Probabilistic production cost simulation

EG2200 Lectures 12–14, autumn 2015

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Course objectives

Apply probabilistic production cost simulation to calculate the expected operation cost and risk of power deficit in an electricity market, and to use the results of a simulation to judge the consequences of various actions in the electricity market.
Probabilistic production cost simulation (PPC)

- Development started in the late 1960s.
- Analytical method.
- Limited possibilities to consider the electricity market in detail.
Electricity market model

Assume

• Perfect competition
• Perfect information
• Load is not price sensitive
• Neglect grid losses and limitations
• Neglect start-up costs and ramp rate limitations
• All inputs are independent

(Some of these assumptions can be avoided with clever modelling.)
Calculation of system indices

Basic concept

- Assume all power plants have 100% availability.
- System indices can then be calculated directly from the load duration curve (LDC).

Example

- Known load duration curve.
- Two power plants, 300 MW each, always available, variable operation costs $\beta_1$ and $\beta_2$ respectively, $\beta_1 \leq \beta_2$. 
Calculation of system indices

Basic concept

\[ LOLP = LDC(600) = 876 \text{ h/year} = 10\%. \]
Calculation of system indices

Basic concept

\[
EENS = \int_{600}^{\infty} LDC(x) \, dx.
\]

\[
EENS = \int_{600}^{\infty} LDC(x) \, dx.
\]
Calculation of system indices

Basic concept

\[ ETOC = \beta_1 EG_1 + \beta_2 EG_2 \]

Perfect competition \( \Rightarrow \) Unit 1 is used first as \( \beta_1 \leq \beta_2 \)
Load model

- We only consider the total load of the system.
- The load is assumed to be price insensitive.

⇒

- The load is modelled by one input, $D$.
- The probability distribution of the load is represented by a duration curve.
Load model

How is the load duration curve determined?
The probability distribution of the load cannot be computed but must be estimated from historical data and forecasts for the future.

• **Alternative 1.** Use a standard probability distribution (for example the normal distribution) with the same statistical properties as historical data.

• **Alternative 2.** Create a duration curve directly from historical data (or forecasts).
Load model

Duration curve from historical data

**Definition 6.11.** The load curve states the load at a certain time, i.e.,

\[ D(k) = \text{load hour } k \ [\text{MWh/h}] \]

**Definition 6.12.** The real load duration curve states the load level that is exceeded for a certain time, i.e.,

\[ LDC_R(k) = \text{load level that is exceeded during } k \text{ hours} \ [\text{MWh/h}] \]
Load model

Duration curve from historical data

Exempel 6.9:

The expectation value of the load is equal to the area below the load curve.

\[
E[D] = 12 600 \text{ MWh/day}
\]
Load model

Duration curve from historical data

Exempel 6.9:

\[ D(k), \quad LDC_R(k) \]

The area below the real duration curve is as large as the area below the load curve.

\[ E[D] = 12 600 \text{ MWh/day} \]
Load model

Duration curve from historical data

**Definition 6.13.** The inverted load duration curve states how long time a certain load level is exceeded, i.e.,

\[ LDC(x) = \text{number of hours when the load is exceeding } x \text{ [h]} \].

Dividing \( LDC(x) \) by the length of the studied time period, \( T \), gives the normalised load duration curve.

The normalised duration curve states the probability that a certain load level is exceeded (cf. definition C.3).
Load model

Duration curve from historical data

Exempel 6.10:

We have only switched the $y$ and $x$ axes; hence, the area below the curve has not changed.
Load model

Duration curve from historical data

Exempel 6.10:

Now, the y-axis has been scaled by $1/T$; the expected load is therefore the area below the curve multiplied by $T$. 
Load model

Discrete approximation

• The load is in reality a continuous random variable.
• To compute expected energy etc. we need to integrate the load duration curve, which in general will be done using numerical methods.
• It might therefore be necessary to use a discrete approximation of the probability distribution of the load.
Load model

Discrete approximation
Model of thermal power plants

The following properties are included in the model of thermal power plants:

- **Installed capacity,** $\hat{G}_g$.
  - Ramp rates are neglected.
- **Generation cost,** $C_{Gg}(G_g) = \alpha + \beta G_g$.
  - The variable cost is independent of generation level, i.e., constant efficiency.
  - Start-up costs are neglected.
- **Availability,** $p_g$.

