AUTOMATIC CONTROL

KTH

EL2745 Principles of Wireless Sensor Networks

Exam 08:00–13:00, October 28, 2015

Aid:

Printed slides from the course, reading material such as 'An Introduction to Wireless Sensor Networks" or similar text approved by course responsible are approved; Mathematical handbook (e.g., "Beta Mathematics Handbook" by Råde & Westergren) and pocket calculators are approved. The course compendium, your notes to the exercise lectures, other textbooks, handbooks, exercises, solutions, smartphones, tablets, etc. may **not** be used.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page and write on one side per sheet.
- Each answer must be motivated.
- Specify the total number of handed-in pages on the cover.
- Each subproblem is marked with its maximum credit.

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available on your "my pages" between one and two weeks from the exam.

Responsible: Carlo Fischione, `carlofi@kth.se`

Good Luck!

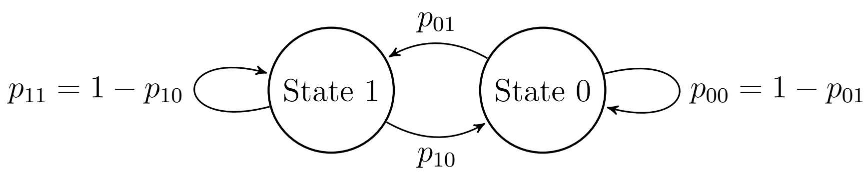

Figure 1: Two state Gilbert-Elliot model. State 0 represents a "message reception" event. State 1 represents a "message loss" event.

1. **Bernoulli and Gilbert-Elliot Model of the Wireless Channel**

   In Wireless Sensor Networks (WSNs) communication problems, the physical layer of the communication between a transmitter and a receiver node, which includes the communication channel, and modulation schemes, is modelled by the Bernoulli and Gilbert-Elliot models. The Bernoulli model assumes that the event of loosing a message has same probability p, independent of the messages. Instead, Gilbert-Elliot model uses a two state Markov chain to capture bursty error behavior of a wireless channel, as illustrated in Figure 1. A message is received in Reception state, state 0, whereas it will be lost in Loss state, state 1. The probabilities to transit from a state to another are depicted in Figure 1.

(a) [2p] Find the steady-state message loss rates for Bernoulli and Gilbert-Elliot models. When the Gilbert-Elliot model reduces to the Bernoulli model?

(b) [2p] When l messages are consecutively lost over the wireless channel, we say that a burst of size l occurs. Find the probability of having a burst of size l for the Bernoulli and Gilbert-Elliot models and comment the differences.

(c) [3p] Find the average burst length for both models.

(d) [3p] Consider the Bernoulli model. Messages of f bits are transmitted over a Rayleigh channel and received with an Additive White Gaussian Noise (AWGN). Derive the expression of the probability of message loss of the Bernoulli model for the given channel at high SNR regimes. [hint: At high SNR regimes one can use the approximation $(1+x)^{1/2} \sim 1 + x/2$.]

2. **Slotted ALOHA with Forward Error Correction**

In a slotted ALOHA network, a "delay sensitive message" is transmitted as follows: once a node has the message to transmit, it attempts transmission in the upcoming time slot; if collision happens at the receiver, the message is lost and will not be retransmitted, since it would arrive late to the receiver anyway and this is not compatible with the "delay sensitive message" assumption. Suppose a large number of nodes are in the network, and therefore from the point of view of a receiver node, altogether they generate messages according to a Poisson process with arrival intensity λ. Suppose that message transmission times are the same as the time slot length T.

(a) [2p] Assume that the receivers can tolerate a message loss probability of 0.1. What is the maximum allowed arrival intensity of messages at the transmitter if $T = 1$? [Hint: recall that the probability of "no arrival" in a time slot of duration T is $e^{-T\lambda}$]

(b) [2p] Assume again that the receivers can tolerate a message loss probability of 0.1. Suppose that the time slots have the duration of two time units, $2T$ with $T = 1$. What is the maximum allowed arrival intensity for the receivers to experience the message loss probability of 0.1?

(c) [3p] To cope with the message losses, a so called forward error correction (FEC) is introduced in the ALOHA system. In FEC, information is transmitted twice, in two different messages separated by a random delay. This still gives a Poisson distribution modeling the message generation process. Give the probability that a message is successfully received.

(d) [3p] Argue for which arrival intensity the FEC described above increases the probability that messages are successfully received.

3. **Detection**

Suppose that a sensor node makes an observation x that has the following conditionally uniform density

$$p(x|\theta) = \begin{cases} \frac{1}{\theta}, & 0 < x \leq \theta \\ 0, & \text{otherwise}, \end{cases}$$

where θ is a random variable with density

$$p(\theta) = \begin{cases} \theta \exp(-\theta), & \theta \geq 0 \\ 0, & \text{otherwise}. \end{cases}$$

(a) [2p] Find the Maximum a Posteriori (MAP) estimator of θ. [Hint: A useful formula: for $\nu \geq 0$, $\int_\nu^\infty u \exp(-u) du = (\nu + 1)\exp(-\nu)$]

(b) [2p] Find the Minimum Mean Square Error (MMSE) estimator of θ.

