Royal Institute of Technology

MACHINE LEARNING 2 – DGM, CH 8

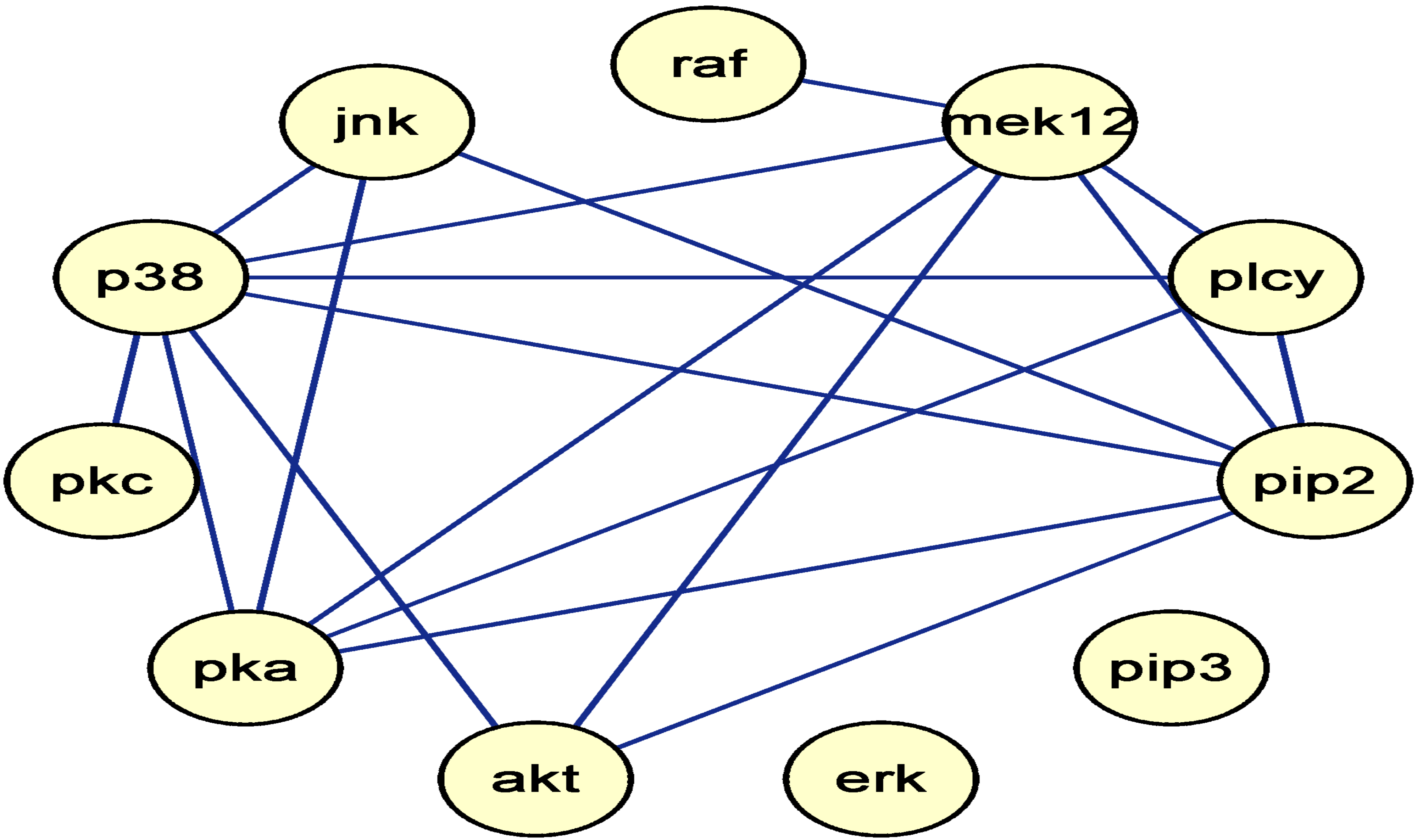
Lecture 6

Royal Institute of Technology

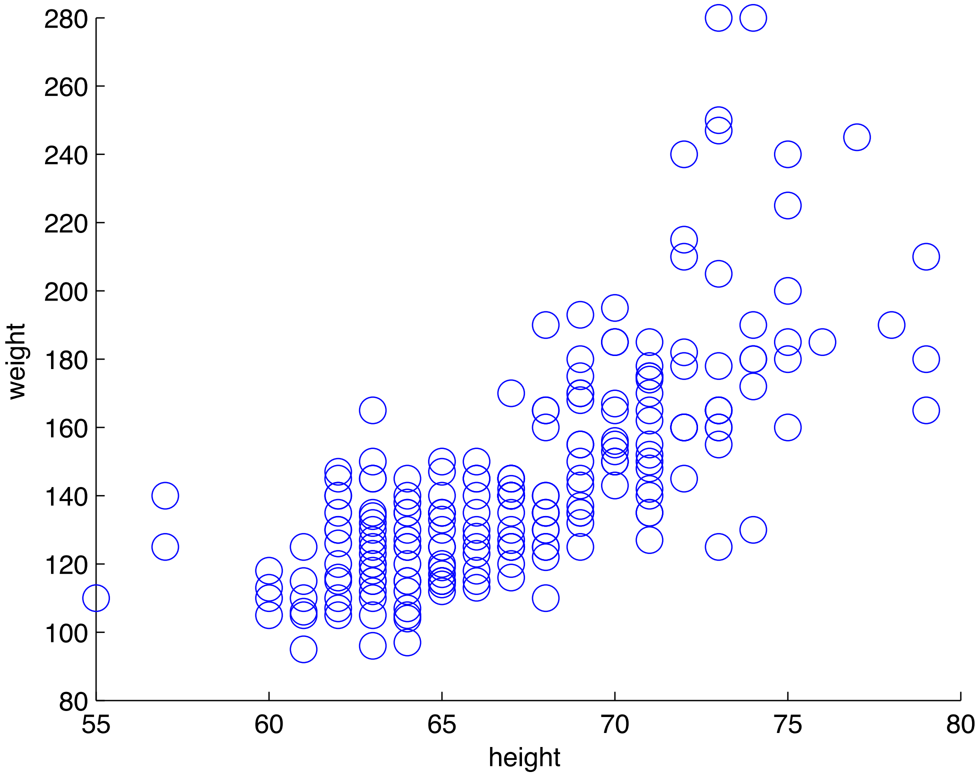
Computational Biology

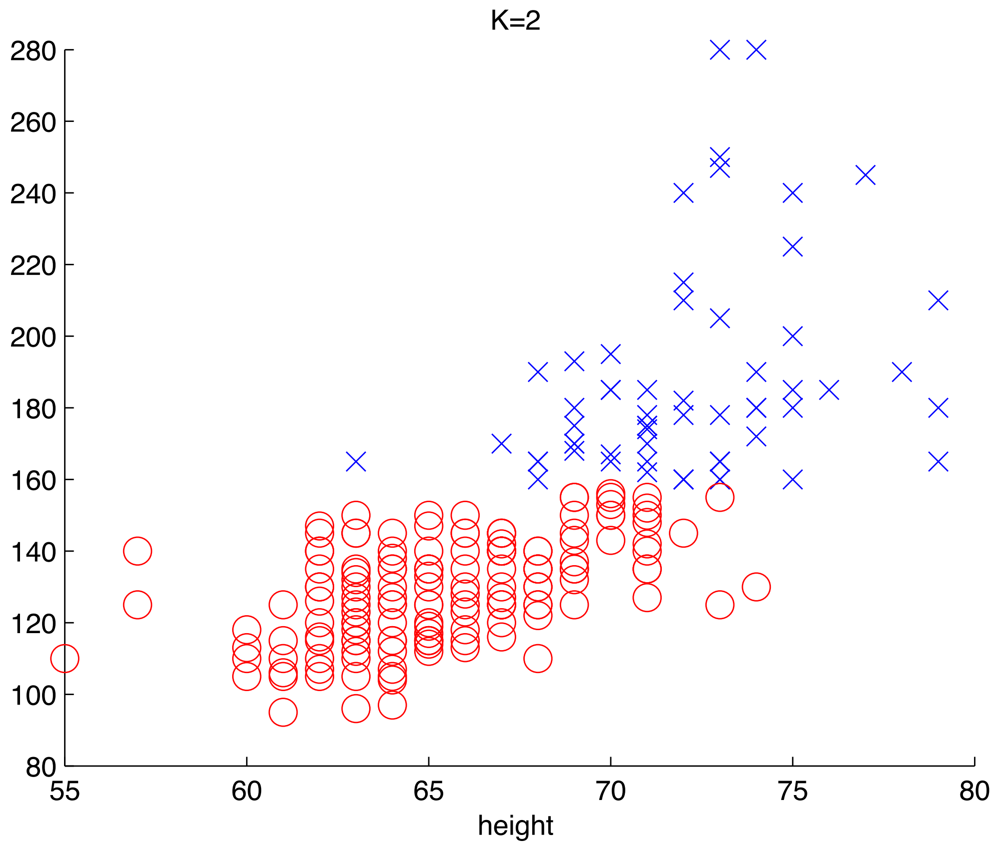
Machine Learning – a main tool

Jens Lagergren



PROTEIN-PROTEIN INTERACTION NETWORK





- Each subset should contain similar points
- Pairs of subsets should have dissimilar points.

CLUSTERING

K-MEANS

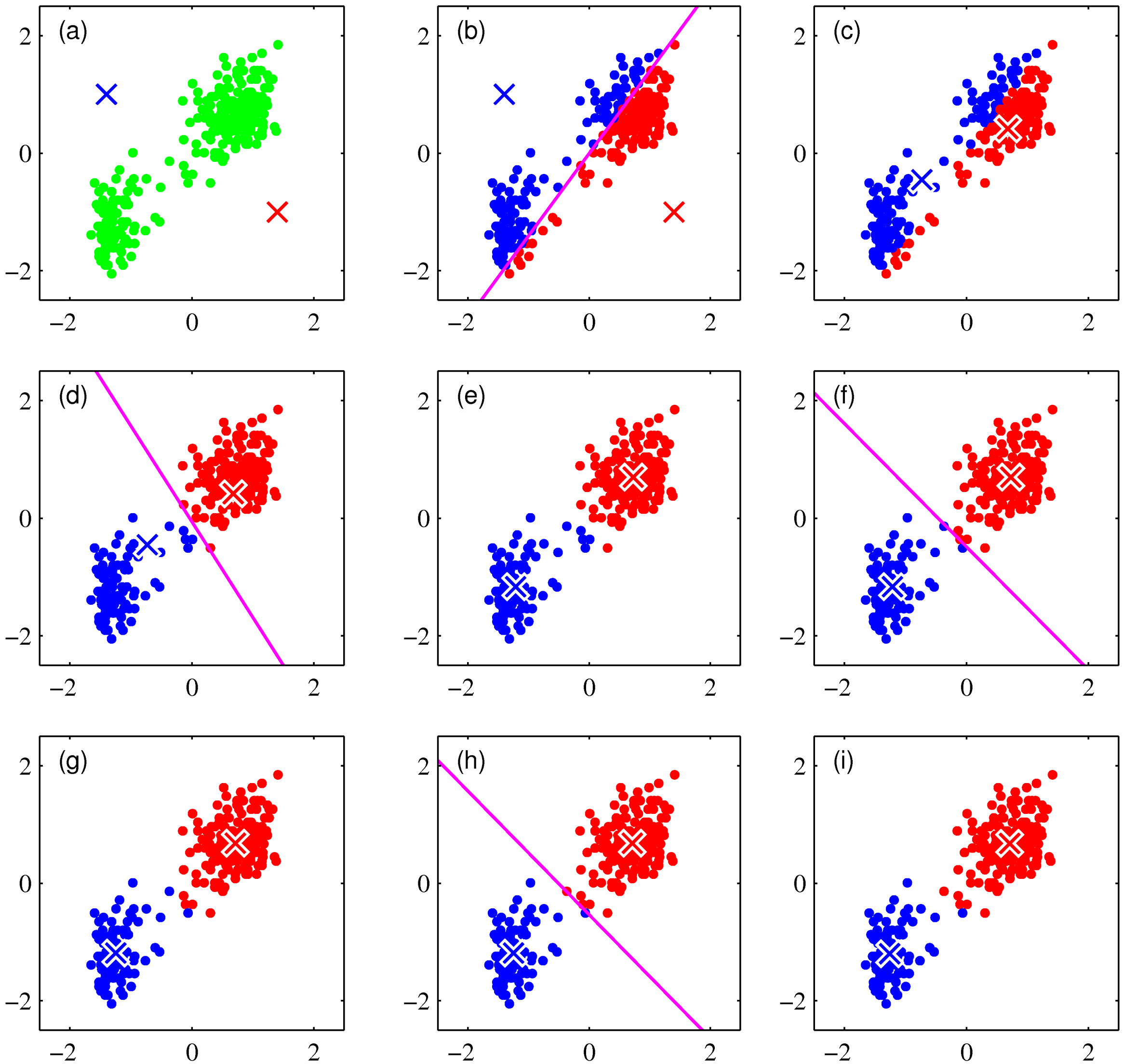
- Data vectors $D=\{x_1,\ldots,x_N\}$
- Randomly selected classes $z_1,\ldots,z_N$
- Iteratively do

$$\boldsymbol{\mu}_c = \frac{1}{N_c} \sum_{n:z_n=c} \boldsymbol{x}_n, \qquad \text{where } N_c = |\{n : z_n = c\}|$$

$$z_n = \text{argmin}_c ||\boldsymbol{x}_n - \boldsymbol{\mu}_c||_2$$

- One step O(NKD), can be improved

ASSIGN POINT TO MEANS



THIS LECTURE

- Probability?
- DGM
- Basic definitions
- Examples
- Learning parameters - given complete data
- Illustrating a known model

BERNOULLI AND CATEGORICAL

$$\mathrm{Ber}(x|\theta) = \begin{cases} \theta & \text{if } x = 1 \\ 1-\theta & \text{if } x = 0 \end{cases}$$

$$\mathrm{Cat}(x|\boldsymbol{\theta}) = \theta_x$$

- ★ One or several (unordered) coin tosses
- ★ A dice (possibly biased)