Moreover, it is assumed that failures are independent of the load and the state of other power plants.
Model of thermal power plants

- A thermal power plant is modelled by one input, $\bar{G}_g$ (available generation capacity) and a constant variable operation cost, $\beta_{Gg}$.
- It is assumed that the available generation capacity has a two-point distribution.
Model of thermal power plants

How are the parameters of the two point distribution determined?
The probability distribution of the available generation capacity cannot be computed but must be estimated from historical data and forecasts for the future.
Model of thermal power plants

Availability from historical data

**Definition 6.14.** The Mean Time To Failure is calculated by

\[ MTTF = \frac{1}{K} \sum_{k=1}^{K} t_u(k), \]

where \( K \) are the number of periods when the power plant is available and \( t_u(k) \) is the duration of each of these periods.
Model of thermal power plants

Availability from historical data

**Definition 6.15.** The Mean Time To Repair is calculated by

\[
MTTR = \frac{1}{K} \sum_{k=1}^{K} t_d(k),
\]

where \( K \) are the number of periods when the power plant is unavailable and \( t_d(k) \) is the duration of each of these periods.
Definition 6.16. The failure rate is the probability that an available unit will fail and can be estimated by

$$\lambda = \frac{1}{MTTF}.$$  

Definition 6.16. The repair rate is the probability that an unavailable unit will be repaired and can be estimated by

$$\mu = \frac{1}{MTTR}.$$
Definition 6.18. The availability is the probability that a power plant will be available and can be estimated as the share of a longer time period during which the unit is available:

\[ p = \frac{MTTF}{MTTF + MTTR} = \frac{\mu}{\mu + \lambda}. \]

Definition 6.18. The unavailability is the probability that a power plant will be unavailable and can be estimated by

\[ q = 1 - p = \frac{MTTR}{MTTF + MTTR} = \frac{\lambda}{\mu + \lambda}. \]
Model of thermal power plants

Availability and utilisation

- **Availability.** Probability that a power plant can be used.
- **Capacity factor.** The average available capacity divided by the installed capacity.

\[
\text{capacity factor} = \frac{1}{TG} \sum_{t=1}^{T} \bar{G}_t.
\]

- **Utilisation factor.** The average real generation divided by the installed capacity.

\[
\text{utilisation factor} = \frac{1}{TG} \sum_{t=1}^{T} G_t \leq \text{capacity factor}.
\]

- **Full load hours.** The number of hours a unit would be able generate its full capacity without exceeding the real annual generation.

\[
\text{full load hours} = \frac{1}{G} \sum_{t=1}^{T} G_t = T \cdot \text{utilisation factor}.
\]
Equivalent load

- We have seen that it is straightforward to compute the system indices in a power system where all power plants have 100% availability.
- What do we do if this is not the case?
- **Brilliant idea:** Consider outages in power plants as load increase instead of lower available generation capacity.
Definition 6.14. The equivalent load is given by

\[ E_g = D + \sum_{k=1}^{g} O_k, \]

where

\[ E_g = \text{equivalent load seen by power plant } g+1, \]
\[ D = \text{real load}, \]
\[ O_k = \text{outage in power plant } k. \]
Equivalent load

Example

![Graphs](image-url)
Calculation of system indices

• The system indices $LOLP$, $EENS$ and $ETOC$ can be computed from the equivalent load duration curve, $\tilde{F}_g(x)$.

• Introduce the symbols

\[
\bar{G}_g^{\text{tot}} = \sum_{k=1}^{g} \bar{G}_k, \quad \hat{G}_g^{\text{tot}} = \sum_{k=1}^{g} \hat{G}_k \quad \text{and} \quad \bar{O}_g^{\text{tot}} = \sum_{k=1}^{g} \bar{O}_k.
\]
Calculation of system indices

Risk of power deficit

- Power deficit occurs when the load is higher than the available generation capacity, i.e.,

\[
LOLP_g = P(D > \overline{G}_g^{\text{tot}}) = P(D > \hat{G}_g^{\text{tot}} - O_g^{\text{tot}}) = \\
P(D + O_g^{\text{tot}} > \hat{G}_g^{\text{tot}}) = P(E_g > \hat{G}_g^{\text{tot}}) = \tilde{F}_g(\hat{G}_g^{\text{tot}}).
\]
Calculation of system indices

