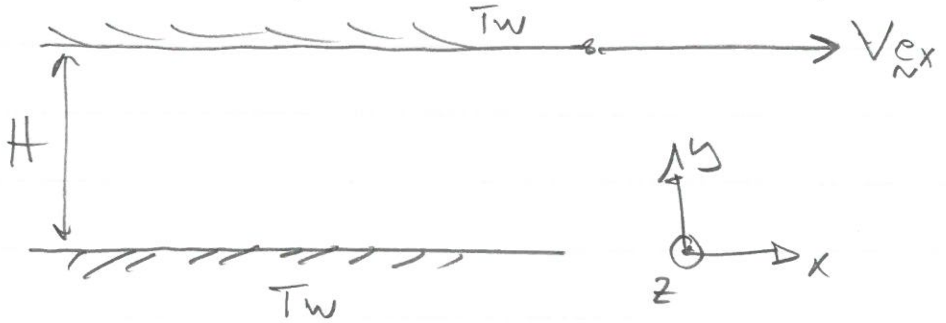
2015-11-10

Example to study for Friday, November 13

Consider flow at large Knudsen number, $Kn = \lambda / H \gg 1$, between two plates.



Both plates have the same temperature, T_w, but different velocities according to the figure.

The boundary condition on the distribution function f requires that molecules colliding with the lower wall accomodates to the temperature of the wall giving a Maxwell distribution for particles with positive y-component of the velocity, $c_y > 0$,

$$f^+ = n_w \beta_w^3 \pi^{-3/2} e^{-\beta_w^2 (c_x^2 + c_y^2 + c_z^2)} \quad ; \quad c_y > 0$$

where $\beta_w^2 = \dfrac{m}{2kT_w}$.

Particles with negative y-component accomodates to the temperature of the upper wall, T_w, and to the velocity of the upper wall $V\underset{\sim}{e}_x$ giving a Maxwell distribution shifted with the mean according to

$$f^- = n_w \beta_w^3 \pi^{-3/2} e^{-\beta_w^2[(c_x - V)^2 + c_y^2 + c_z^2]} \quad ; \; c_y < 0$$

Since at $Kn \gg 1$ no collisions occur between the molecules themselves, the distribution function is given by f^+ & f^- at any position between the plates.

a) Show that the number density of the gas, n, equals the given constant n_w in the expressions for f^+ & f', i.e. $n = n_w$.

b) Calculate the average macroscopic velocity components V_x, V_y and V_z.

c) Introduce the thermal velocity $\underset{\sim}{c}'$

i.e. $c_x' = c_x - V_x$, $c_y' = c_y - V_y$, $c_z' = c_z - V_z$

Calculate the mean thermal energy

$$\frac{m}{2}\overline{c'^2} = \frac{m}{2}\left(\overline{c_x'^2} + \overline{c_y'^2} + \overline{c_z'^2}\right)$$

d) Identify the temperature T of the gas (not the temp. of the walls T_w)

from $\frac{\overline{c'^2}}{3} = \frac{p}{\varrho} = RT = \frac{kT}{m}$ and show

that $T = T_w + \frac{mV^2}{12k}$

e) From your calculations in d) you should be able to identify

$\overline{c_x'^2}$, $\overline{c_y'^2}$ and $\overline{c_z'^2}$.

Identify the kinetic "temperatures" T_x, T_y, T_z from

$\frac{1}{2}kT_x = m\frac{\overline{c_x'^2}}{2}$, $\frac{1}{2}kT_y = m\frac{\overline{c_y'^2}}{2}$, $\frac{1}{2}kT_z = m\frac{\overline{c_z'^2}}{2}$

f) Calculate the macroscopic shear stress τ_{xy}

$\tau_{xy} = -\varrho\overline{c_x' c_y'}$

Integrals you may need:

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2} \quad ; \quad \int_0^\infty x e^{-x^2}\,dx = \frac{1}{2}$$

$$\int_0^\infty x^2 e^{-x^2}\,dx = \frac{\sqrt{\pi}}{4}$$