PRODUCT RULE: CONDITIONING

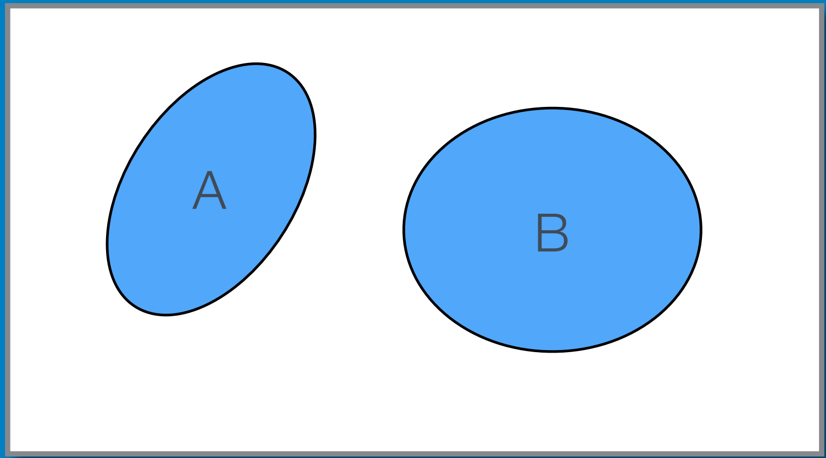
$$p(x, y) = p(y)p(x|y)$$

or

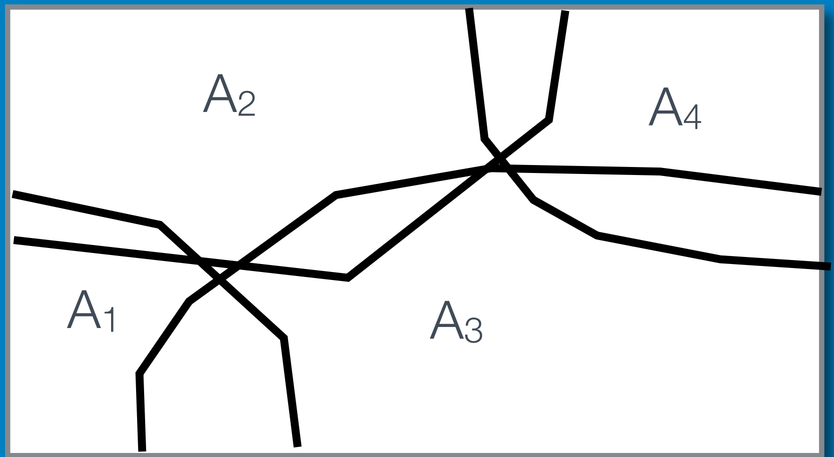
$$p(x|y) = \frac{p(x, y)}{p(y)}$$

EXCLUSIVE & EXHAUSTIVE

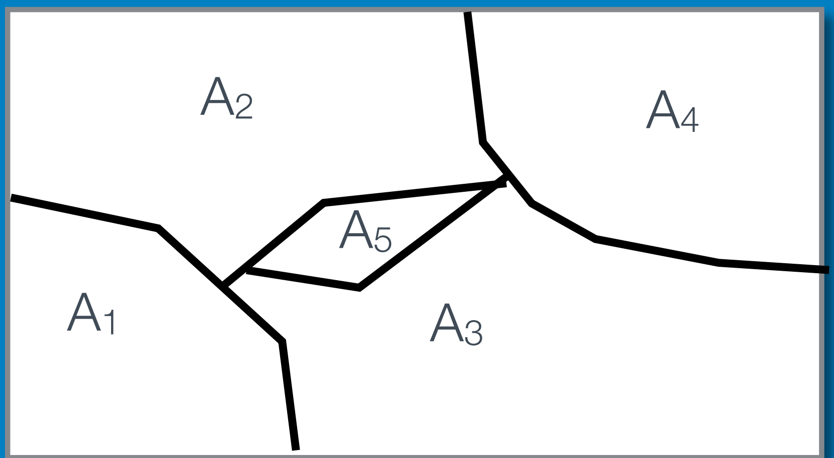
Exclusive



Exhaustive



Exclusive & exhaustive



- Exclusive

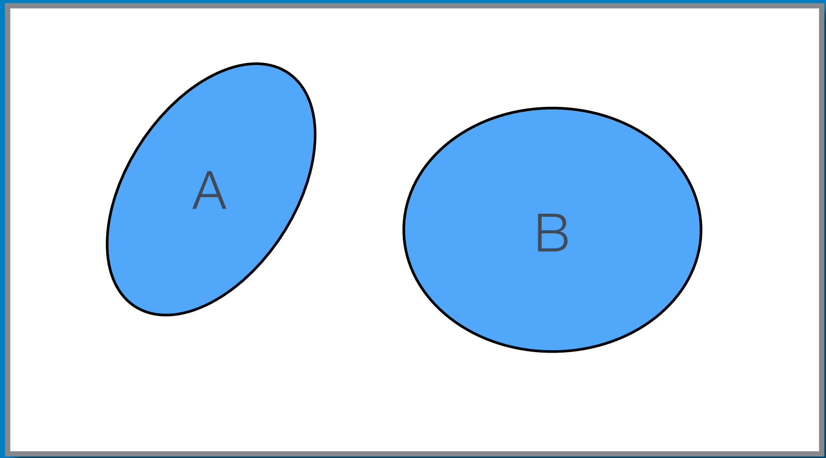
$$p(A \text{ or } B) = p(A) + P(B)$$

- Exclusive & exhaustive

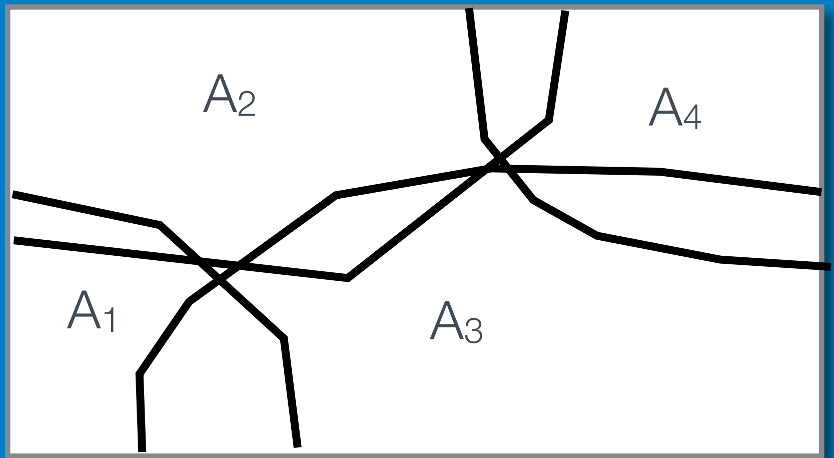
$$\sum_i p(A_i) = 1$$

SUM RULE: EXCLUSIVE & EXHAUSTIVE

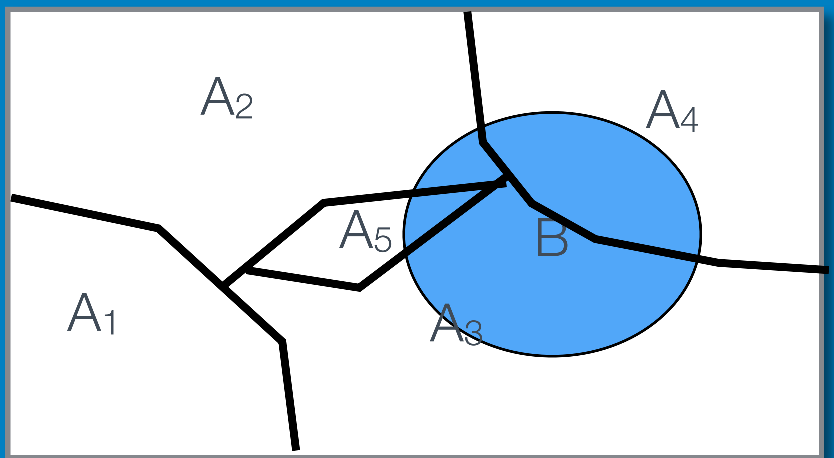
Exclusive



Exhaustive



Exclusive & exhaustive



- Exclusive

$$p(A \text{ or } B) = p(A) + P(B)$$

- Exclusive & exhaustive

$$p(B) = \sum_i p(B, A_i) = \sum_i p(A_i)p(B|A_i)$$

BAYES RULE

$$p(X|Y) = \frac{p(X,Y)}{p(Y)} = \frac{p(X)p(Y|X)}{\sum_x p(x)p(Y|x)}$$

A TABLE

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

A TABLE

Table A-1 The Standard Normal Distribution

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
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1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
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1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
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2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

VISUAL ACCESSIBILITY

Table A-1 The Standard Normal Distribution

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
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1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
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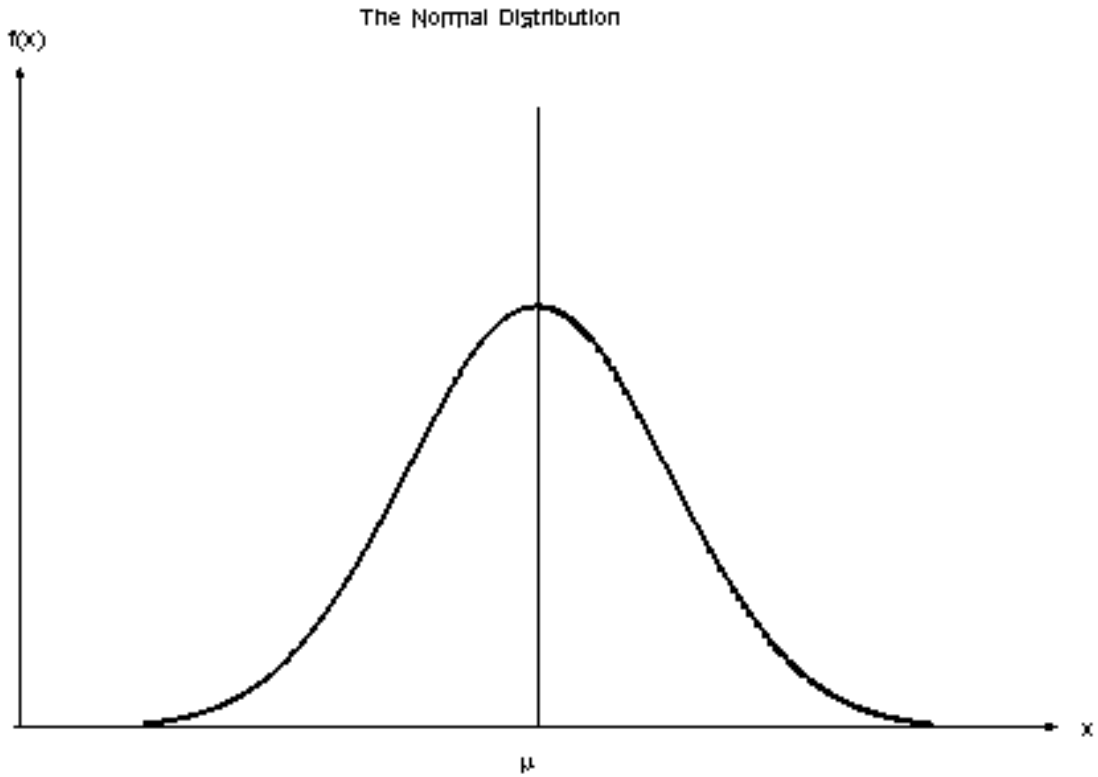
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010



MATHEMATICAL TREATMENT

Table A-1 The Standard Normal Distribution

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1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
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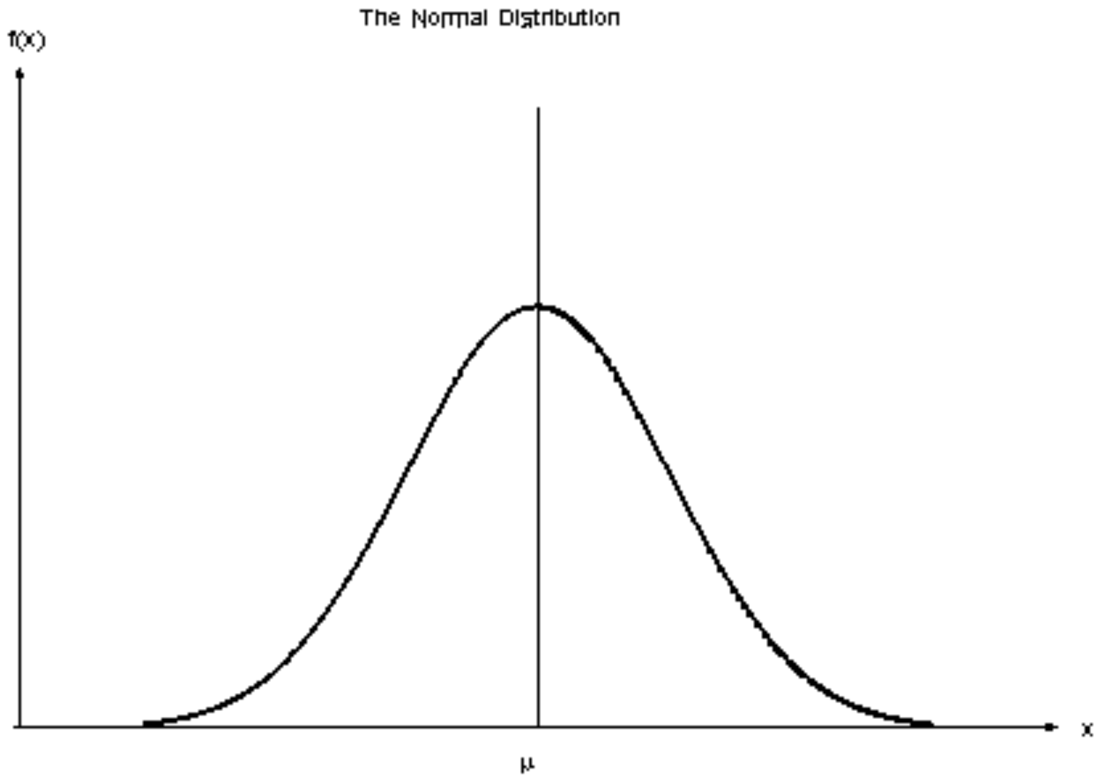
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

