Department of Mathematics

SF1626
Several Variable Calculus
Academic year 2015/2016, Period 2

Seminar 3

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a hand-in of one of the problems. Solve problems 1-4 below and write down your solutions on separate sheets. Write your name and personal number on the top of each page. When the seminar starts you will be informed about which problem to hand in. Before starting with the seminar problems you should solve the recommended exercises from the text book Calculus by Adams and Essex (8th edition). These exercises are:

Section	Recommended problems
12.8	13, 17
12.9	1, 3, 5, 7, 11
13.1	5, 7, 9, 19, 23, 25
13.2	3, 5, 9, 15
13.3	3, 9, 11, 15
13.4	1, 3

PROBLEMS

Problem 1. Let f be the function given by $f(x, y) = x^2y^3 + x^2 - 4y^3$ for all (x, y) in $\mathbb{R}^2$.

(a) Determine all critival points of f.
(b) Determine the degree two Taylor polynomial of f at all critival points.
(c) Determine the type of each critical point.

Problem 2. The function f is given by

$$f(x, y) = (\sin x + \sin y)^2$$

for all (x, y) in $\mathbb{R}^2$.

(a) Determine all critival points of f.

(b) Draw the critical points of f together with the level curve $f(x, y) = 0$. Mark which of the critical points that are local maxima, which of them that are local minima and which of them that are neither.

Problem 3. Consider the problem of finding the maximal and minimal value of the function given by $f(x, y, z) = 2x - 2y + z$ under the conditions that $x^2 + y^2 + z^2 \leq 2$ and $x^2 + y^2 \leq z$.

(a) Sketch the domain D given by the two conditions.

(b) Search for critical points of f in the interior of D.

(c) Look for possible extreme values on the boundary of D by parametrization of the boundary.

(d) Look for possible extreme values on the boundary of D by means of Lagrange's method.

(e) Draw a conclusion about the maximal and minimial values for f. How can we be sure that the method leads us to the correct answer?

Problem 4. Consider the function f given by

$$f(x, y) = \frac{1 + 3xy}{1 + x^2 + y^2}$$

for all (x, y) in the closed circular disk C with radius 2 and center in $(x, y) = (0, 0)$ in $\mathbb{R}^2$.

(a) Determine all inner critical points of f.

(b) Use a parametrization of the boundary of C to determine the extre values of f on the boundary.

(c) Use Lagrange's method to find candidates for extreme values on the boundary of C.

(d) Use the results from (a), (b) and (c) to determine the maximum and minimum value for f in C.