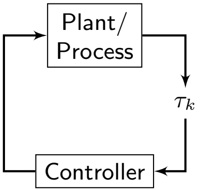
Assume that the controller takes a decision proportional to the state

$$u(t) = -Lx(t)$$

- L is chosen accordingly in order to achieve stability
- In general $u(t) = -L\hat{x}(t)$ where $\hat{x}(t)$ is an estimate of $x(t)$ based on $y(t)$
- Assume $x(t)$ is estimated perfectly

WSNCS with constant network delay

Let τ_k be the delay introduced by the network



Assume $0 \leq \tau_k \leq h$ where h the sampling time

When is the closed loop control system stable despite τ_k and given $u\left(kh\right) = -Lx\left(kh\right)$?

Network Delays

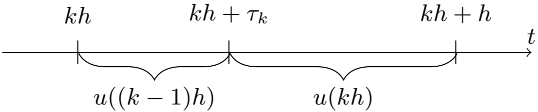
Our model becomes:

$$\dot{x}(t) = Ax(t) + Bu(t - \tau_k), \quad t \in [kh, kh + h) \tag{2}$$
$$y(t) = Cx(t) + Du(t)$$

where $u(t)$ is the input signal in the absence of delay, i.e. $u(t) = u(kh)$ for $t \in [kh, kh + h)$.

In the presence of delay τ_k, the control command $u(hk - h)$ is used until time $t = hk + \tau_k$.

WSNCS with constant network delay



From lecture 12, the solution of (1) is

$$x\left(kh+h\right)=e^{Ah}x\left(kh\right)+\int\limits_{kh}^{kh+h}e^{A(hk+h-\tau)}Bu\left(\tau\right)d\tau=$$

$$=e^{Ah}x\left(kh\right)+\int\limits_{kh}^{kh+\tau_k}e^{A(hk+h-\tau)}Bu\left(\left(k-1\right)h\right)d\tau+\int\limits_{kh+\tau_k}^{kh+h}e^{A(hk+h-\tau)}Bu\left(kh\right)d\tau=$$

by changing variable $s=kh+h-\tau$, we obtain

$$x(kh+h)=\phi x(kh)+\Gamma_0(\tau_k)u(kh)+\Gamma_1(\tau_k)u(kh-h) \qquad (3)$$

$$\phi=e^{Ah} \qquad \Gamma_0\left(\tau_k\right)=\int\limits_{0}^{h-\tau_k}e^{As}Bds \qquad \Gamma_1\left(\tau_k\right)=\int\limits_{h-\tau_k}^{h}e^{As}Bds$$

Network Delays

Consider the linear control input $u(x(kh)) = -Lx(kh), \quad L \in \mathbb{R}^{n \times n}$
Then, we can write

$$x\left(kh+h\right) = \phi x\left(kh\right) - \Gamma_0\left(\tau_k\right) Lx\left(kh\right) - \Gamma_1\left(\tau_k\right) Lx\left(\left(k-1\right)h\right) \tag{4}$$

By defining the augmented state vector $z = \begin{bmatrix} x(kh) \\ u(kh-h) \end{bmatrix}$ and the matrix $\overline{\phi}(\tau_k) = \begin{bmatrix} \phi - \Gamma_0(\tau_k)L & \Gamma_1(\tau_k) \\ -L & 0 \end{bmatrix}$.

We obtain the equivalent form

$$z(kh+h) = \overline{\phi}(\tau_k)z(kh) \tag{5}$$

If the maximum eigenvalue of $\overline{\phi}$, $\rho\left(\overline{\phi}\right) < 1$, then the closed loop system is asymptotically stable.

Example

Suppose that we are given a simple scalar system that is subject to a constant network delay τ and governed by the following equation:

$$\dot{x}(t) = u(t)$$

Assume that the controller decision is $u(t) = -Lx(t)$.