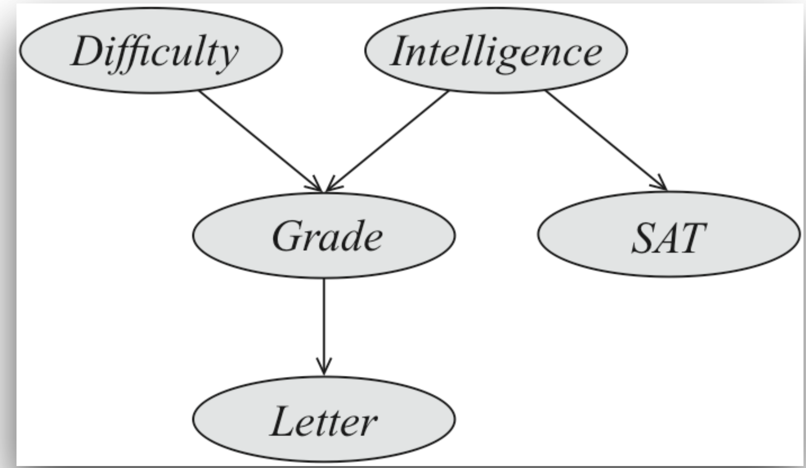


$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

REPRESENTING AND WORKING WITH DISTRIBUTIONS

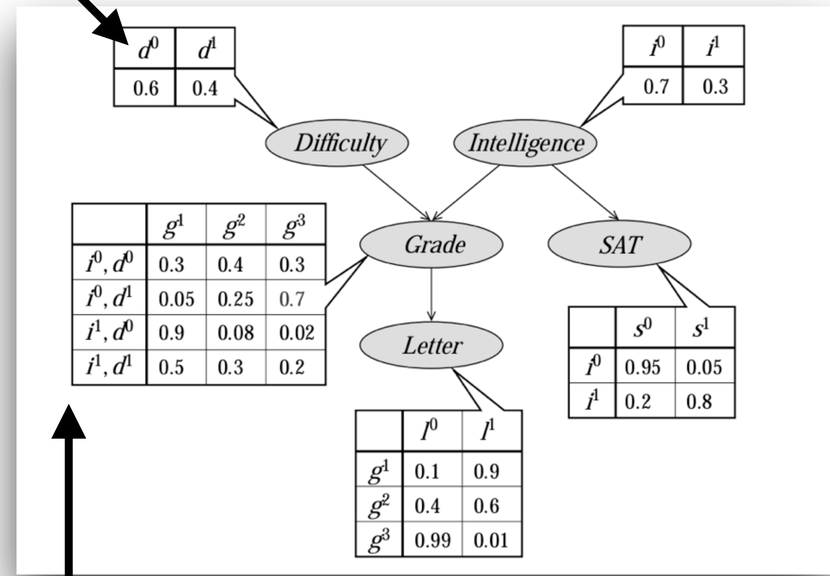
- For all but the smallest n, the explicit representation of the joint distribution is *unmanageable from every perspective*.
  - Computationally, it is very *expensive to manipulate* and generally *too large to store* in memory.
  - Cognitively, it is *impossible to acquire so many numbers* from a human expert; moreover, the numbers are very small and *do not correspond to events that people can reasonably contemplate*.
  - Statistically, if we want to learn the distribution from data, we would *need ridiculously large amounts of data to estimate* this many parameters robustly.
- These problems were the *main barrier* to the adoption of probabilistic methods for expert systems *until the development of the methodologies we now will consider.*

DGM - GRAPH AND CPDS VS JOINT



P(D,I,G,S,L)

d has value 0



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	g^1	g^2	g^3
i^0, d^0	0.3	0.4	0.3
i^0, d^1	0.05	0.25	0.7
i^1, d^0	0.9	0.08	0.02
i^1, d^1	0.5	0.3	0.2

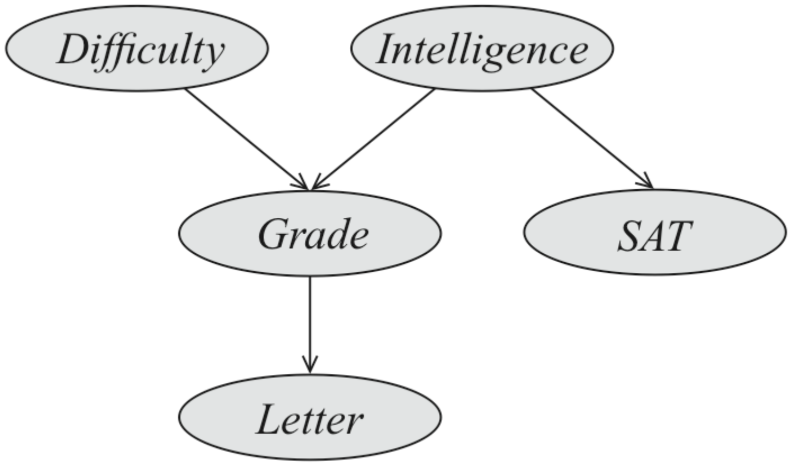
	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

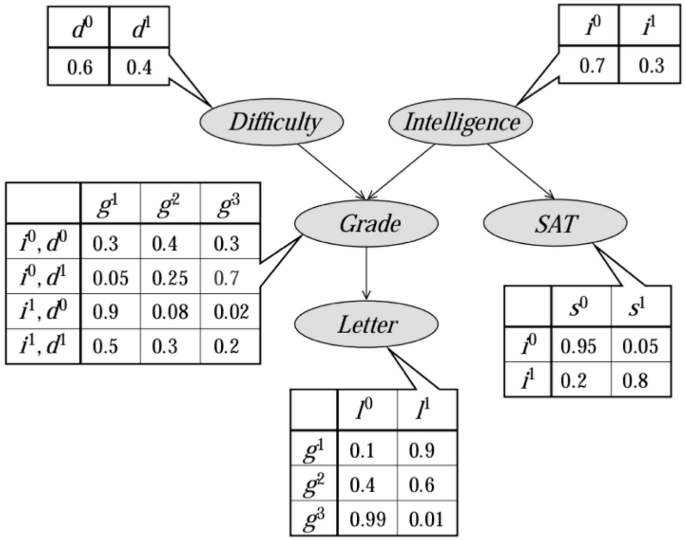
	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01

CPD

- CPT - table, i.e., categorical
- Gaussian

THREE LEVELS OF COMPUTATIONAL PROBLEMS





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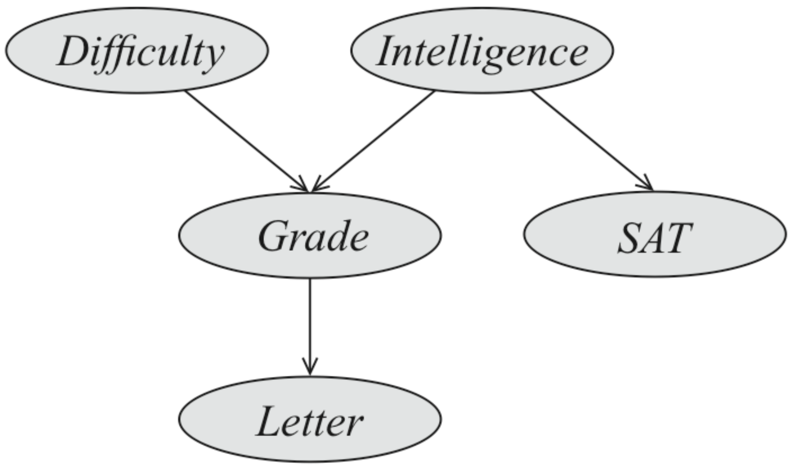
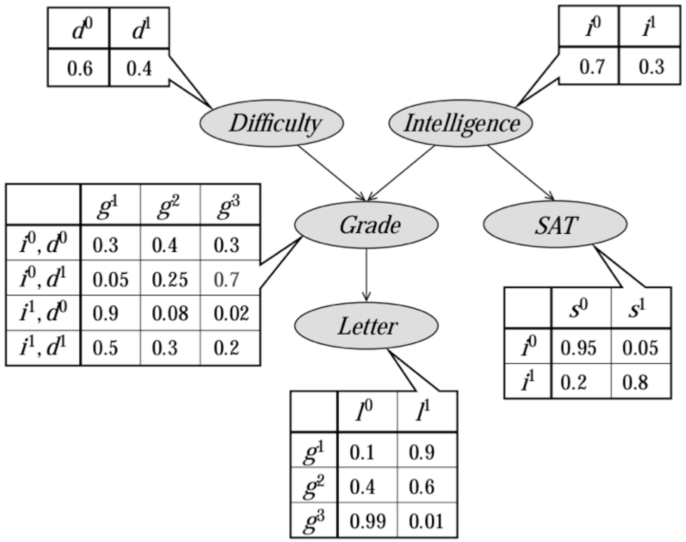
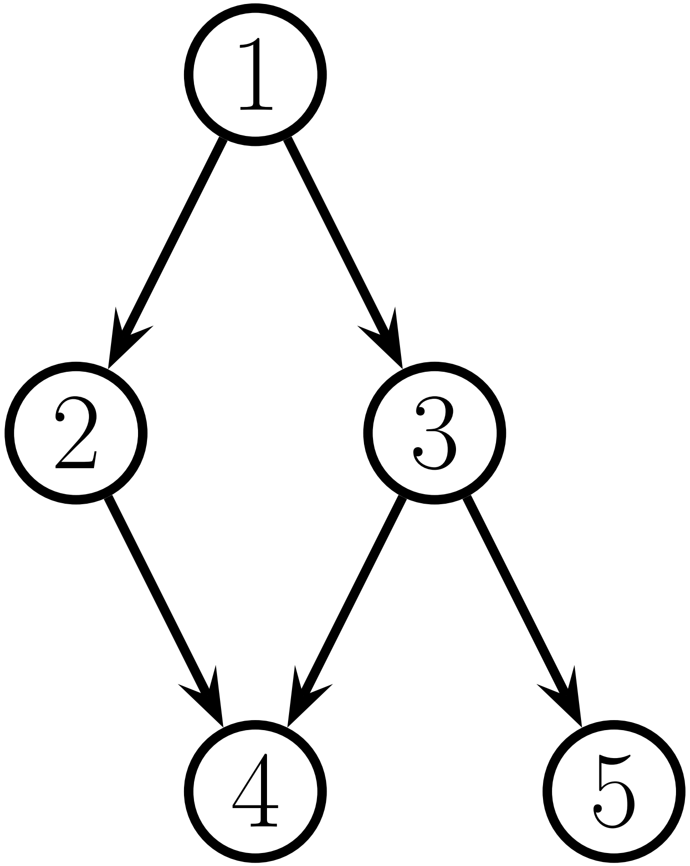
- Inference: given G and θ, compute probabilities or marginalize
- Parameter learning: given G and D, learn θ
- Structure learning: given D, learn G and θ

THREE LEVELS





- Inference: given G and θ, compute probabilities or marginalize — Marginalize often hard
- Parameter learning: given G and D, learn θ — Easy for observable data
- Structure learning: given D, learn G and θ — Hard unless tree-like, doable in practice for observable



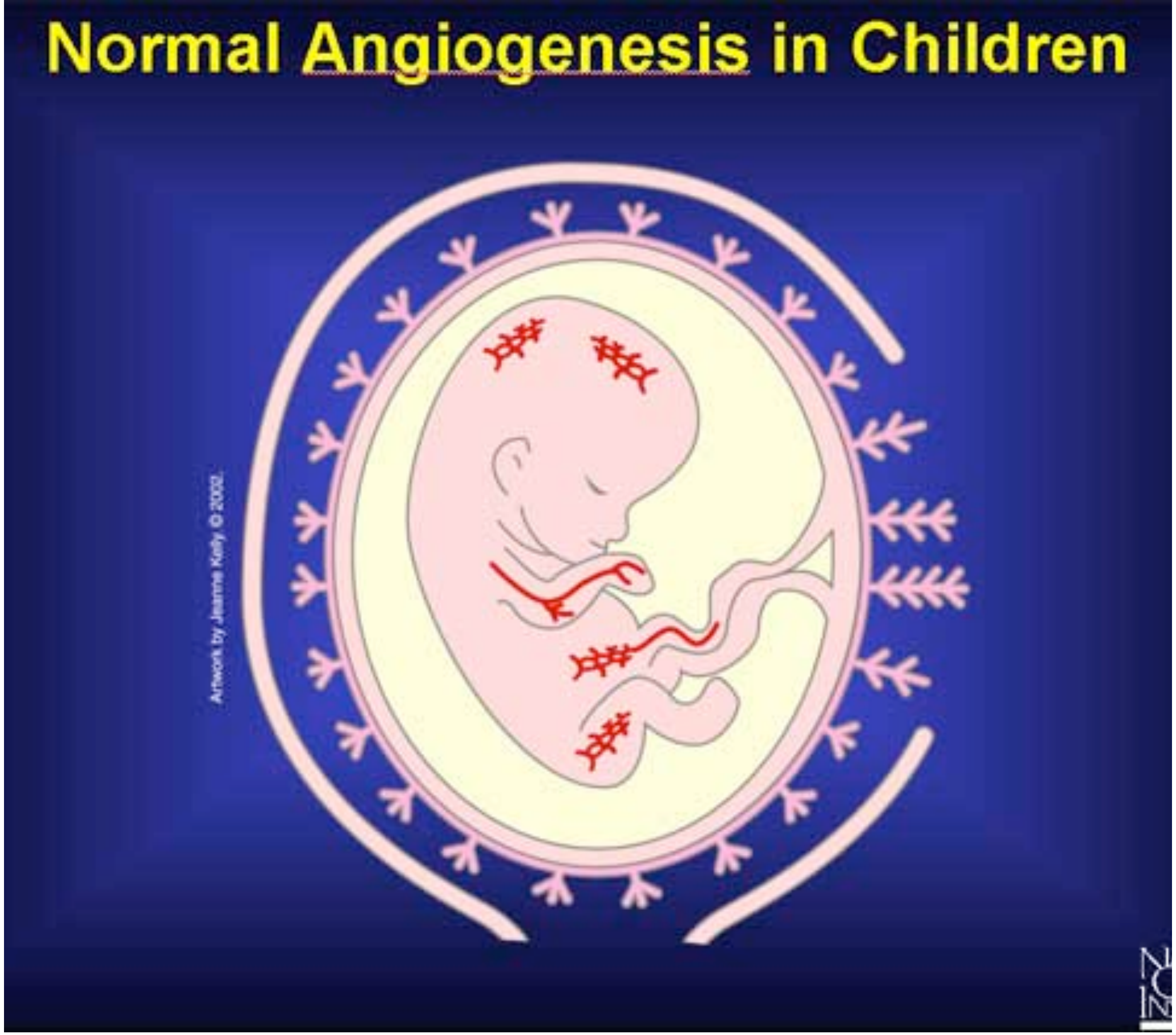
NOT SO MUCH SEMANTICS

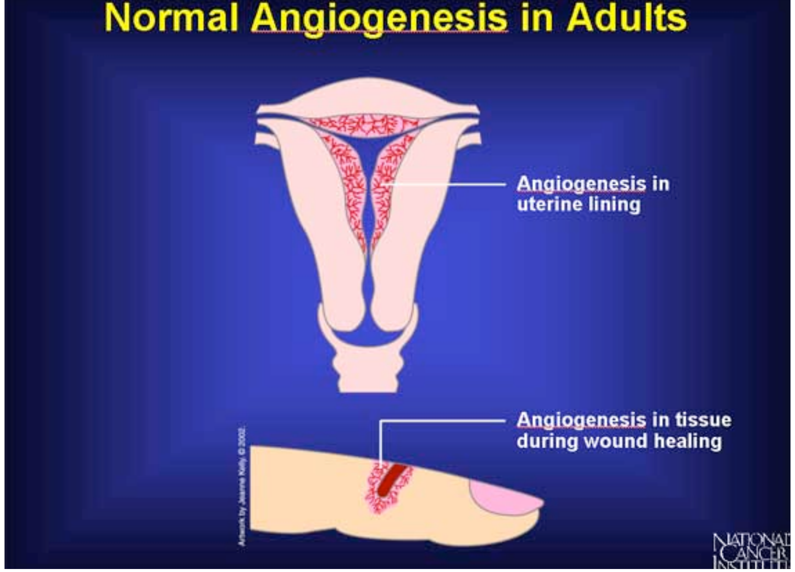
★What is the meaning of the underlying DAG? what is the semantics?

