Department of Mathematics
SF2729 Groups and Rings
Period 2, 2015

HOMEWORK 2

Solving homework problems is an important practice and can improve your grade. It is therefore especially important that your work is:

- legible and written in an understandable way with full sentences and complete arguments,
- original, i.e. not copied or paraphrased from another source, and
- handed in on time.

Submission. The solutions can be hand-written or typed and should be submitted before the deadline, Monday November 16, 2pm. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `boij@kth.se`. If submitted by email, the homework should be in one pdf-file and typed or scanned with high contrast.

Scoring. The maximal total score from all twelve sets of homework is 36 and the total score will be divided by nine and rounded up when counted towards the first part of the final exam. For each set of homework problems, the maximal score is 3 which correponds to $2/3$ of the points of the problems, i.e., $\min\{3, \Sigma/2\}$.

Problem 1. Show that $\mathrm{Gl}_2(\mathbb{Z}_2) \cong D_6 \cong S_3$ by giving explicit isomorphisms

$$\Phi\colon\ \mathrm{Gl}_2(\mathbb{Z}_2) \xrightarrow{\sim} D_6 \quad \text{and} \quad \Psi\colon D_6 \xrightarrow{\sim} S_3.$$

(Don't forget to verify that the proposed maps are isomorphisms.) **(3 p)**

Problem 2. Let $G \subseteq \mathrm{Gl}_2(\mathbb{Z}_3)$ be the set of upper triangular invertible matrices with entries in $\mathbb{Z}_3$.

(a) Show that the order of G is 12. **(1 p)**
(b) Determine whether G is isomorphic to the cyclic group C_{12}, the dihedral group D_{12} or to the symmetry group of a tetrahedron. **(2 p)**

Problem 3. Let $\mathrm{SL}_2(\mathbb{Z})$ be the group of 2×2-matrices with determinant 1 and with entries in $\mathbb{Z}$ and let $\mathbb{H} \subseteq \mathbb{C}$ be the upper halfplane consisting of all complex numbers with positive imaginary part.

Show that $\mathrm{SL}_2(\mathbb{Z})$ acts on $\mathbb{H}$ by the following action

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot z = \frac{az+b}{cz+d}, \qquad \forall z \in \mathbb{H}, \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathrm{SL}_2(\mathbb{Z}).$$

and compute the kernel of this action. **(3 p)**