In order to study the stability of the delay system, the matrix $\overline{\phi}$ needs to be constructed. Thus, since $A = 0$ and $B = 1$,

$$\phi = e^0 = 1 \qquad \Gamma_0 = \int\limits_0^{h-\tau} ds = h - \tau \qquad \Gamma_1 = \int\limits_{h-\tau}^{h} ds = \tau$$

and the matrix becomes

$$\overline{\phi} = \begin{bmatrix} \phi - \Gamma_0(\tau) L & \Gamma_1(\tau) \\ -L & 0 \end{bmatrix} = \begin{bmatrix} 1 - hL + \tau & \tau \\ -L & 0 \end{bmatrix}$$

Example

To compute the eigenvalues of $\overline{\phi}$, we need to solve the equation:

$$\det\left(\overline{\phi} - \lambda I\right) = 0 \Leftrightarrow (1 - hL + \tau L - \lambda)(-\lambda) + \tau L = 0 \Leftrightarrow$$

$$\Leftrightarrow \lambda^2 - \lambda(1 - hL + \tau L) + \tau L = 0 \Leftrightarrow$$

$$\Leftrightarrow \lambda_{1,2} = \frac{1 - hL + \tau L \pm \sqrt{(1 - hL + \tau L)^2 - 4\tau L}}{2}$$

The sampled system state is asymptotically stable iff $|\lambda_1|, |\lambda_2| < 1$.

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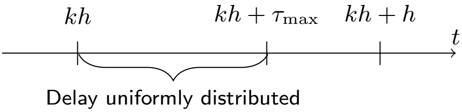
WSNCS with random network delay



- We now assume that the delay is not constant, but a random variable of maximum value $\tau_{\max}$
- Using $\tau_{\max}$ as in the previous case, would give conservative results
- We could exploit the randomness to get better results

When is the system stable despite random delay?

WSNCS with random network delay

Assume for simplicity $h = 1$

$$x(k+1) = \phi x(k) + \Gamma u(k)$$

$$y(k) = Cx(k) + Du(k)$$

We rewrite the system of equations in terms of z-transform,

$$zX(z) - X(0) = \phi X(z) + \Gamma U(z)$$

$$Y(z) = CX(z) + DU(z)$$

where $X(z) = \sum_{k=0}^{\infty} x(k)z^{-k}$, $U(z) = \sum_{k=0}^{\infty} u(k)z^{-k}$ and $Y(z) = \sum_{k=0}^{\infty} y(k)z^{-k}$

Note: the z-transform converts discrete-time signals into a frequency domain representation (similarly to Laplace transformation for continuous-time signals).

WSNCS with random network delay

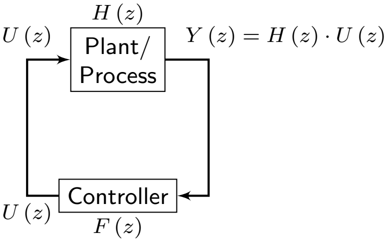
Combining the equations and solving for Y,

$$\begin{aligned} Y\left(z\right) &= C(zI-\phi)^{-1} zX\left(0\right) + \left[C(zI-\phi)^{-1}\Gamma + D\right] U\left(z\right) \\ &= C(zI-\phi)^{-1} zX\left(0\right) + H(z) U\left(z\right) \end{aligned}$$

where $H(z)$ is defined as the **pulse transfer function** of the system

$$H\left(z\right) \triangleq \left[C(zI-\phi)^{-1}\Gamma + D\right]$$

WSNCS with random network delay



The control law is given as

$$u(k) = \sum_{n=0}^{k} f(n)\, y(k-n) \xrightarrow{Z\{\}} U(z) = F(z) \cdot Y(z)$$

$F(z)$ can be properly designed in order to affect the controller's decision

WSNCS with random network delay

Theorem

Consider the WSNCS with a random uniform distributed delay with $\tau_{\max} \leq h$ *and* $U(z) = F(z) \cdot Y(z)$*. The closed loop system is stable if*

$$\left| \frac{F(z) H(z)}{1 + F(z) H(z)} \right| \leq \frac{1}{\tau_{\max} \cdot |z - 1|} \quad \forall \omega$$

where $z = e^{i\omega}$

- Sufficient but not necessary condition for stability
- $F(z)$ and/or $\tau_{\max}$ can be tuned to achieve closed loop stability.