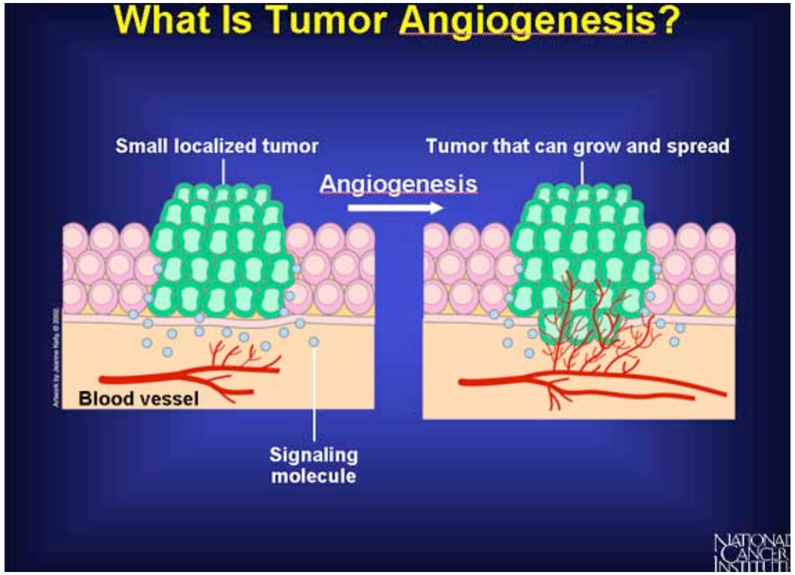
★Which DAGs can represent a given distribution?

VISUALIZATION

- Another application
  - describe and visualize a “designed model” or a distribution and, in particular, its dependencies







ABERRATION DEPENDENCIES - EX. ANGIOGENESIS

SOMATIC EVOLUTION

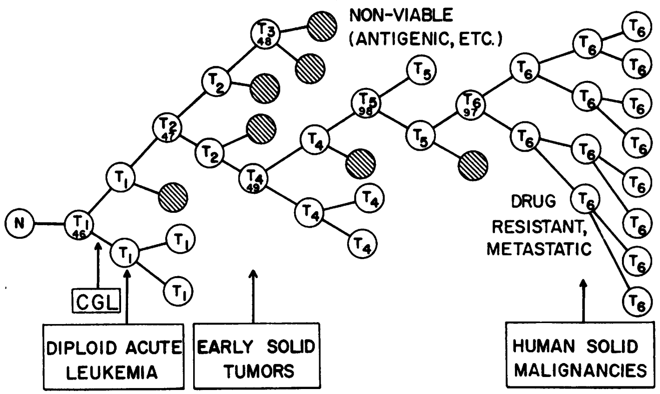
The Clonal Evolution of Tumor Cell Populations

Acquired genetic lability permits stepwise selection of variant sublines and underlies tumor progression.

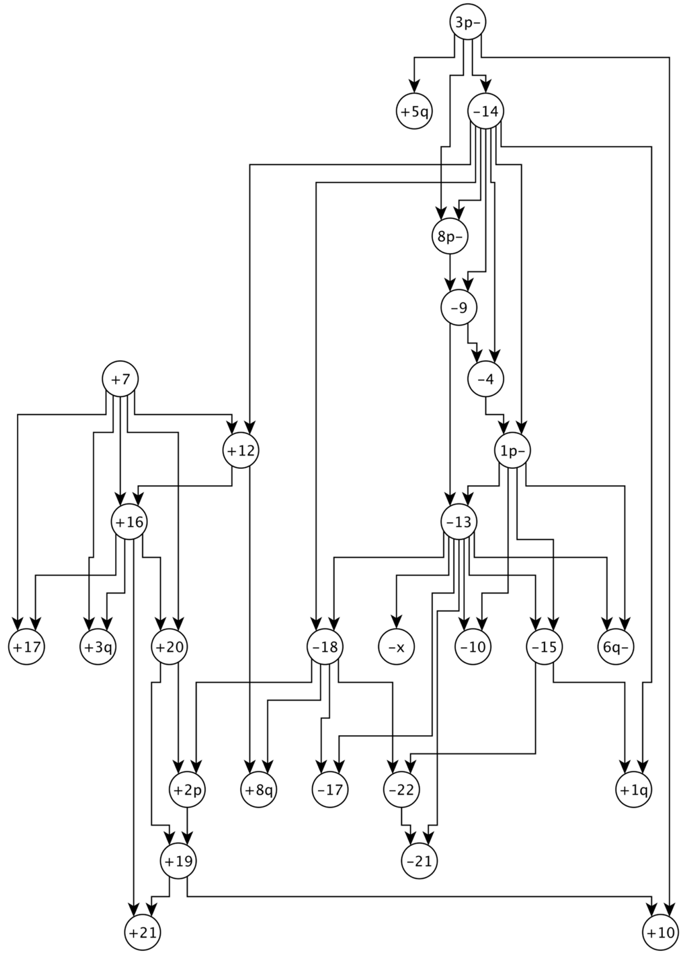
Peter C. Nowell

The author is professor of pathology, School of Medicine, University of Pennsylvania, Philadelphia 19174.

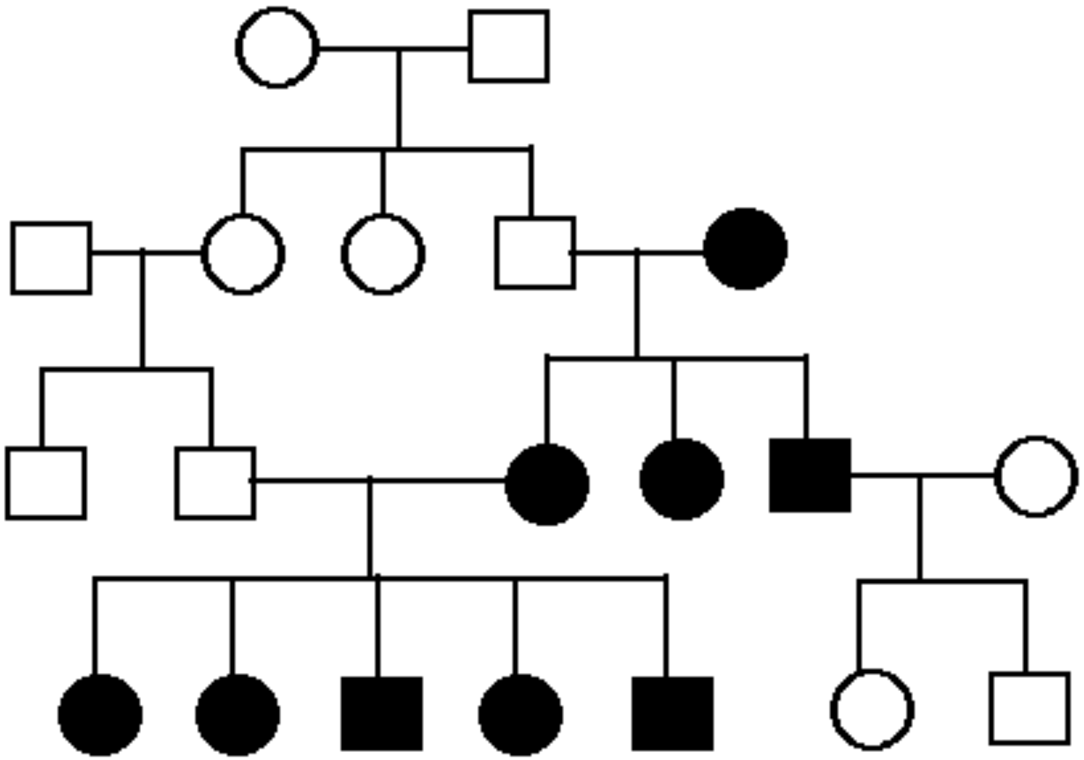
1 OCTOBER 1976 SCIENCE, VOL. 194

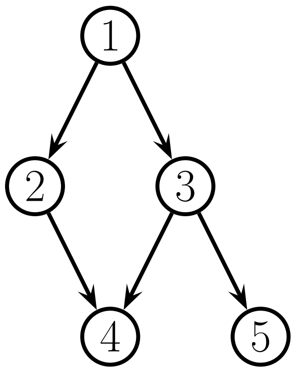


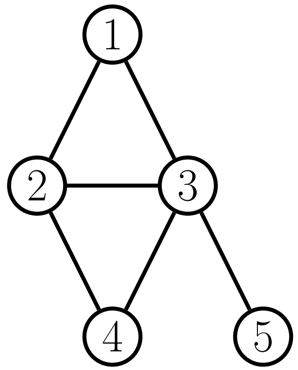
Oncogenetic network



A PEDIGREE







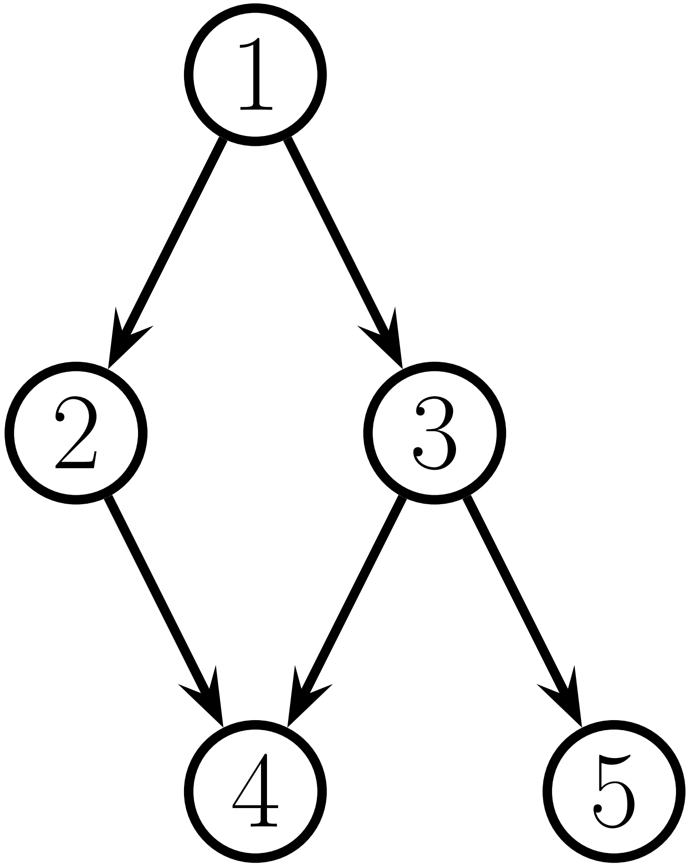
Directed graphical model

- DAG
- vertices r.v.s
- equipped with local CPDs
- allows causal like dependencies

Undirected graphical model - Markov Random Fields

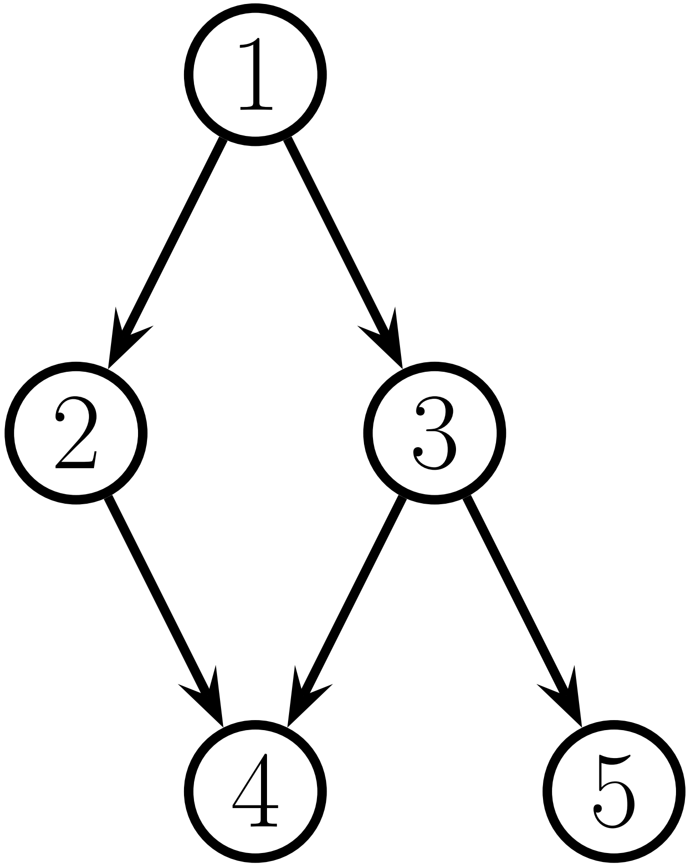
- graph
- vertices r.v.s
- equipped with local "factors"

GRAPHICAL MODELS



TERMINOLOGY

- ★ Parent
- ★ Child
- ★ Family
- ★ Root
- ★ Leaf
- ★Neighbors



TERMINOLOGY

- ★ Degree (in and out)
- ★ Cycle (directed or not)
- ★ Directed Acyclic Graph (DAG)
- ★ Topological order (parents < child)
- ★ Path (directed or not)
- ★ Ancestors

CPD - BERNOULLI OR CATEGORICAL

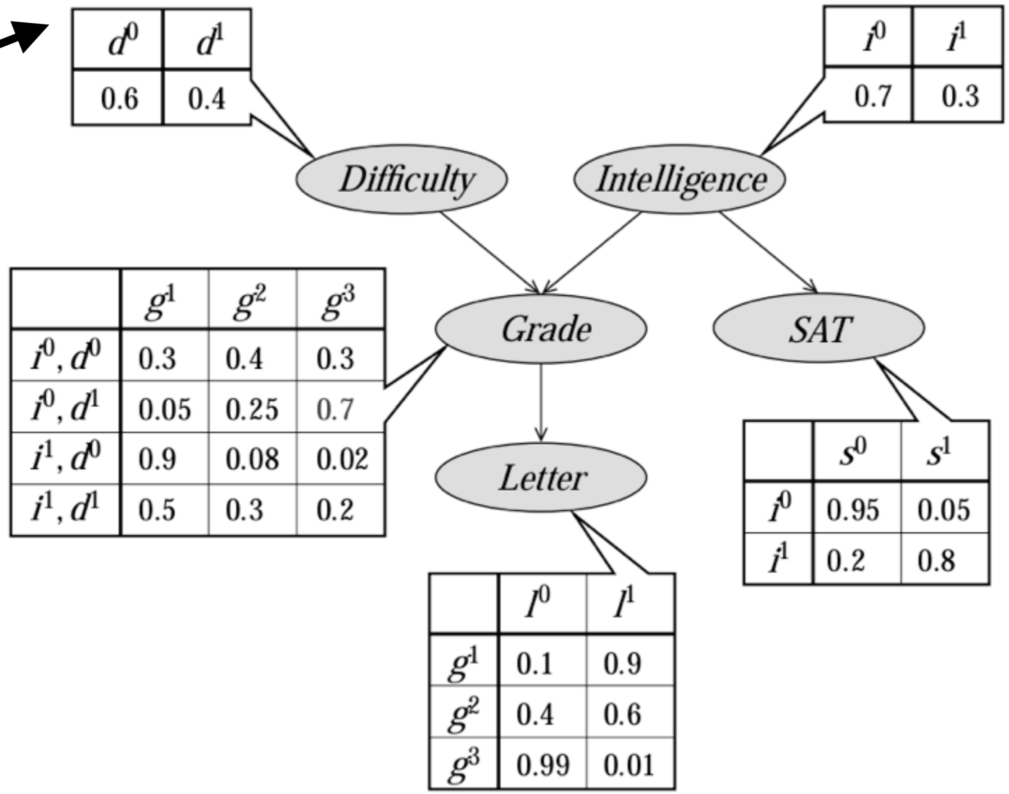
$$\mathrm{Ber}(x|\theta) = \begin{cases} \theta & \text{if } x = 1 \\ 1 - \theta & \text{if } x = 0 \end{cases}$$

$$\mathrm{Cat}(x|\boldsymbol{\theta}) = \theta_x$$

- ★ One or several (unordered) coin tosses
- ★ A dice (possibly biased)

