SF1625 Calculus in one variable
Exam
Tuesday 27:th of October 2015

Time: 08:00-13:00
No calculators, formula sheets etc allowed
Examiner: Lars Filipsson

This exam consists of nine problems, each worth four points, hence the maximal score is 36. Part A consists of the three first problems. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically. You can check how many bonus points you have on your results page.

The following three problems constitute part B and the last three problems part C. You need a certain amount of points from part C to obtain the highest grades.

The grading will be performed according to this table:

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

To obtain a maximal 4 for a solution to a problem on the exam, your solution must be well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

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PART A

1. Let the function f be given by $f(x) = 1 + x + \dfrac{4}{(x-2)^2}$.

   A. Determine the domain of definition of f.
   B. Find the intervals where f is increasing and decreasing, respectively.
   C. Find all local extreme values of f.
   D. Find all asymptotes to the graph $y = f(x)$.
   E. Sketch, using the above, the graph $y = f(x)$.

2. Compute the integrals:

   A. $\displaystyle\int_0^2 \frac{x}{(x^2+4)^{1/3}}\,dx$ (you may use the substitution $u = x^2 + 4$)
   B. $\displaystyle\int_1^4 \sqrt{x}\ln x\,dx$ (you may want to integrate by parts)

3. What is the largest possible area of a right triangle, if the hypotenuse and one of the other sides together has a total length of 1 meter?

PART B

4. Let $f(t) = e^{-t} + \sin t - \cos t$..
   A. Find the Taylor polynomial of degree 2 around the point $t = 0$ to the function f.
   B. State the error term (in some form)
   C. Compute the limit $\lim_{t \to 0} \dfrac{f(t)}{t^2}$..

5. Compute the integral $\displaystyle\int_0^1 \arcsin x \, dx$.

   (For a maximum score, an exact computation is required, but an approximate computation might give some points. Simplify your answer.)

6. The charge $q(t)$ in the capacitor in a certain alternating current circuit satisfies the differential equation

$$\frac{d^2q}{dt^2} + 2\frac{dq}{dt} + 4q = 39\cos t$$

   with initial values $q(0) = 0$ och $q'(0) = 0$.
   A. Determine the charge in the capacitor at time t.
   B. Let us say that $q(t)$ has the long tem behavior $f(t)$ if $\lim_{t\to\infty} |q(t) - f(t)| = 0$. Determine the long term behavior of the charge in the capacitor.

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PART C

7. This is about the theory of differentiation and integration.
   A. Formulate the product rule of differentiation.
   B. Prove the product rule of differentiation.
   C. Formulate the rule for integration by parts.
   D. Prove the rule for integration by parts.

8. Let $F(x) = \int_0^x e^{-t^2} \cos t \, dt$ with domain of definition $D = [0, \pi]$.
   A. On what intervals of D is F increasing and decreasing, respectively.
   B. Find points a och b in D such that
      $F(a) \leq F(x)$ for all $x \in D$,
      $F(b) \geq F(x)$ for all $x \in D$.

9. Compute the limit $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n^2} \arctan \frac{k}{n}$.