



HOMWORK 1

Solving homework problems is an important practice and can improve your grade. It is therefore especially important that your work is:

- legible and written in an understandable way with full sentences and complete arguments,
- original, i.e. not copied or paraphrased from another source, and
- handed in on time.

Submission. The solutions can be hand-written or typed and should be submitted before the deadline, Monday November 9, 2pm. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `boij@kth.se`. If submitted by email, the homework should be in one pdf-file and typed or scanned with high contrast.

Scoring. The maximal total score from all twelve sets of homework is 36 and the total score will be divided by nine and rounded up when counted towards the first part of the final exam. For each set of homework problems, the maximal score is 3 which corresponds to $2/3$ of the points of the problems, i.e., $\min\{3, \Sigma/2\}$.

Problem 1. Let $G = \mathbb{R}_+ \times \mathbb{R} = \{(x, y) : x > 0\} \subseteq \mathbb{R}^2$ and define the binary operation $*$ by

$$(x_1, y_1) * (x_2, y_2) = (x_1 x_2, x_1 y_2 + x_2^{-1} y_1),$$

for (x_1, y_1) and (x_2, y_2) in G .

- (a) Show that G is a group under $*$. (2 p)
- (b) Can we replace $\mathbb{R}_+ \times \mathbb{R}$ by $\mathbb{R}_+ \times V$ where V is any vector space over \mathbb{R} and draw the same conclusion? Explain why or why not. (1 p)

Problem 2. Let G be the dihedral group of order $2n$, where $n > 2$. Solve the system of equations

$$\begin{cases} a^4 = b^5, \\ aba = bab, \end{cases}$$

in G . (3 p)

Problem 3. Let $M_2(\mathbb{Z}_2)$ be the set of 2×2 -matrices with entries in \mathbb{Z}_2 , i.e., the integers modulo 2. Matrix multiplication and matrix inverse satisfy the same rules that we are used to from matrices over the real numbers.

Determine all subsets $G \subseteq M_2(\mathbb{Z}_2)$ that form groups with matrix multiplication as the group operation. (3 p)