NOTATION

d has value 0



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	g^1	g^2	g^3
i^0, d^0	0.3	0.4	0.3
i^0, d^1	0.05	0.25	0.7
i^1, d^0	0.9	0.08	0.02
i^1, d^1	0.5	0.3	0.2

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

	l^0	l^1
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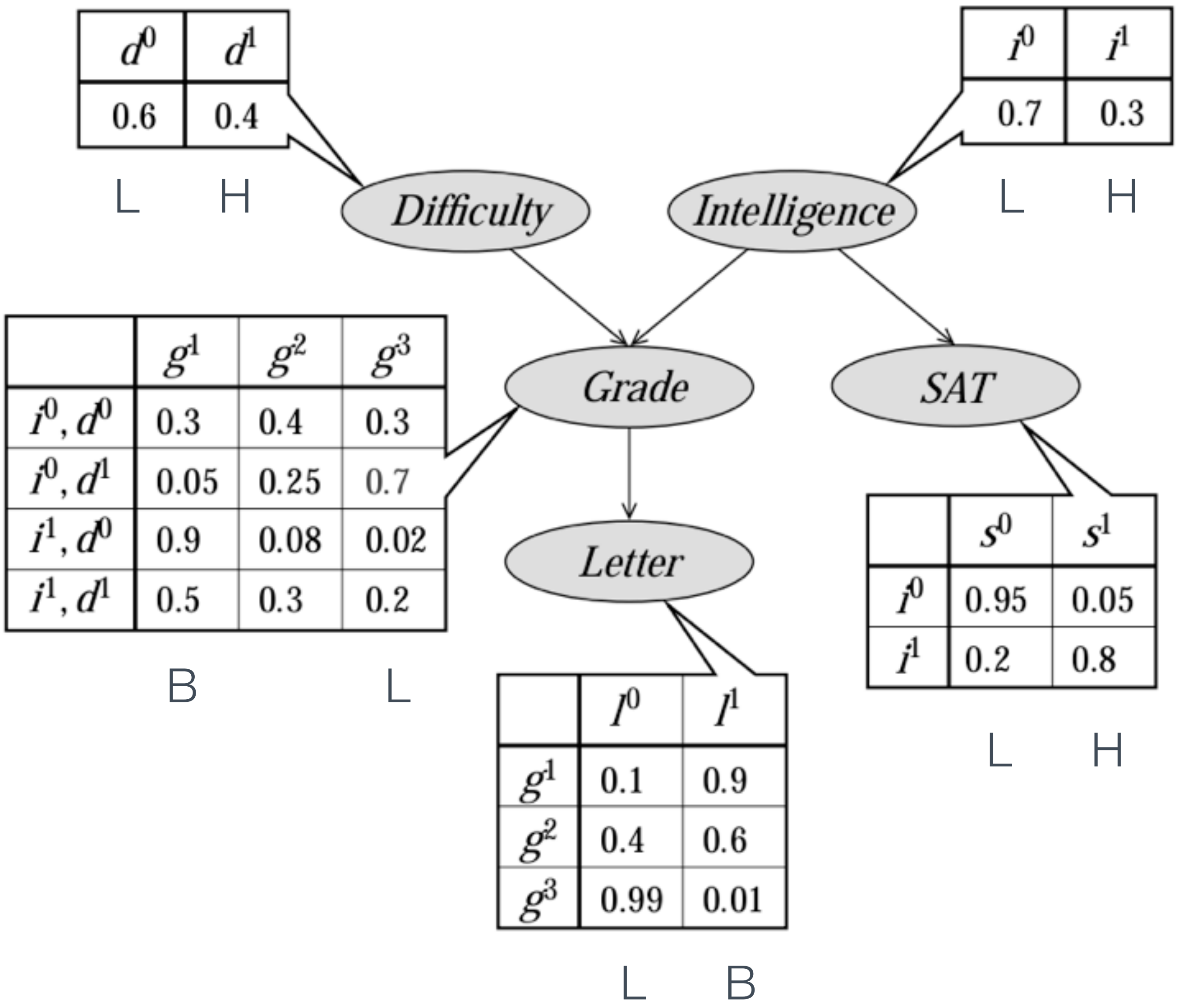
g^3	0.99	0.01

EXTENDED STUDENT EXAMPLE



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B - better

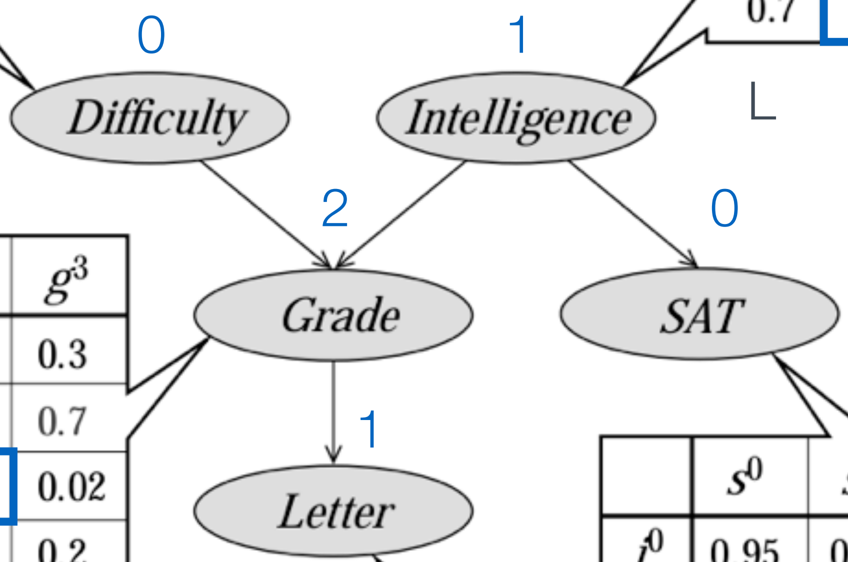
H - higher

L - less

EXTENDED STUDENT EXAMPLE

d^0	d^1
0.6	0.4
L	H

i^0	i^1
0.7	0.3
L	H



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i^1, d^1	0.5	0.3	0.2
	B		L

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8
	L	H

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01
	L	B

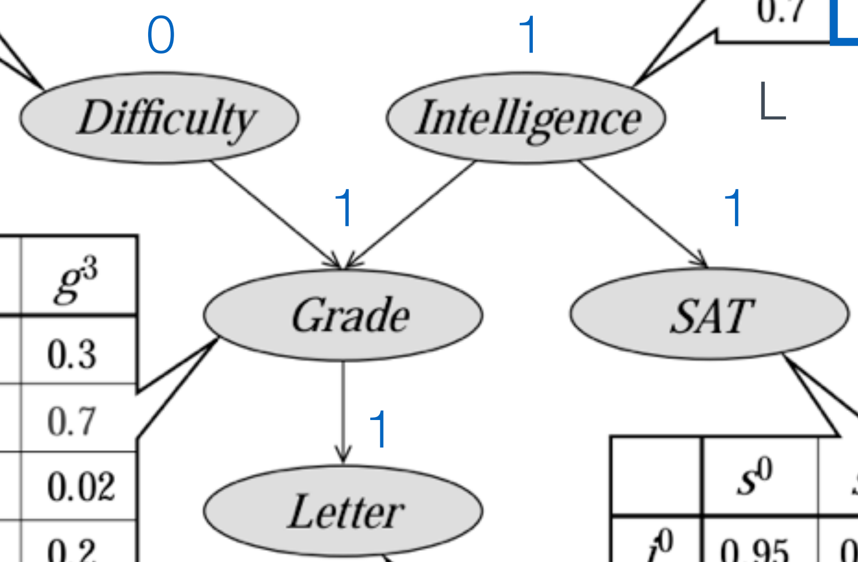
P(D=0,I=1,G=2,L=1,S=0) =
P(D=0)P(I=1)P(G=2|D=0,I=1)
P(L=1|G=2)P(S=0|I=1)=
0.6*0.3*0.08*0.6*0.2

B - better
H - higher
L - less

EXTENDED STUDENT EXAMPLE

d^0	d^1
0.6	0.4
L	H

i^0	i^1
0.7	0.3
L	H



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i^1, d^1	0.5	0.3	0.2
	B		L

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8
	L	H

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01
	L	B

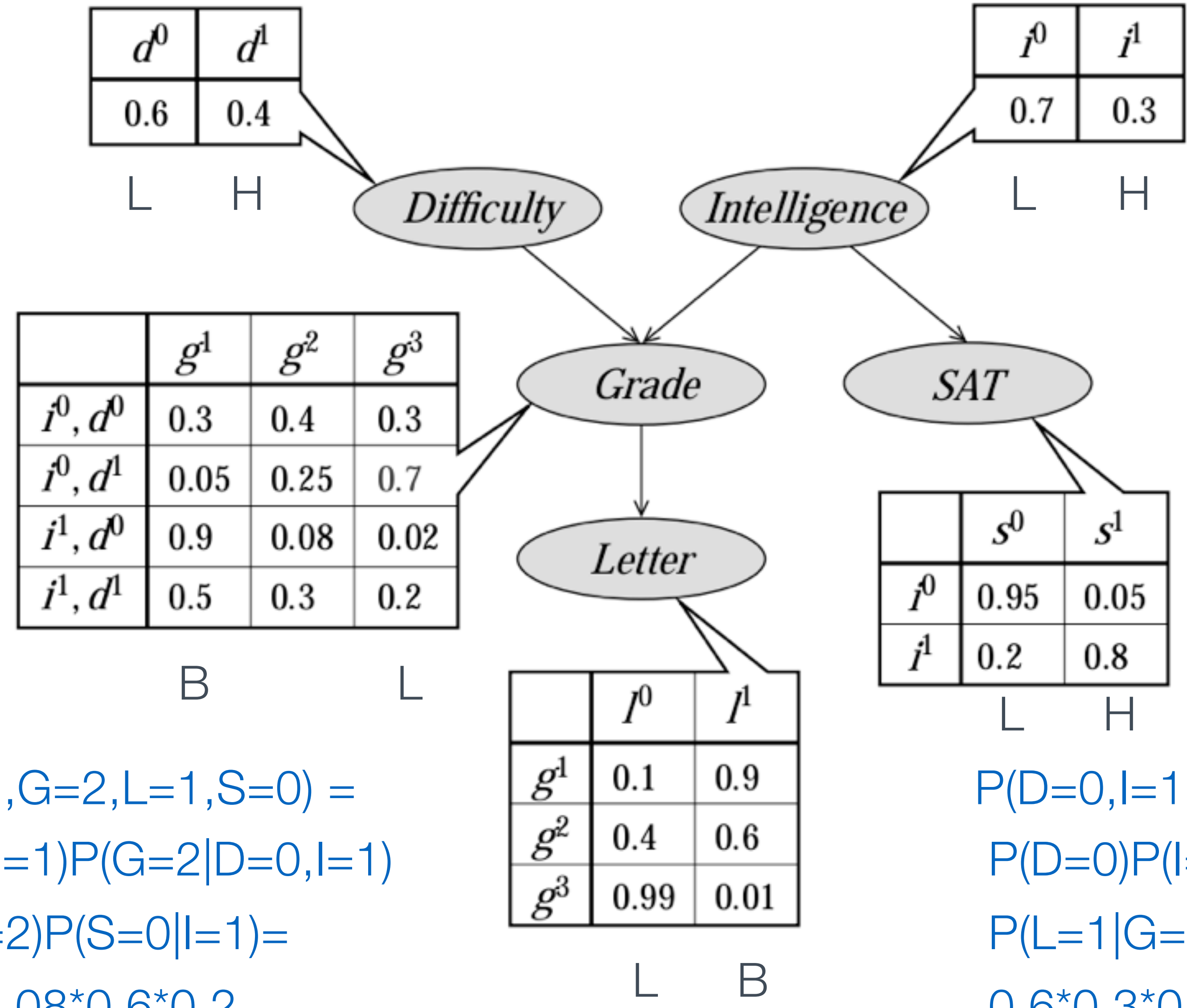
P(D=0,I=1,G=1,L=1,S=1) =

P(D=0)P(I=1)P(G=1|D=0,I=1)

P(L=1|G=1)P(S=1|I=1)=

0.6*0.3*0.9*0.9*0.8

EXTENDED STUDENT EXAMPLE



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P(D=0,I=1,G=1,L=1,S=1) =
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P(L=1|G=1)P(S=1|I=1)=
0.6*0.3*0.9*0.9*0.8

INFERENCE – THE CHAIN RULE

$$p(\underbrace{\boldsymbol{x}_{[V]}}_{\boldsymbol{x}_1,\ldots,\boldsymbol{x}_V}) = p(\boldsymbol{x}_1)p(\boldsymbol{x}_2|\boldsymbol{x}_1)p(\boldsymbol{x}_3|\boldsymbol{x}_1,\boldsymbol{x}_2)\cdots p(\boldsymbol{x}_V|\boldsymbol{x}_{[V-1]})$$

- Assuming binary r.v., $p(X_V \mid X_{[V-1]})$ has 2^{V-1} parameters
- Total # parameters $\sum_{1\leq i\leq V} 2^{i-1} = 2^V-1$

CONDITIONAL INDEPENDENCE

- X and Y are conditionally independent given Z iff

  $p(X, Y|Z) = P(X|Z)\, P(Y|Z)$

- Implies

  $p(X|Y, Z) = p(X, Y|Z)/p(Y|Z) = p(X|Z)$

EX. WHERE IND. OBVIOUSLY FACILITATES

$$p(\underbrace{\boldsymbol{x}_{[V]}}_{\boldsymbol{x}_1,\ldots,\boldsymbol{x}_V}) = p(\boldsymbol{x}_1)p(\boldsymbol{x}_2|\boldsymbol{x}_1)p(\boldsymbol{x}_3|\boldsymbol{x}_1,\boldsymbol{x}_2)\cdots p(\boldsymbol{x}_V|\boldsymbol{x}_{[V-1]})$$