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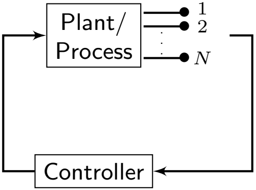
WSNCS with asynchronous events

Consider a WSNCS where the same state, $x(k)$, is observed at different time instants with different measurement functions



The system is described by the set of difference equations

$$x(k+1) = f_i(x(k)) \quad i = 1, ..., N$$

where $1, \ldots, N$ the set of discrete states with the respective associated rates $r_1, r_2, ..., r_N$. These rates are the fraction of time that each discrete state occurs, that is

$$r_i = \frac{1}{t_i} \quad i = 1, ..., N$$

WSNCS with asynchronous events

The stability condition of such model is given by the following theorem

Theorem

Given a WSNCS as defined above, if there exists a function $V(x) : \mathbb{R}^n \to \mathbb{R}_+$ *such that*

$$V(x)|_{x=0} = 0, \quad V(x)|_{x \neq 0} > 0, \quad \dot{V}(x) < 0,$$

and scalars $\alpha_1, \alpha_2, ..., \alpha_N$ *corresponding to each rate such that*

$$\alpha_1^{r1} \cdot \alpha_2^{r2} \cdot \dots \cdot \alpha_N^{rn} > \alpha > 1$$

and

$$V(x(k+1)) - V(x(k)) \leq ({\alpha_i}^{-2} - 1)V(x(k)) \quad i = 1, \dots, N$$

then the WSNCS is asymptotically stable.

Note, $V(x)$ is called Lyapunov function of x.
α defines the decay rate of x.

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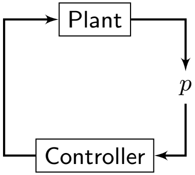
Packet losses



The input $u(t)$ will depend on whether a packet drop has occurred or not and is given by: $u(t) = -L\overline{x}(kh)$ for $t \in [kh, kh+h)$, where

$$\overline{x}(kh) = \begin{cases} x(kh-h) & \text{the } k\text{:th packet was lost} \\ x(kh) & \text{otherwise} \end{cases}$$

The probability of a packet loss is associated to the **packet loss probability** p

WSNCS with packet losses

The characteristic equations of the closed loop system are

$$x\left((k+1)h\right) = \phi x(kh) + \Gamma u(kh)$$

$$u(kh) = -L\bar{x}(kh)$$

where recall that

$$\bar{x}(kh) = \begin{cases} x(kh) & \text{if no packet losses} \\ \bar{x}\left((k-1)h\right) & \text{otherwise} \end{cases}$$

When is the system stable?

WSNCS with packet losses

Theorem

Suppose that the closed loop system is stable in the case of no packet losses. Then

1. *If the open loop system is stable (i.e., $\rho(\phi)$ is stable) then the closed loop system (with packet losses) is stable for every p*
2. *If ϕ is unstable, then the closed loop system (with packet losses) is stable when*

$$\frac{1}{1-\frac{\gamma_1}{\gamma_2}} < 1 - p$$

where $\gamma_1 = \log \lambda^2_{\max}(\phi - \Gamma L)$, $\gamma_2 = \log \lambda^2_{\max}(\phi)$ and $\lambda_{\max}$ the maximum eigenvalue of the corresponding matrix

We have the following parameters:

- The packet loss probability p, that depends on PHY, MAC, routing
- The controller L