(c) [3p] Consider the mean absolute error between an estimator and θ. Compute the estimator of θ that gives the minimum of such an error.

(d) [3p] Suppose now that θ is constant, i.e., it does not follow the density $p(\theta)$ given above. Prove under which conditional distribution $p(x|\theta)$ the MAP and MMSE estimators are identical.

4. **Localization and Synchronization**

In this exercise, you will consider different problems of localization and synchronization.

(a) [2p] Suppose a sensor node observe the Cartesian position of a target that is known to be located on the unit circle. The sensor measurement model for this problem can be written

$$y = \begin{bmatrix} \cos(x) \\ \sin(x) \end{bmatrix} + e\,, \qquad\qquad e \sim \mathcal{N}(0, \sigma^2 I)\,.$$

Show that the Maximum Likelihood Estimate (MLE) for the polar position (angle x) is given by

$$\hat{x} = \arctan(y_2/y_1)\,.$$

(b) [3p] Consider a sensor network with $M = 3$ sensors located at $(0,0)$, $(3,0)$ and $(6,0)$, respectively. The true target position is at $x_0 = (3,4)$. Each sensor provides a Time of Arrival (TOA) range measurement $r_i = \|p_i - x_0\| + r_0 + e_i$, where $R = \mathrm{Cov}(e_i) = 1$ and r_0 is an unknown constant. In vector form, this reads $y = (r_1, r_2, r_3)^T = h(x) + e$. Derive the expression of the sensor model for the Time Difference of Arrival (TDOA) observations $r_i - r_j$ for all $i \neq j$ in the generic form $\bar{y} = \bar{h}(\bar{x}) + \bar{e}$. What is the covariance matrix $\bar{R} = \mathrm{Cov}(\bar{e})$ and $\bar{h}(x_0)$ numerically in this case?

(c) [3p] Consider a sensor network where two sensors at known positions (x_i, y_i), $i = 1, 2$. Triangulate the position of an object in unknown position (x, y) based on the two angle measurements φ_1 and φ_2. Then, assume that the angle measurements are perfect. Derive an expression for (x, y) when $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (1, 0)$.

(d) [2p] Suppose that node i has software clock $C_i(t)$, and wants to establish the relative time between it and some time server. The local clock of node i can be represented relative to the server as follow:

$$C_i(t) = a_0 + a_1 C_s(t)\,,$$

where a_0 and a_1 are the relative clock offset and drift, respectively. Assume that a_0 and a_1 are constant. Consider the following measurements for $C_i(t)$ are 2700, 2810, and 2920, while those for $C_s(t)$ are 2000, 2100, and 2200. Find the Minimum Mean Square Estimator (MMSE) of a_0 and a_1.

5. **Networked Control System**

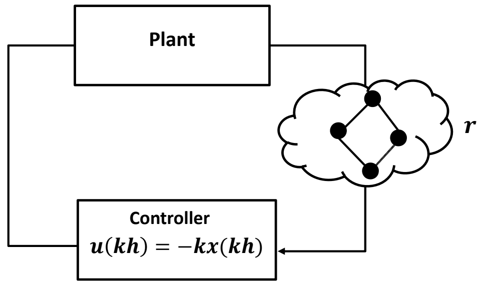

Figure 2: Closed loop system over a WSN.

Consider the Wireless Sensor Network Control System (WSN-CS) in Figure 2. The system consists of a continuous plant

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1a)$$
$$y(t) = Cx(t)\,. \quad (1b)$$

The system is sampled with sampling time h, and the discrete controller is given by

$$u(kh) = -Kx(kh), \qquad k = 0, 1, 2, \dots,$$

where K is a constant. Assume that the network does not introduce any delay and any message losses, unless explicitly stated.

(a) [2p] Consider the special form of the system (1) above, given by

$$\dot{x}(t) = -ax(t) + bu(t)$$
$$y(t) = cx(t)\,,$$

where a, b, and c are scalars. Let the input be constant over periods of length h. Sample the system and discuss how the stability of the discrete time system vary with the sampling time h.

(b) [2p] Now consider the special form of the system (1) above, given by

$$\dot{x}(t) = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(t)$$
$$y(t) = x(t)\,.$$

Sample this system with sampling period $h = 1$.

(c) [2p] Consider the system given in item (b). Assume that $u(k) = -Kx(k)$, where $K = [0.3059, 0.0227]^T$. Analyze the stability of the system.

(d) [4p] Now, suppose that the network unfortunately introduces message losses. Give and discuss sufficient conditions for which the closed loop system is stable. If these conditions are not satisfied, discuss what can be done at the network level (protocol parameters) or at the controller level so to still ensure closed loop stability.