Unserved energy

- Everything to the right of $\hat{G}_g^{tot}$ in the equivalent load duration curve corresponds to load that cannot be served due to power deficit.

\[ EENS_g = T \int_{\hat{G}_g^{tot}}^{\infty} \tilde{F}_g(x) dx. \]

- Remember to scale by the time period considered—see page 16!
Calculation of system indices

Expected generation

• Remember that in the equivalent load we are assuming that all power plants are always available and add load instead.
• The expected generation can therefore not be computed directly from the equivalent load duration curve.
• However, it can be computed indirectly by comparing the unserved energy with and without a power plant:

\[ EG_g = EENS_{g-1} - EENS_g. \]
Calculation of system indices

Expected generation

• For two-state generating units, the expected generation of power plant $g$ can be computed directly from $F_{g-1}(x)$:

$$EG_g = T \cdot p_g \int \tilde{F}_{g-1}(x) dx.$$
Calculation of system indices

Expected total operation cost

• The expected total operation cost is computed from the expected generation in each power plant:

\[
ETOC_g = \sum_{k = 1}^{g} \beta_k E G_k.
\]

• If there is a cost for load that is disconnected then that can be included as well:

\[
ETOC_g = \sum_{k = 1}^{g} \beta_k E G_k + \beta_{UEENS_g}.
\]
Calculation of system indices

Example

- Run-of-the river hydro (500 kW, 0 €/kWh, 100% availability).
- Diesel generator set (200 kW, 1 €/kWh, 90% availability).
- Load ($N(400,80)$-distributed, not price sensitive).
Calculation of system indices

Example

\[
EENS_1 = 1 \cdot \int_{0}^{\infty} \tilde{F}_1(x) \, dx = 3.89 + 0.16 = 4.05 \text{ kWh/h.}
\]

\[
= \frac{1}{500} \cdot \int_{0}^{\infty} \tilde{F}_1(x) \, dx = 3.89 + 0.16 = 4.05 \text{ kWh/h.}
\]
Calculation of system indices

Example

\[ EENS_2 = 1 \cdot \int \tilde{F}_2(x) \, dx = 0.39 + 0.02 = 0.41 \text{ kWh/h.} \]

\[ 700 \]
Calculation of system indices

Example

\[ \text{LOLP} = \tilde{F}_2(700) \approx 1\%. \]

\[ EG_2 = EENS_1 - EENS_2 = 4.05 - 0.41 = 3.64 \text{ kWh/h}. \]

\[ ETOC = 1 \cdot EG_2 = 3.64 \text{ €/h}. \]
Equivalent load duration curve

Derivation

- \( \tilde{F}_g(x) = P(E_g > x) = P(D + O_1 + \ldots + O_{g-1} + O_g > x) \).
- The probability distribution of a sum of independent random variables can be computed using convolution.
- The formula for computing equivalent load duration curves can also be understood as follows:
  - Assume that the equivalent load duration curve without power plant \( g \), \( \tilde{F}_{g-1}(x) \), is known.
  - What is the probability that the equivalent load exceeds \( x \) if power plant \( g \) is available?
  - What is the probability that the equivalent load exceeds \( x \) if power plant \( g \) is unavailable?
  - Combine the results.
Equivalent load duration curve

Derivation

Power plant $g$ is available

- $E_g = D + O_1 + ... + O_{g-1} + O_g$.
- $O_g = 0 \implies E_g = D + O_1 + ... + O_{g-1} = E_{g-1}$.
- Hence,

$$P(E_g > x) \big|_{\bar{G}_g = \hat{G}_g} = P(E_{g-1} > x) = \tilde{F}_{g-1}(x).$$
Equivalent load duration curve

Derivation

Power plant $g$ is unavailable

- $E_g = D + O_1 + \ldots + O_{g-1} + O_g$.
- $O_g = \hat{G}_g \Rightarrow E_g = D + O_1 + \ldots + O_{g-1} + \hat{G}_g = E_{g-1} + \hat{G}_g$.
- Hence,

$$
P(E_g > x) \bigg|_{\hat{G}_g = 0} = P(E_{g-1} + \hat{G}_g > x) =$$

$$= P(E_{g-1} > x - \hat{G}_g) = \tilde{F}_{g-1}(x - \hat{G}_g).$$
Equivalent load duration curve

Derivation

• The probability that the equivalent load exceeds $x$ is then

$$P(E_g > x) = p_g \cdot P(E_g > x) \bigg|_{\bar{G}_g = \hat{G}_g} + q_g \cdot P(E_g > x) \bigg|_{\bar{G}_g = 0} =$$

$$= \tilde{F}_g(x) = p_g \tilde{F}_{g-1}(x) + q_g \tilde{F}_{g-1}(x - \hat{G}_g).$$

• The formula above is an example of a convolution formula.

