Department of Mathematics

SF1626
Several Variable Calculus
Academic year 2015/2016, Period 2

Seminar 1

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a hand-in of one of the problems. Solve problems 1-4 below and write down your solutions on separate sheets. Write your name and personal number on the top of each page. When the seminar starts you will be informed about which problem to hand in. Before starting with the seminar problems you should solve the recommended exercises from the text book Calculus by Adams and Essex (8th edition). These exercises are:

Section	**Recommended problems**
10.1:	11, 25, 27, 29, 31, 33, 35, 37, 39
10.6:	3, 5, 9, 13
11.1:	17, 21, 33
11.2:	3
11.3:	5, 7, 11, 13, 15
12.1:	5, 9, 13,15, 17, 23, 27, 33
12.2:	5, 7, 9, 11, 15

PROBLEMS

Problem 1. Consider the sets in the xy-plane given by

$$\begin{aligned} D_1 &= \{(x,y)\colon x^2+y^2<3,\ 2x+3y=5\} \\ D_2 &= \{(x,y)\colon x^2+y^2>2\} \\ D_3 &= \{(x,y)\colon 2x+3y=5\} \end{aligned}$$

(a) Sketch the sets D_1, D_2 och D_3.
(b) Mark the *inner points* of the sets.
(c) Mark the *boundary points* of the sets.
(d) Determine which of the sets that are *open*, *closed* or neither *open* nor *closed*.

Problem 2. A *great circle* on a sphere is a circle that is the intersection of the sphere with a plane through the centre of the sphere. Consider the unit sphere S with equation $x^2+y^2+z^2 = 1$ and the great circle C that is the intersection with the plane $x+y+z = 0$.

(a) Express the equations of the great circle C by means of *spherical coordinates*.
(b) Express the equations of the great circle C by means of *cylindrical coordinates*.
(c) There are many possible parametrizations o a given curve. Determine one parametrization of C by writing

$$\mathbf{r}(t) = \mathbf{u}\cos t + \mathbf{v}\sin t,$$

where $\mathbf{u}$ and $\mathbf{v}$ are two ortogonal unit vectors in the plane that cuts out C from S.
(d) Determine expressions for the velocity and the speed of a particle that travels along the great circle C according to the parametrizationfrom part (c), where the parameter is considered as time.

Problem 3. A particle with mass $m = 9.1$ g travels in a spiral-shaped orbit that is described by

$$\mathbf{r}(t) = (R\cos\omega t, R\sin\omega t, kt),$$

där $R = 7.2$ cm, $k = 0.53$ m/s och $\omega = 4.7$ radianer/s are constants.

(a) Calculate the velocity $\mathbf{r}'(t)$.
(b) Calculate the acceleration $\mathbf{r}''(t)$.
(c) Show that the velocity and the acceleration are ortogonal to each other.
(d) The kinetic energy of the particle is given by $\frac{m}{2}|\mathbf{r}'(t)|^2$. Calculate this energy.

Problem 4. Let $f(x, y) = 3x^2 + 4xy + 3y^2$ for all (x, y) in $\mathbb{R}^2$. In order to study f it can be useful to consider the change of variables given by $u = x + y$ och $v = x - y$.

(a) Sketch some of the level curves of the function f.
(b) Sketch the graph of the function f.
(c) Determine a parametrization of the curve that is given by the intersection of the graph of the function f and the plane given by the equation $z = x + 3y$.