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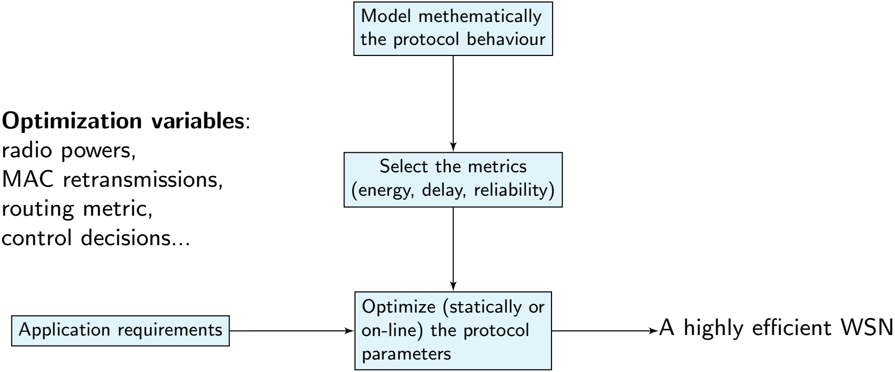
WSNs design

Optimization variables:
radio powers,
MAC retransmissions,
routing metric,
control decisions...



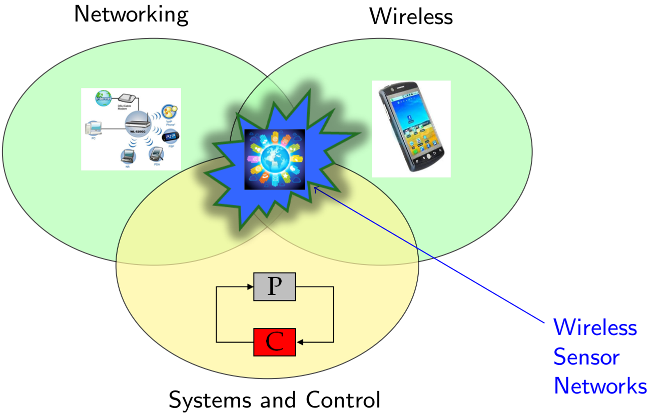
The role of mathematical modeling and optimization is central

WSNCS design

$$
\begin{aligned}
&\min_{x} J(x) \\
&\text{s.t. } \Pr(\text{succ}) \geq 1 - p \\
&\qquad \Pr(\text{delay} \leq \tau_{\max}) \geq \delta
\end{aligned}
$$

The cost function $J(x)$ represent the WSNCS cost, e.g., energy consumption $E(x)$. The optimization variable x collects both communication and control parameters.

Wireless Sensor Networks



Summary

- We saw that there is no need to design WSNs that minimize the delay and maximize the packet reception probability
- The controllers can tolerate a certain degree of delay and packet losses
- The efficient design of a wireless sensor network control system can be posed by optimization problems

Exam, October 28, 08:00-13:00, M33

- 5 exercises chosen on every part of the course, inspired from the exercises of compendium and homework
- 5 hours to complete the exam
- **Allowed** to bring PRINTED lecture slides and draft book, and basic books on math, e.g., Mathematics Handbook by Rade & Westergren
- **Not allowed** to bring exercise lecture notes
- **Not allowed** to bring compendium with exercises and solutions
- Results available after 2-3 weeks

Master thesis projects

- Theoretical, practical, or business oriented
- Conduct forefront research
- Possible collaboration with industry (e.g., Ericsson Research)
- Interaction with Professors, Research Associates, and PhD students
- You can propose the topic, or ask for a project on
  - Distributed optimization over WSNs
  - Distributed detection and estimation
  - Design of wireless sensor networked control systems
  - Future wireless networks
  - Internet of Things
  - MAC, Routing
  - Smart grids
  - Privacy
  - ...

PhD in Electrical Engineering

- For motivated and talented students, possibility of continuing towards a PhD
- International collaborations UC Berkeley, Stanford University, MIT, Caltech,...
- Conferences, workshops, summer schools around the world
- Competitive salary
- World-wide job market (academia or companies)
- Access to senior positions in research-oriented industries
- Research (50%), courses (30%), teaching (20%), fun (100%)
- 4-5 years to earn the PhD