• Starting from the load duration curve, $\tilde{F}_0(x)$ (which is an input to the simulation), the equivalent load duration curves can be computed by adding one power plant at a time.

• Power plants should be added according to merit order, i.e., according to increasing variable costs (as we assume perfect competition).
Probabilistic production cost simulation
Example I

Problem description

• Load
  - 200 kWh/h Mon–Fri 5–19, Sat–Sun 10–17
  - 100 kWh/h all other times

• Power plants
  - Diesel generator set 150 kWh/h, 90% availability, 10 ¤/kWh.
  - Diesel generator set 100 kWh/h, 80% availability, 12 ¤/kWh.

• Compute $ETOC$ and $LOLP$. 
Probabilistic production cost simulation

Example I

Solution

• The frequency function of the load can be computed from the given data:

\[ f_0(x) = P(D = x) = \begin{cases} 
84/168 = 0.5 & x = 100, \\
84/168 = 0.5 & x = 200, \\
0 & \text{all other } x.
\end{cases} \]

• From theorem C.4 we then have

\[ \tilde{F}_0(x) = \sum_{t > x} f_0(t) = \begin{cases} 
0.5 + 0.5 = 1 & x < 100, \\
0.5 & 100 \leq x < 200, \\
0 & 200 \leq x.
\end{cases} \]
Probabilistic production cost simulation

Example 1

\[ EENS_0 = 1 \cdot \int_0^\infty \tilde{F}_0(x) \, dx = 100 \cdot 1 + 100 \cdot 0.5 = 150 \text{ kWh/h}. \]
The larger diesel generator set is first in the merit order:

\[
\tilde{F}_1(x) = 0.9 \cdot \tilde{F}_0(x) + 0.1 \cdot \tilde{F}_0(x - 150) =
\begin{align*}
0.9 \cdot 1 + 0.1 \cdot 1 &= 1 & x < 100, \\
0.9 \cdot 0.5 + 0.1 \cdot 1 &= 0.55 & 100 \leq x < 200, \\
0.9 \cdot 0 + 0.1 \cdot 1 &= 0.1 & 200 \leq x < 250, \\
0.9 \cdot 0 + 0.1 \cdot 0.5 &= 0.05 & 250 \leq x < 350, \\
0 &= 0 & 350 \leq x.
\end{align*}
\]
Probabilistic production cost simulation

Example I

\[
EENS_1 = 1 \cdot \int \tilde{F}_1(x) \, dx = \frac{50 \cdot 0.55 + 50 \cdot 0.1 + 100 \cdot 0.05}{150} = 37.5 \text{ kWh/h.}
\]
Probabilistic production cost simulation

Example I

Next, we add the smaller diesel generator set:

\[
\tilde{F}_2(x) = 0.8 \cdot \tilde{F}_1(x) + 0.2 \cdot \tilde{F}_1(x - 100) =
\begin{align*}
0.8 \cdot 1 + 0.2 \cdot 1 &= 1 & x < 100, \\
0.8 \cdot 0.55 + 0.2 \cdot 1 &= 0.64 & 100 \leq x < 200, \\
0.8 \cdot 0.1 + 0.2 \cdot 0.55 &= 0.19 & 200 \leq x < 250, \\
0.8 \cdot 0.05 + 0.2 \cdot 0.55 &= 0.15 & 250 \leq x < 300, \\
0.8 \cdot 0.05 + 0.2 \cdot 0.1 &= 0.06 & 300 \leq x < 350, \\
0.8 \cdot 0 + 0.2 \cdot 0.55 &= 0.01 & 350 \leq x < 450, \\
0 &= 0 & 450 \leq x.
\end{align*}
\]
Probabilistic production cost simulation