★Assume first order Markov property $\boldsymbol{x}_t \perp \boldsymbol{x}_{[t-2]}|\boldsymbol{x}_{t-1}$

i.e., if time ordered, future independent of past given present

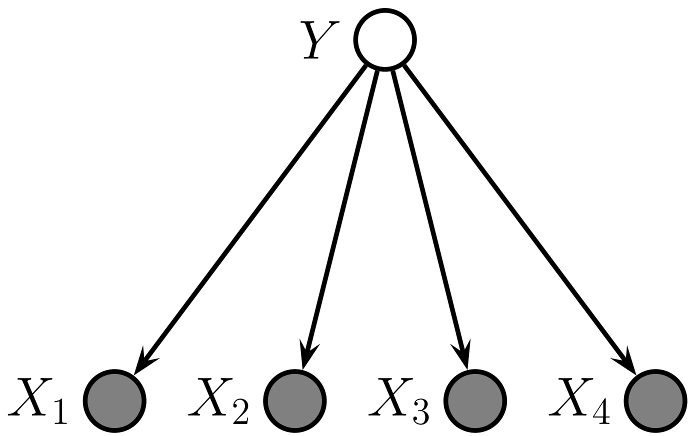
★Then

$$p(\boldsymbol{x}_{[V]}) = p(\boldsymbol{x}_1)\prod_{t=1}^{V-1} p(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t)$$

SPECIAL CASE: HIDDEN MARKOV MODEL (HMM)

$$
\begin{array}{ccccccc}
z_1 & \longrightarrow & z_2 & \longrightarrow & z_3 & \cdots & z_V \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
x_1 & & x_2 & & x_3 & & x_V
\end{array}
$$

- Z_i hidden
- X_i observable
- Hidden often not observable when training, never when applying



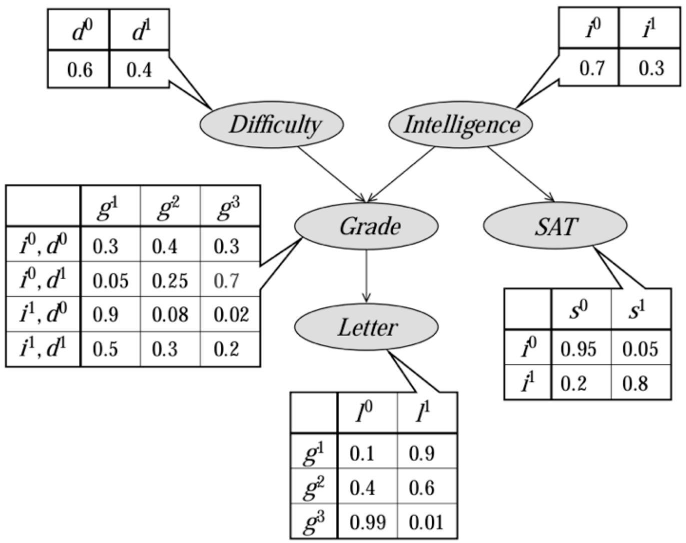
SPECIAL CASE: NAIVE BAYES CLASSIFIER

$$p(\boldsymbol{x}, y) = p(y) \prod_{t=1}^{4} p(x_t | y)$$

FACTORIZATION - A BINARY EXAMPLE

Given data and GM with CPDs (new CPDs on a need to know basis)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1



FACTORIZATION - AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$\begin{aligned} L(\boldsymbol{\theta}; \mathcal{D}) = & \, p(0,1,1,1,1|\boldsymbol{\theta}) p(1,1,1,0,0|\boldsymbol{\theta}) \\ & p(1,1,0,0,1|\boldsymbol{\theta}) p(1,0,0,0,0|\boldsymbol{\theta}) \\ & p(1,1,0,0,1|\boldsymbol{\theta}) \end{aligned}$$

AN EXAMPLE

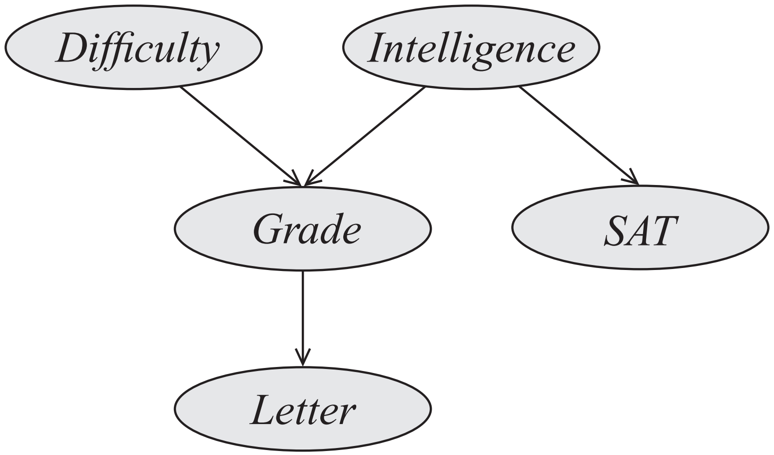
Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$\begin{aligned} L(\boldsymbol{\theta};\mathcal{D}) = & \ p(0,1,1,1,1|\boldsymbol{\theta})p(1,1,1,0,0|\boldsymbol{\theta}) \\ & p(1,1,0,0,1|\boldsymbol{\theta})p(1,0,0,0,0|\boldsymbol{\theta}) \\ & p(1,1,0,0,1|\boldsymbol{\theta}) \\ = & \ p(D=(0,1,1,1,1)|\boldsymbol{\theta}_D)\dots \end{aligned}$$

AN EXAMPLE

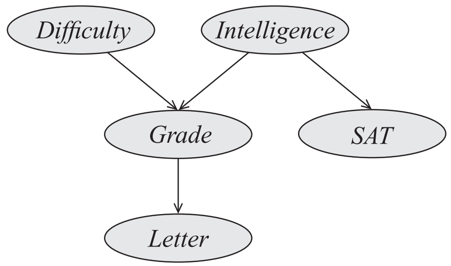
Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)



D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$
\begin{aligned}
L(\boldsymbol{\theta};\mathcal{D}) = & \; p(D=(0,1,1,1,1)|\boldsymbol{\theta}_D) \\
& p(I=(1,1,1,0,1)|\boldsymbol{\theta}_I) \\
& p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S) \\
& p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G) \\
& p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)
\end{aligned}
$$

AN EXAMPLE



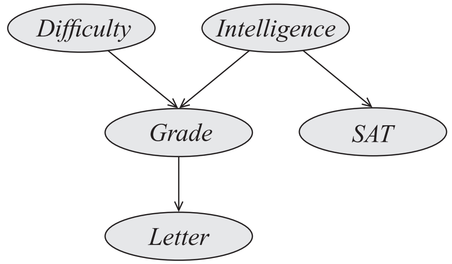
Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ_D	D=0	D=1
	2/5	3/5

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$\begin{aligned} L(\boldsymbol{\theta};\mathcal{D}) = & \, p(D=(0,1,1,1,1)|\boldsymbol{\theta}_D) \\ & p(I=(1,1,1,0,1)|\boldsymbol{\theta}_I) \\ & p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S) \\ & p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G) \\ & p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L) \end{aligned}$$

AN EXAMPLE



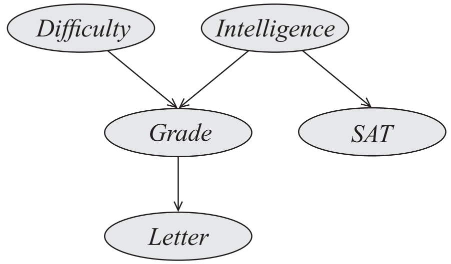
Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ$_D$	D=0	D=1
	2/5	3/5

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$
\begin{aligned}
L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \\
p(I = (1,1,1,0,1)|\boldsymbol{\theta}_I) \\
p(S = (1,1,0,0,0)|I = (1,1,1,0,1), \boldsymbol{\theta}_S) \\
p(G = (1,0,0,0,0)|D = (0,1,1,1,1), I = (1,1,1,0,1), \boldsymbol{\theta}_G) \\
p(L = (1,0,1,0,1)|G = (1,0,0,0,0), \boldsymbol{\theta}_L)
\end{aligned}
$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ_I	I=0	I=1
	1/4	3/4

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4$$

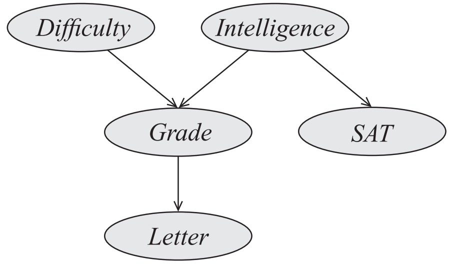
$$p(I = (1,1,1,0,1)|\boldsymbol{\theta}_I)$$

$$p(S = (1,1,0,0,0)|I = (1,1,1,0,1), \boldsymbol{\theta}_S)$$

$$p(G = (1,0,0,0,0)|D = (0,1,1,1,1), I = (1,1,1,0,1), \boldsymbol{\theta}_G)$$

$$p(L = (1,0,1,0,1)|G = (1,0,0,0,0), \boldsymbol{\theta}_L)$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ_I	I=0	I=1
	1/4	3/4

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

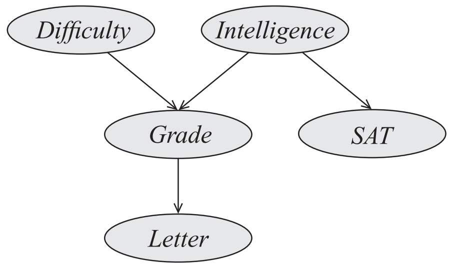
$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4$$

$$p(S = (1,1,0,0,0) | I = (1,1,1,0,1), \boldsymbol{\theta}_S)$$

$$p(G = (1,0,0,0,0) | D = (0,1,1,1,1), I = (1,1,1,0,1), \boldsymbol{\theta}_G)$$

$$p(L = (1,0,1,0,1) | G = (1,0,0,0,0), \boldsymbol{\theta}_L)$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

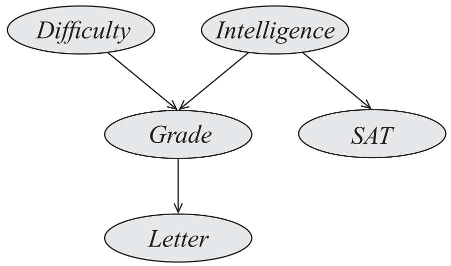
θ_S	S=0	S=1
I=0	1	0
I=1	1/6	5/6

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4$$

$$p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S)$$
$$p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G)$$
$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ_S	S=0	S=1
I=0	1	0
I=1	1/6	5/6

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$p(G = (1,0,0,0,0) | D = (0,1,1,1,1), I = (1,1,1,0,1), \boldsymbol{\theta}_G)$$

$$p(L = (1,0,1,0,1) | G = (1,0,0,0,0), \boldsymbol{\theta}_L)$$

AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ_G	G=0 (Less)	G=1 (Better)
D=0, I=0	1/2	1/2
D=1, I=0	3/5	2/5
D=0, I=1	1/10	9/10
D=1, I=1	2/5	3/5

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$p(G=(1,0,0,0,0)|D=(0,1,1,1,1), I=(1,1,1,0,1), \boldsymbol{\theta}_G)$$

$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0), \boldsymbol{\theta}_L)$$

AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ_G	G=0	G=1
D=0, I=0	1/2	1/2
D=1, I=0	3/5	2/5
D=0, I=1	1/10	9/10
D=1, I=1	2/5	3/5

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}$$
$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ_L	L=0	L=1
G=0	2/3	1/3
G=1	0	1

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}$$

$$p(L = (1, 0, 1, 0, 1) | G = (1, 0, 0, 0, 0), \boldsymbol{\theta}_L)$$

AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

θ_L	L=0	L=1
G=0	2/3	1/3
G=1	0	1

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$$

FACTORIZATION - AN EXAMPLE

"Row wise"

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta}; \mathcal{D}) = p(0,1,1,1,1|\boldsymbol{\theta})p(1,1,1,0,0|\boldsymbol{\theta}) \\ p(1,1,0,0,1|\boldsymbol{\theta})p(1,0,0,0,0|\boldsymbol{\theta}) \\ p(1,1,0,0,1|\boldsymbol{\theta})$$

"Column wise"

$$L(\boldsymbol{\theta}; \mathcal{D}) = p(D = (0,1,1,1,1)|\boldsymbol{\theta}_D) \\ p(I = (1,1,1,0,1)|\boldsymbol{\theta}_I) \\ p(S = (1,1,0,0,0)|I = (1,1,1,0,1), \boldsymbol{\theta}_S) \\ p(G = (1,0,0,0,0)|D = (0,1,1,1,1), I = (1,1,1,0,1), \boldsymbol{\theta}_G) \\ p(L = (1,0,1,0,1)|G = (1,0,0,0,0), \boldsymbol{\theta}_L)$$

THE LIKELIHOOD FACTORIZES

- ★ Complete data

$$\mathcal{D} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$$

$$\boldsymbol{x}_n = \{\boldsymbol{x}_{n1}, \ldots, \boldsymbol{x}_{nV}\}$$

- ★ Likelihood

$$p(\mathcal{D}|\boldsymbol{\theta}) = \prod_{n=1}^{N} p(\boldsymbol{x}_n|\boldsymbol{\theta}) = \prod_{n=1}^{N} \prod_{v=1}^{V} p(\boldsymbol{x}_{nv}|\boldsymbol{x}_{n,\mathrm{pa}(v)}, \boldsymbol{\theta})$$

$$= \prod_{v=1}^{V} \prod_{n=1}^{N} p(\boldsymbol{x}_{nv}|\boldsymbol{x}_{n,\mathrm{pa}(v)}, \boldsymbol{\theta}) = \prod_{v=1}^{V} p(\mathcal{D}_v|\boldsymbol{\theta}_v)$$

where D_v is values of v together with its parents and θ_v is v's CPD

- ★ Called: decomposable likelihood (factorizes into family-factors)

MLE FOR CATEGORICAL

- Likelihood $$p(D) = \prod_{i \in [k]} \theta_i^{N_i}$$
- where $$\sum_{i \in [k]} \theta_i = 1$$
- as well as loglikelihood $$p(D) = \sum_{i \in [k]} N_i log \theta_i$$
- is maximized by $$\theta_i = \frac{N_i}{\sum_{i \in [k]} N_i}$$

CATEGORICAL – NOTATION

- For a v ∈ [V],

  values $k \in [K_v]$

  combined values $c \in C_v = \prod_{s \in \mathrm{pa}(v)} [K_s]$ ← Cartesian product

