



Seminar 2

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a quiz on a variant of one of the recommended exercises from the text book *Calculus* by Adams and Essex (8th edition) which are marked by boldface in the following list:

Avsnitt	Rekommenderade uppgifter
12.3	5 , 7, 13 , 23
12.4	5 , 7, 11 , 15, 17
12.5	7 , 11, 17 , 21
12.6	3, 5 , 17 , 19
12.7	3 , 5, 13, 17 , 25

In the seminar the following problems will be discussed.

PROBLEMS

Problem 1. When looking at the intersection between two graphs of functions $f(x, y)$ and $g(x, y)$ it could define a curve in three-space. For example the intersection of the graphs of the functions

$$f(x, y) = \sqrt{x^2 + y^2} \quad \text{och} \quad g(x, y) = 6 + x - y$$

gives a *conic section*, C , i.e., a degree two curve in the plane $x - y - z + 6 = 0$.

- Determine the tangent planes of the two graphs in the example at the point $(x, y, z) = (3, 4, 5)$.
- Compute normal vectors of the two graphs at the same point.
- How can we know that the tangent line to the conic section is orthogonal two the normal vectors of both graphs?
- Determine the tangent line to the curve C at the point $(x, y, z) = (3, 4, 5)$, for example by means of the cross product.

Problem 2. The vector values function \mathbf{f} is given by $\mathbf{f}(\mathbf{r}) = \mathbf{v} \times \mathbf{r}$ where \mathbf{v} is a constant vector $\mathbf{v} = (2, -2, 3)$.

- Compute the Jacobi matrix $D\mathbf{f}$.
- Show that \mathbf{f} is a *linear* function.
- What relation does the matrix of \mathbf{f} as a linear map have to the Jacobi matrix $D\mathbf{f}$?

Problem 3. A function $f(x, y)$ is called *harmonic* if it satisfies the Laplace equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

- Use the chain rule to show that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (a^2 + b^2) \left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} \right)$$

if

$$\begin{cases} u = ax - by, \\ v = bx + ay, \end{cases}$$

where a and b are constants.

- Use (a) to show that the function $e^{3x+4y} \sin(4x - 3y)$ is harmonic.
- Show that the product of two harmonic functions is harmonic if their gradients are orthogonal to each other at every point.

Problem 4. The curve C with equation

$$y^2 = x^3 + x^2$$

can be parametrized by $\mathbf{r}(t) = (x(t), y(t)) = (t^2 - 1, t^3 - t)$, where t is a real parameter.

- Sketch the curve C or use a suitable software to plot it.
- Check that $\mathbf{r}(t)$ lies on the curve C for all t . How can we be sure that the parametrization reaches all points of C ?
- Check that $\mathbf{r}'(t)$ is orthogonal to the gradient $\nabla f(\mathbf{r}(t))$ where

$$f(x, y) = y^2 - x^3 - x^2.$$

- Discuss why the vectors in (c) are orthogonal to each other.