Example I

\[ EENS_2 = 1 \cdot \int_{\mathbb{R}} \tilde{F}_2(x) \, dx = \frac{50 \cdot 0.15 + 50 \cdot 0.06 + 100 \cdot 0.01}{250} = 11.5 \text{ kWh/h.} \]
Probabilistic production cost simulation

Example I

\[
EG_1 = EENS_0 - EENS_1 = 112.5 \text{ kWh/h}
\]

\[
EG_2 = EENS_1 - EENS_2 = 26.0 \text{ kWh/h}
\]

\[
ETOC = 10EG_1 + 12EG_2 = 1437 \text{ €/h}
\]

\[
LOLP = \tilde{F}_2(250) = 15\%
\]
Model of wind power

The following properties are included in the model of wind power plants:

- Installed capacity, $\hat{W}_n$.
- Generation cost, $C_{Wn}(W_n) = \alpha + \beta W_n$.
  - The variable cost is generally negligible.
- Availability, $p$.
- Wind speed distribution, $\tilde{F}_v$.

Moreover, it is assumed that failures and wind speed are independent of each other as well as of the load and the state of other power plants.
Model of wind power

• A wind power plant or a wind farm (i.e., a group of wind power plants) can be represented by a single input, $\bar{W}_n$ (available generation capacity) and a constant variable operation cost, $\beta_{Wn}$ (which can generally be assumed as zero).

• The density function for $\bar{W}_n$ can be computed by combining the probability distribution of the wind speed by the power curve (the wind power generation as a function of wind speed), i.e., $W_n(v)$.
  - See the course compendium for details.
Model of wind power

- The probability distribution of the wind speed must be estimated from historical data or forecasts (similar to the probability distribution of the load).
- A standard probability distribution that is commonly used for wind speeds is the Rayleigh distribution.

\[ \mu_v = \text{mean wind speed} \]
Model of wind power

- The power curve depends on the type of wind power plant.
Model of wind power

Discrete approximation

- The available wind power generation capacity is a continuous random variable.
- To apply the convolution formula it is preferable to have a discrete probability distribution.
- It is therefore preferable to use a discrete approximation of the probability distribution of the available wind power generation capacity.
Model of wind power

Discrete approximation
Equivalent load duration curve

General formula

- The convolution formula for the equivalent load duration curve can be generalised as follows:

\[
\tilde{F}_g(x) = \sum_{i=1}^{N_g} p_{g,i} \tilde{F}_{g-1}(x - x_{g,i}),
\]

where

\( N_g \) = number of states for power plant \( g \),

\( p_{g,i} = f_{\hat{G}_g} (G_g) = f_{\hat{G}_g} (\hat{G}_g - x_{g,i}) \) = probability of state \( i \),

\( x_{g,i} \) = outage (compared to installed capacity) in state \( i \).
Equivalent load duration curve

General formula

Verification for thermal power plants:

\( N_g = 2 \)

\( p_{g,1} = p_g, x_{g,1} = 0 \)

\( p_{g,2} = q_g, x_{g,2} = \hat{G}_g \)

Using these values in the general formula for the equivalent load duration curve yields

\[
\tilde{F}_g(x) = \sum_{i=1}^{N_g} p_g, i \tilde{F}_{g-1}(x - x_{g,i}) =
= p_g \tilde{F}_{g-1}(x) + q_g \tilde{F}_{g-1}(x - \hat{G}_g),
\]
i.e., the same expression as we used earlier.
Probabilistic production cost simulation

Example II

Problem description

• Load
  - 200 kWh/h Mon–Fri 5–19, Sat–Sun 10–17
  - 100 kWh/h all other times

• Power plants
  - Diesel generator set 150 kWh/h, 90% availability, 10 $/kWh.
  - Diesel generator set 100 kWh/h, 80% availability, 12 $/kWh.
  - Wind power plant 100 kWh/h (20%), 50 kWh/h (30%), 0 kWh/h (50%), negligible variable costs.