- Cat CPDs

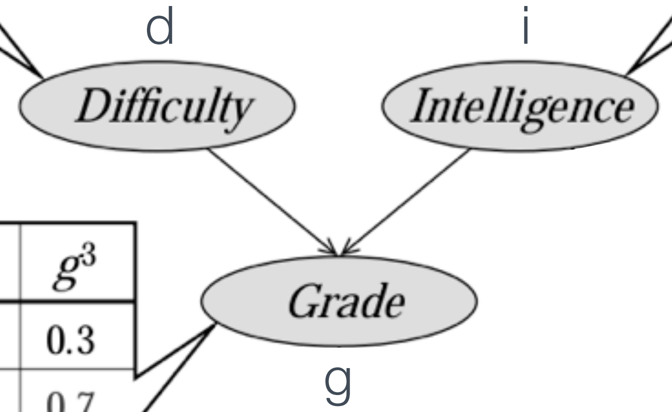
  where $P(x_v | x_{\mathrm{pa}(v)} = c) = \mathrm{Cat}(\boldsymbol{\theta}_{vc})$

  and $\boldsymbol{\theta}_{vck} = P(x_v = k | x_{\mathrm{pa}(v)} = c)$

NOTATION EXAMPLE

$K_d = \{0, 1\}$

$K_i = \{0, 1\}$



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

		g^1	g^2	g^3
$\mathrm{Cat}(\boldsymbol{\theta}_{g\langle 0,0\rangle})$	i^0, d^0	0.3	0.4	0.3
$\mathrm{Cat}(\boldsymbol{\theta}_{g\langle 0,1\rangle})$	i^0, d^1	0.05	0.25	0.7
$\mathrm{Cat}(\boldsymbol{\theta}_{g\langle 1,0\rangle})$	i^1, d^0	0.9	0.08	0.02
$\mathrm{Cat}(\boldsymbol{\theta}_{g\langle 1,1\rangle})$	i^1, d^1	0.5	0.3	0.2

$K_g = \{1, 2, 3\}$

$\theta_{g\langle 1,1\rangle 2}$

$$C_g = \prod_{s \in \mathrm{pa}(g)} [K_s] = K_i \times K_d = \{\langle 0,0\rangle, \langle 0,1\rangle, \langle 1,0\rangle, \langle 1,1\rangle, \}$$

FACTORIZATION - AN EXAMPLE

$$\begin{aligned} L(\boldsymbol{\theta}; \mathcal{D}) = & \, p(D = (0,1,1,1,1) | \boldsymbol{\theta}_D) \\ & p(I = (1,1,1,0,1) | \boldsymbol{\theta}_I) \\ & p(S = (1,1,0,0,0) | I = (1,1,1,0,1), \boldsymbol{\theta}_S) \\ & p(G = (1,0,0,0,0) | D = (0,1,1,1,1), I = (1,1,1,0,1), \boldsymbol{\theta}_G) \\ & p(L = (1,0,1,0,1) | G = (1,0,0,0,0), \boldsymbol{\theta}_L) \end{aligned}$$

"Column wise"

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$N_{v\mathbf{c}k} = \sum_{n=1}^{N} I(x_{nv} = k, x_{n,\mathrm{pa}(v)} = \mathbf{c})$$

$$N_{v\mathbf{c}} = \sum_{n=1}^{N} I(x_{n,\mathrm{pa}(v)} = \mathbf{c})$$

$$N_{G\langle 1,1 \rangle 0} = ?$$

FACTORIZATION - AN EXAMPLE

$$\begin{aligned} L(\boldsymbol{\theta};\mathcal{D}) = & p(D=(0,1,1,1,1)|\boldsymbol{\theta}_D) \\ & p(I=(1,1,1,0,1)|\boldsymbol{\theta}_I) \\ & p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S) \\ & p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G) \\ & p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L) \end{aligned}$$

"Column wise"

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

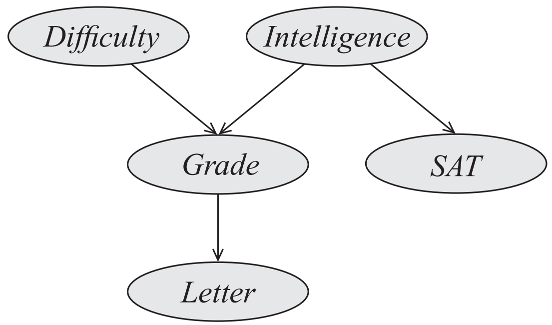
$$N_{v\mathbf{c}k} = \sum_{n=1}^{N} I(x_{nv} = k, x_{n,\mathrm{pa}(v)} = \mathbf{c})$$

$$N_{v\mathbf{c}} = \sum_{n=1}^{N} I(x_{n,\mathrm{pa}(v)} = \mathbf{c})$$

$$N_{G\langle 1,1\rangle 0} = 3$$

MLE FOR CAT CPDS

- Each $P(\mathcal{D}_v|\boldsymbol{\theta}_v)$, i.e., here each $\boldsymbol{\theta}_{v\mathbf{c}}$ can be maximized independently



$$\mathcal{D} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$$

- So, MLE is

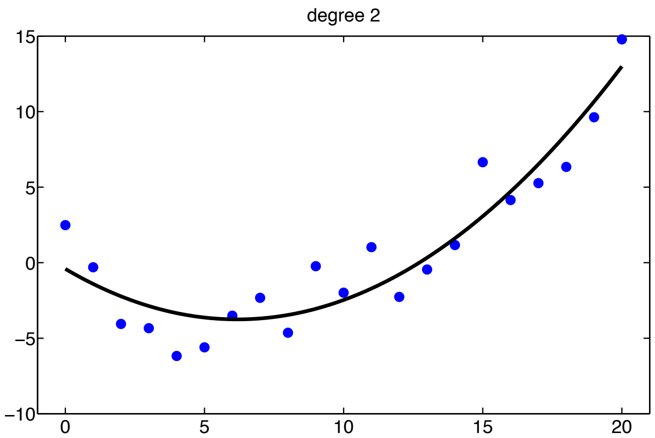
$$\boldsymbol{\theta}_{v\mathbf{c}k} = N_{v\mathbf{c}k}/N_{v\mathbf{c}}$$

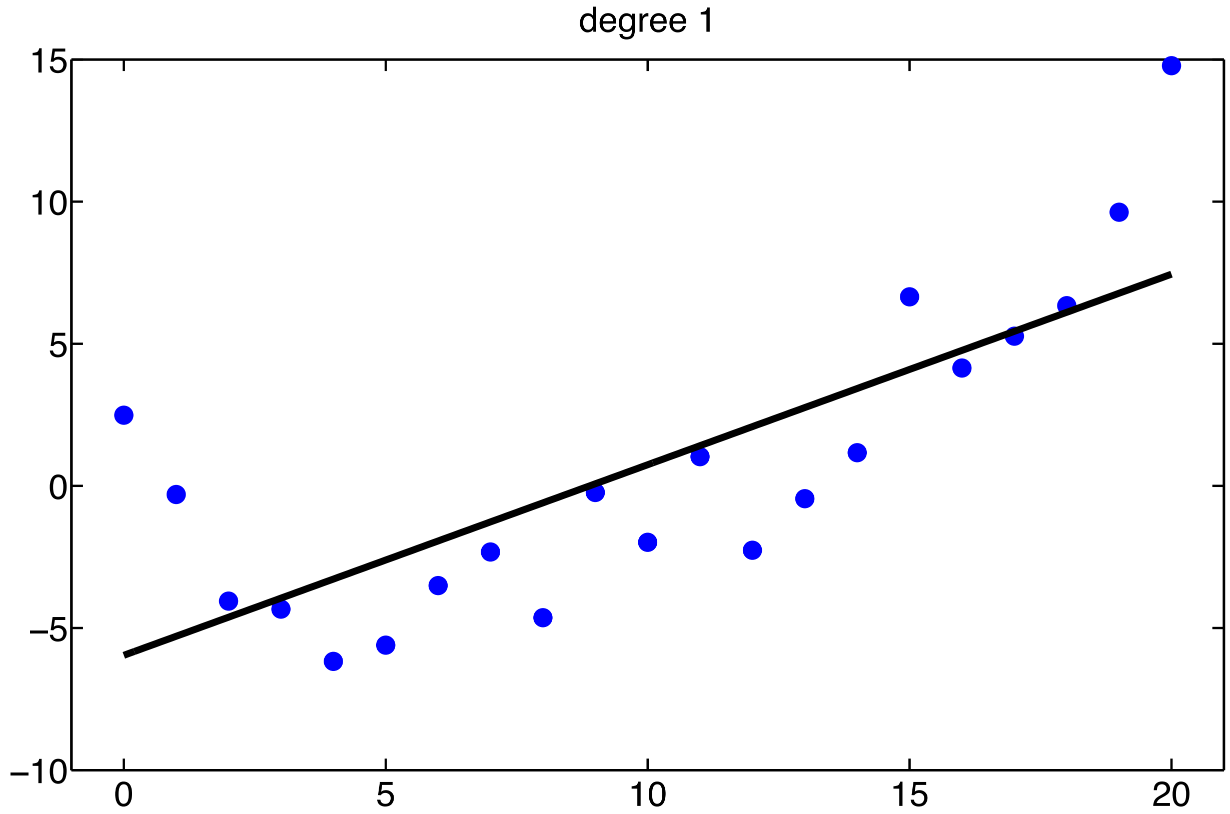
- where

$$N_{v\mathbf{c}k} = \sum_{n=1}^{N} I(x_{nv} = k, x_{n,\mathrm{pa}(v)} = \mathbf{c})$$

