

②  $\Sigma$ -square matrix

eigenvalue decomposition

$$\Sigma = U \Lambda U^T$$

$$\begin{aligned} \Sigma^{-1} &= (U \Lambda U^T)^{-1} = U^{-T} \Lambda^{-1} U^{-1} = \{U^T U = I\} = \\ &= U \Lambda^{-1} U^T \end{aligned}$$

$$\begin{aligned} (x-\mu)^T \Sigma^{-1} (x-\mu) &= (x-\mu)^T U \Lambda^{-1} U^T (x-\mu) = \\ &= (x-\mu)^T U \Lambda^{-\frac{1}{2}} \Lambda^{-\frac{1}{2}} U^T (x-\mu) = \\ &= \{ (AB)^T = B^T A^T \} = \underbrace{(U^T \Lambda^{-\frac{1}{2}} (x-\mu))^T}_{A} (U^T \Lambda^{-\frac{1}{2}} (x-\mu)) \end{aligned}$$

$$A = U^T \Lambda^{-\frac{1}{2}} (x-\mu)$$

↳ This is just a linear mapping of the deviation from the mean.

view 1

$$(U^T \Lambda^{-\frac{1}{2}} (x-\mu))^T I (U^T \Lambda^{-\frac{1}{2}} (x-\mu))$$

- The mapping  $U^T \Lambda^{-\frac{1}{2}}$  maps the data to a representation to a space where the covariance is spherical

view 2

$$(U^T (x-\mu))^T \Lambda^{-1} (U^T (x-\mu))$$

- The mapping  $U^T$  maps the data to a representation to a space where the covariance is diagonal.

IMPORTANT: The transformation makes the dimensions independent.