• Compute $ETOC$ and $LOLP$. 
Probabilistic production cost simulation

Example II

Solution

- The load duration curve, \( \tilde{F}_0(x) \), is the same as in example I.
- The first power plant in the merit order is now the wind power plant.
- Model of the wind power plant:
  - \( N_1 = 3 \)
  - \( p_{1,1} = 0.2, x_{1,1} = 0 \)
  - \( p_{1,2} = 0.3, x_{1,2} = 50 \)
  - \( p_{1,3} = 0.5, x_{1,3} = 100 \)
Probabilistic production cost simulation

Example II

\[ \tilde{F}_1(x) = 0.2 \cdot \tilde{F}_0(x) + 0.3 \cdot \tilde{F}_0(x - 50) + 0.5 \cdot \tilde{F}_0(x - 100) = \]
\[
\begin{align*}
0.2 \cdot 1 + 0.3 \cdot 1 + 0.5 \cdot 1 &= 1 & x < 100, \\
0.2 \cdot 0.5 + 0.3 \cdot 1 + 0.5 \cdot 1 &= 0.9 & 100 \leq x < 150, \\
0.2 \cdot 0.5 + 0.3 \cdot 0.5 + 0.5 \cdot 1 &= 0.75 & 150 \leq x < 200, \\
0.2 \cdot 0 + 0.3 \cdot 0.5 + 0.5 \cdot 0.5 &= 0.4 & 200 \leq x < 250, \\
0.2 \cdot 0 + 0.3 \cdot 0 + 0.5 \cdot 0.5 &= 0.25 & 250 \leq x < 300, \\
0 &= 0 & 300 \leq x.
\end{align*}
\]
Probabilistic production cost simulation

Example II

$$EENS_1 = 1 \cdot \int_{100}^{\infty} \tilde{F}_1(x) \, dx = \frac{50 \cdot 0.9 + 50 \cdot 0.75 + 50 \cdot 0.4 + 50 \cdot 0.25}{100} = 115 \text{ kWh/h}.$$
Probabilistic production cost simulation

Example II

\[ \tilde{F}_2(x) = 0.9 \cdot \tilde{F}_1(x) + 0.1 \cdot \tilde{F}_1(x - 150) = \]

\[
\begin{align*}
  &0.9 \cdot 1 + 0.1 \cdot 1 = 1 & x < 100, \\
  &0.9 \cdot 0.9 + 0.1 \cdot 1 = 0.91 & 100 \leq x < 150, \\
  &0.9 \cdot 0.75 + 0.1 \cdot 1 = 0.775 & 150 \leq x < 200, \\
  &0.9 \cdot 0.4 + 0.1 \cdot 1 = 0.46 & 200 \leq x < 250, \\
  &0.9 \cdot 0.25 + 0.1 \cdot 0.9 = 0.315 & 250 \leq x < 300, \\
  &0.9 \cdot 0 + 0.1 \cdot 0.75 = 0.075 & 300 \leq x < 350, \\
  &0.9 \cdot 0 + 0.1 \cdot 0.4 = 0.04 & 350 \leq x < 400, \\
  &0.9 \cdot 0 + 0.1 \cdot 0.25 = 0.025 & 400 \leq x < 450, \\
  &0 & 450 \leq x.
\end{align*}
\]
Probabilistic production cost simulation

Example II

\[ \tilde{F}_0(x), \tilde{F}_1(x), \tilde{F}_2(x) \]

\[
EENS_2 = 1 \cdot \int_{-\infty}^{\infty} \tilde{F}_2(x) dx = \frac{1}{250} \\
= 50 \cdot 0.315 + 50 \cdot 0.075 + 50 \cdot 0.04 + 50 \cdot 0.025 = 22.75 \text{ kWh/h.}
\]
Probabilistic production cost simulation

Example II

\[ \tilde{F}_3(x) = 0.8 \cdot \tilde{F}_2(x) + 0.2 \cdot \tilde{F}_2(x - 100) = \]

\[
\begin{cases}
0.8 \cdot 1 + 0.2 \cdot 1 = 1 & x < 100, \\
0.8 \cdot 0.91 + 0.2 \cdot 1 = 0.928 & 100 \leq x < 150, \\
0.8 \cdot 0.775 + 0.2 \cdot 1 = 0.82 & 150 \leq x < 200, \\
0.8 \cdot 0.46 + 0.2 \cdot 0.91 = 0.55 & 200 \leq x < 250, \\
0.8 \cdot 0.315 + 0.2 \cdot 0.775 = 0.407 & 250 \leq x < 300, \\
\vdots & \vdots
\end{cases}
\]
Probabilistic production cost simulation

