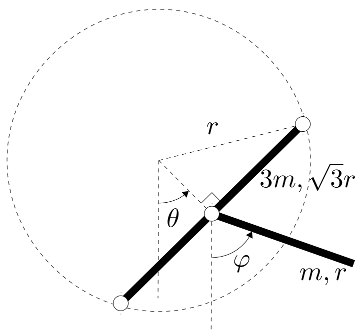
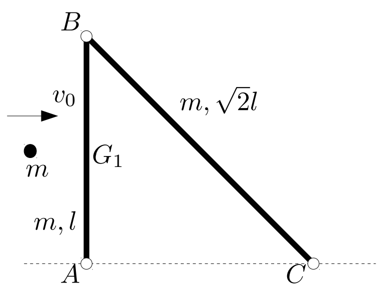
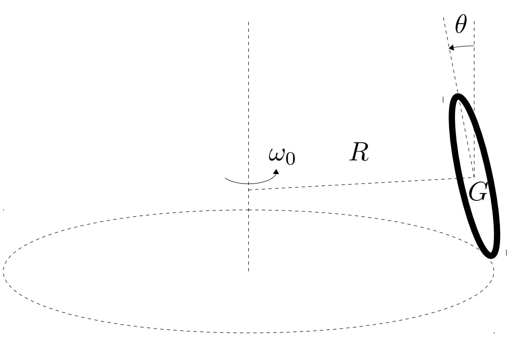
Rigid Body Dynamics (SG2150)
Exam, 2015-10-29, 8.00-12.00



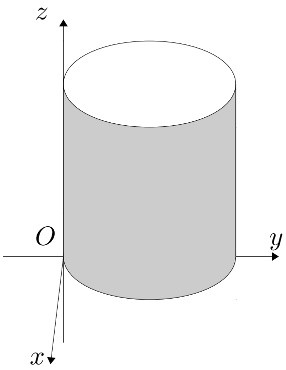
Problem 1. A thin homogenous rod of mass $3m$ and length $\sqrt{3}r$ slides with both end points on a vertical circular smooth track of radius r. The end point of another similar rod of mass m and length r is attached by a smooth hinge joint to the mid point of the longer rod. Find the kinetic and potential energies for this planar system and derive Lagrange's equations. Finaly specialize to motion near the stable equilibrium solution of the system and derive the frequencies of small oscillations.



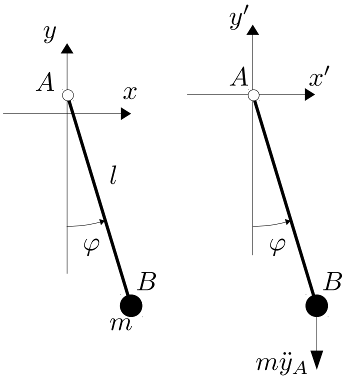
Problem 2. A thin homogenous rod of mass m and length l is attached to a smooth, straight track at one end point A. The rod is initially perpendicular to the track, and the other end point B is attached to another similar rod of mass m and length $\sqrt{2}l$ by a smooth hinge joint. The other end point C of the second rod is also attached to the smooth track. This planar two-link mechanism is at rest when a particle of mass m hits the mid point G_1 of the rod AB with a velocity v_0 parallel to the track. Immediately after this impact, the particle is found to be at rest. Compute the velocity of the point G_1 immediately after the impact.



Problem 3. Consider a thin circular homogenuous ring of radius r and mass m. The ring is rolling (without slipping) in such a way that its center of mass G is describing a circle of constant height and with radius R. The symmetry axis of the ring intersects a vertical axis through the center of this circle, and the symmetry axis makes a constant angle θ with the horizontal plane. The plane containing the vertical axis and the symmetry axis rotates around the vertical axis with angular velocity ω_0. Show that ω_0 is constant, and find a relation between ω_0 and θ.



Problem 4. A thin, homogenous, open-ended cylindrical shell of mass m has radius r and height $2r$. Let O be a point on the lower circular edge. Compute the 3×3 inertia matrix $\boldsymbol{J}_O$ for the axes indicated in the figure, where z is directed along the cylinder axis, and x is tangent to the lower circle. Also find at least one pricipal axis and at least one principal moment of inertia.



Problem 5. A planar particle pendulum consists of a light rod of length l connecting a particle of mass m at the point B with a supension point A. The point A is shaken vertically by a given time function $y_A(t)$. Use the generalized coordinate φ and derive Lagrange's equation.

Next use an accelerated frame of reference that follows the motion of the supension point A. In that frame, the point A is now fixed, but an extra downward inertial force $m\ddot{y}_A(t)$ appears on the particle. This force behaves like an additional time-dependent gravity force and is thus conservative with a potential $m\ddot{y}_A(t)y'_B$. Derive Lagrange's equation again and show that they are the same, even if the Lagrange function is different.

Problem 6. Consider a rigid body moving freely without being subject to any external forces or torques. Let the moments of inertia about three orthogonal principal axes through the center of mass G be J_1, J_2, and J_3 respectively, and let ω_1, ω_2, and ω_3 be the components of the angular velocity vector $\boldsymbol{\omega}$ in these three directions. Start with the angular momentum balance equation $\dot{\boldsymbol{L}}_G = \boldsymbol{0}$, and derive the equations that determine the rate of changes $\dot{\omega}_i$ of the angular velocity components (*Euler's dynamic equations*).

Investigate the conditions for having all $\omega_i(t)$ be constant in time. Separate the cases where all J_i are different, when there is axial rotation symmetry, and when there is sperical rotation symmetry. Can the directions of constant (in the body frame) $\boldsymbol{\omega}$ be characterized?

Each problem gives a maximum of 3 points, so that the total maximum is 18. Grading: 1–3 F; 4–5 FX; 6: E; 7–9 D; 10–12 C; 13–15 B; 16–18 A.

Allowed equipment: Handbook of mathematics and physics. One A4 page with your own compilation of formulae.

AN 2015-10-29