Example II

\[
0.8 \cdot 0.075 + 0.2 \cdot 0.46 = 0.152 \\
0.8 \cdot 0.04 + 0.2 \cdot 0.315 = 0.095 \\
0.8 \cdot 0.025 + 0.2 \cdot 0.075 = 0.035 \\
0.8 \cdot 0 + 0.2 \cdot 0.04 = 0.008 \\
0.8 \cdot 0 + 0.2 \cdot 0.025 = 0.005 \\
0
\]

\[\begin{align*}
&= \begin{cases}
0.152 & \text{if } 300 \leq x < 350, \\
0.095 & \text{if } 350 \leq x < 400, \\
0.035 & \text{if } 400 \leq x < 450, \\
0.008 & \text{if } 450 \leq x < 500, \\
0.005 & \text{if } 500 \leq x < 550, \\
0 & \text{if } 550 \leq x.
\end{cases}
\end{align*}\]
Probabilistic production cost simulation

Example II

\[
\tilde{F}_0(x), \tilde{F}_1(x), \tilde{F}_2(x), \tilde{F}_3(x)
\]

\[
EENS_3 = 1 \cdot \int_{350}^{\infty} \tilde{F}_3(x)dx =
\]

\[
= 50 \cdot 0.095 + 50 \cdot 0.035 + 50 \cdot 0.008 + 50 \cdot 0.005 = 7.15 \text{ kWh/h.}
\]
Probabilistic production cost simulation

Example II

\[ EG_2 = EENS_1 - EENS_2 = 92.25 \text{ kWh/h} \]

\[ EG_3 = EENS_2 - EENS_3 = 15.6 \text{ kWh/h} \]

\[ ETOC = 10EG_2 + 12EG_3 = 1109.7 \text{ €/h} \]

\[ LOLP = F_3(350) = 9.5\% \]
Wind power

Correlation between wind power generation and load

• Available wind power generation, $\bar{W}$, and demand, $D$, are both depending on weather conditions and might thus be correlated.

• **Annual variations.** Some seasons are more windy and colder than others.
  - Cold weather $\Rightarrow$ low load in warm countries.
  - Warm weather $\Rightarrow$ high load in cold countries.

• **Daily variations.** More windy and lower load during night time.

• **Special variations.** Windy and cold $\Rightarrow$ increased load in buildings with electric heating.
Example: Daily variations

Problem

• Day time 7:00-21:00, night time 21:00-7:00.
• Load
  - Day time: 250 kW (10%), 200 kW (30%), 150 kW (30%), 100 kW (30%)
  - Night time: 150 kW (20%), 100 kW, (40%), 50 kW (40%)
• Diesel generator set 200 kW, 95% availability
• Wind power
  - Day time: 200 kW (30%), 100 kW (40%), 0 kW (30%)
  - Night time: 200 kW (40%), 100 kW (50%), 0 kW (10%)
Example: Daily variations

Solution

\[ \tilde{F}_0, \tilde{F}_0 \]

\[ \tilde{F}_\text{day}, \tilde{F}_\text{night} \]
Example: Daily variations

Solution

• Compute $LOLP$ for day time:

\[
LOLP_{\text{day}} = \tilde{F}_2(400) = 0.95\tilde{F}_1(400) + 0.05\tilde{F}_1(400 - 200)
\]

\[
= 0.95(0.3\tilde{F}_0(400) + 0.4\tilde{F}_0(400 - 100) + 0.3\tilde{F}_0(400 - 200))
\]

\[
+ 0.05(0.3\tilde{F}_0(200) + 0.4\tilde{F}_0(200 - 100) + 0.3\tilde{F}_0(200 - 200))
\]

\[
= 0.95(0.3\cdot 0 + 0.4\cdot 0 + 0.3\cdot 0.1) + 0.05(0.3\cdot 0.1 + 0.4\cdot 0.7 + 0.3\cdot 1)
\]

\[
= 0.059 = 5.9\%.
\]
Example: Daily variations

Solution

• Compute $LOLP$ for night time:

$$LOLP_{night} = \ldots = 2.5\%.$$ 

• To compute the system $LOLP$ regardless of time, the results above are weighted according to the duration of the corresponding time period:

$$LOLP = \frac{14}{24}LOLP_{day} + \frac{10}{24}LOLP_{night} = 4.5\%.$$