$$N_{v\mathbf{c}} = \sum_{n=1}^{N} I(x_{n,\mathrm{pa}(v)} = \mathbf{c})$$

c

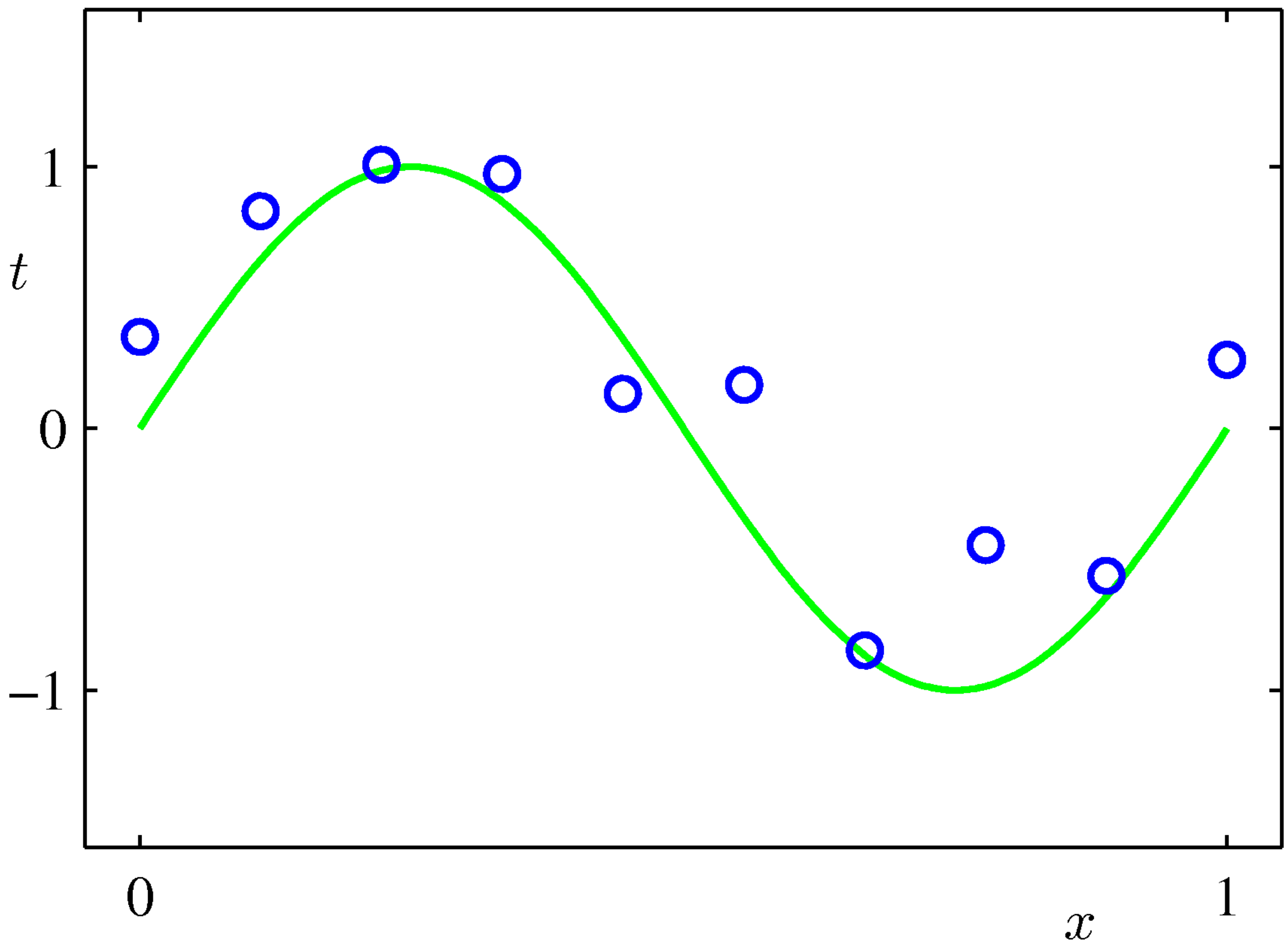




- Size
- Floor
- Location

REGRESSION

EX POLYNOMIAL FITTING - THE DATA

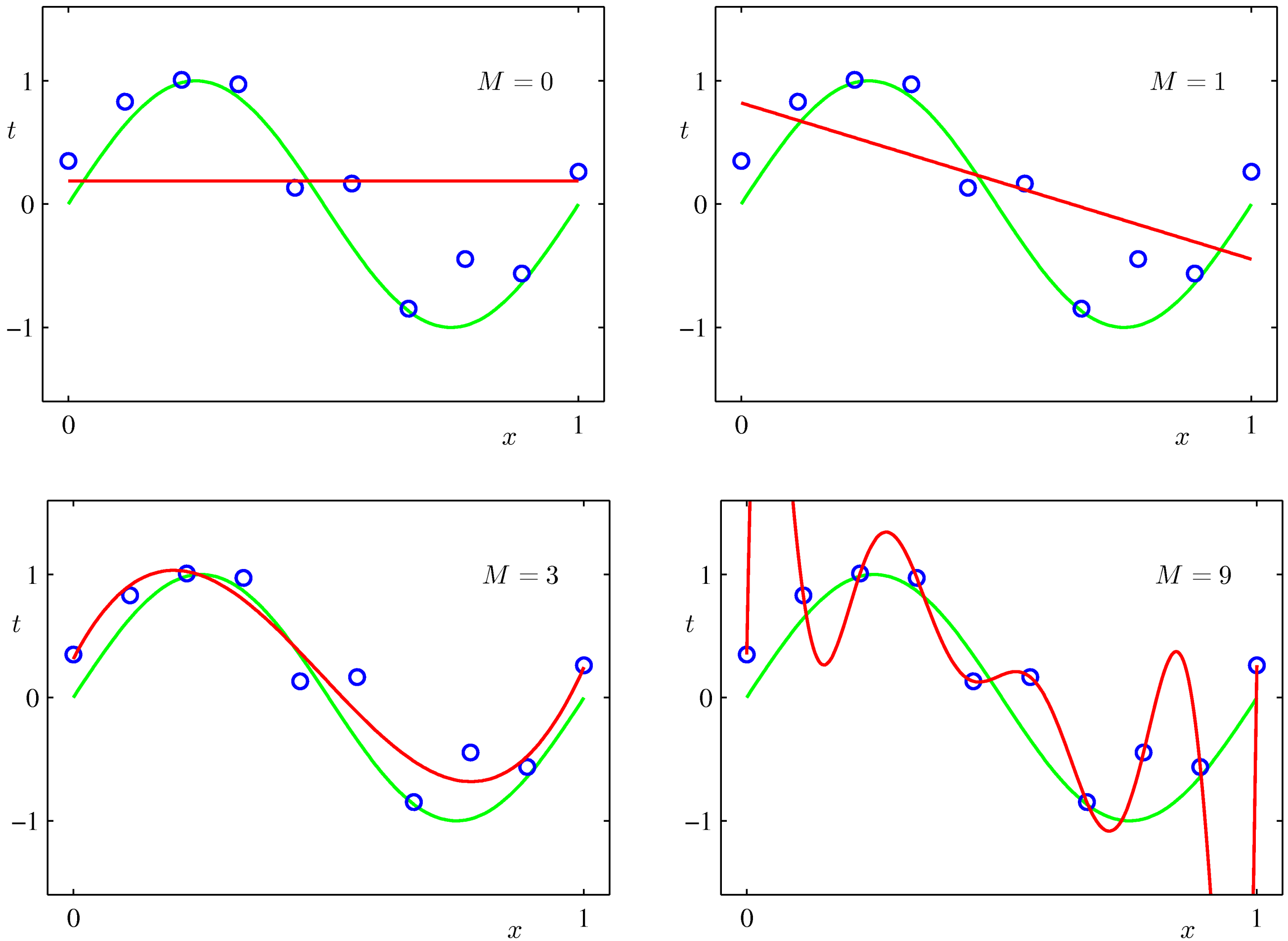


$\sin(2\pi x)$

with additive noise

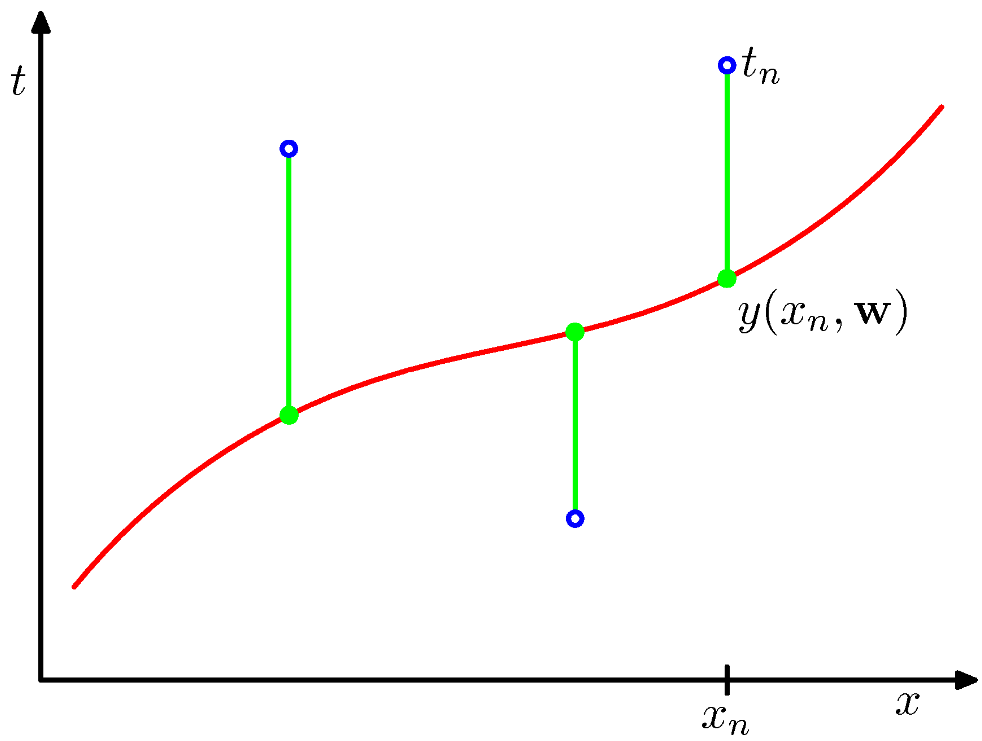
MODELS & PARAMETERS

$$y(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + \ldots + w_M x^M = \sum_{j=0}^{M} w_j x^j$$



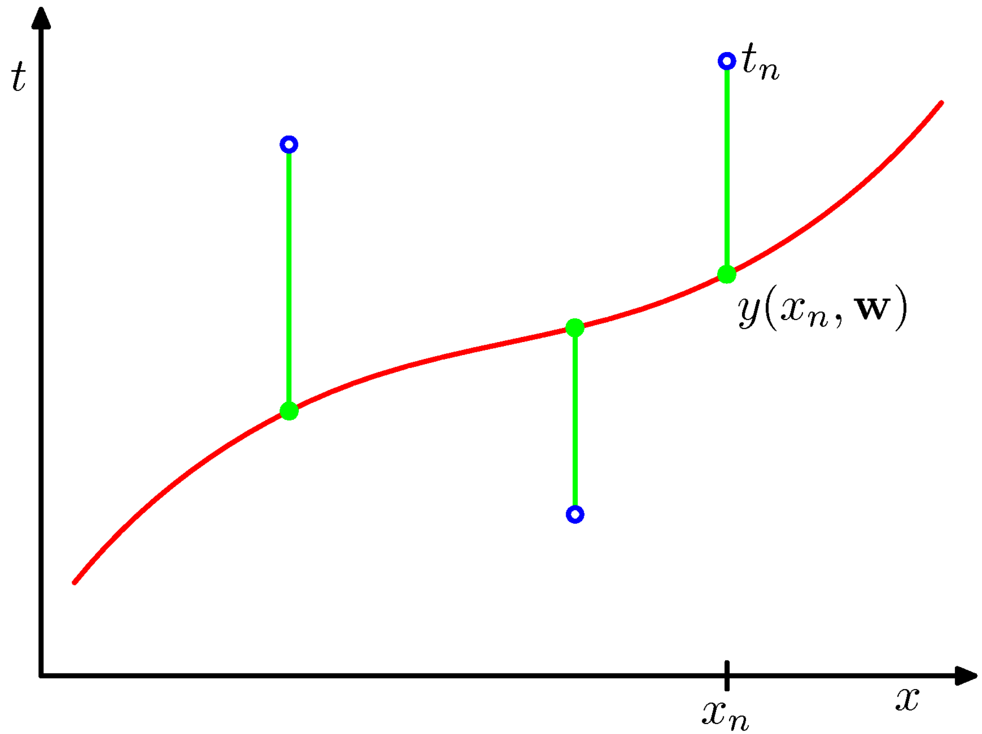
GENERATIVE VIEW

For N data points, t_n is $y(x_n, \mathbf{w})$ plus additive noise from $\mathcal{N}(0, \sigma)$



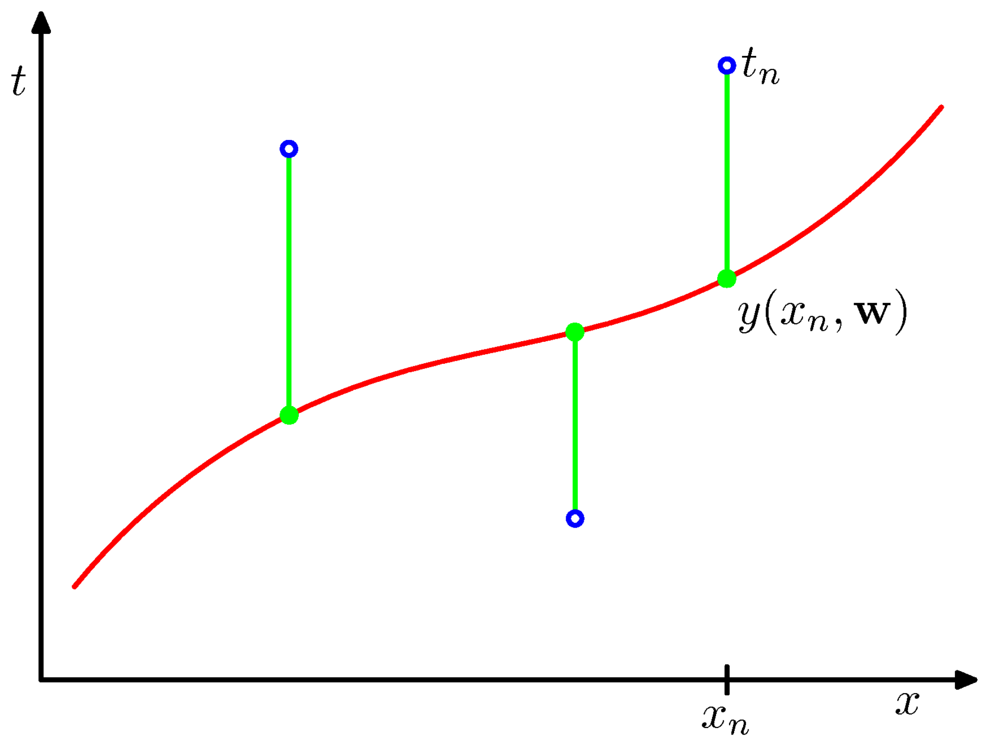
LIKELIHOOD

For N data points $$\prod_{n=1}^{N} \mathcal{N}(y(x_n, \mathbf{w}) - t_n | \sigma)$$



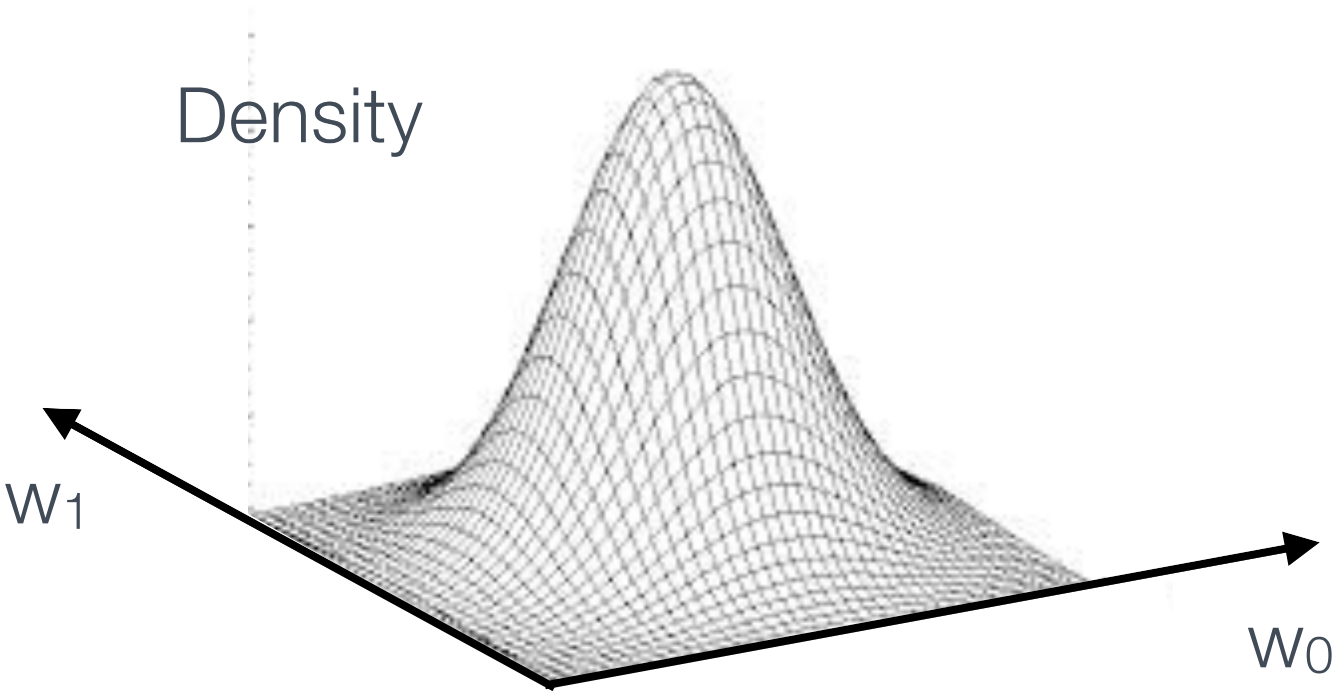
MEASURING ERROR

Sum of squares $E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathbf{w}) - t_n\}^2$



PRIOR ON W- TWO PARAMETER CASE

$$y(x, \mathbf{w}) = w_0 + w_1 x$$



VISUALISING A MODEL-POLYNOMIAL REGRESSION

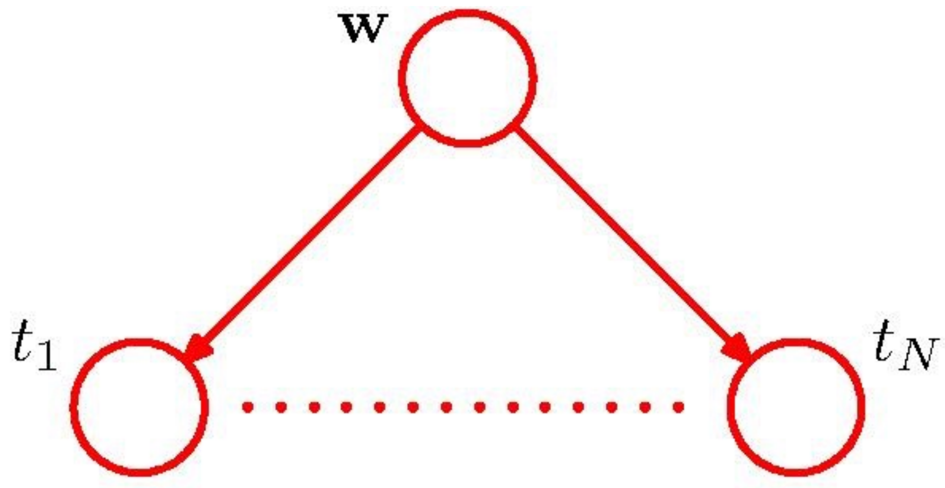
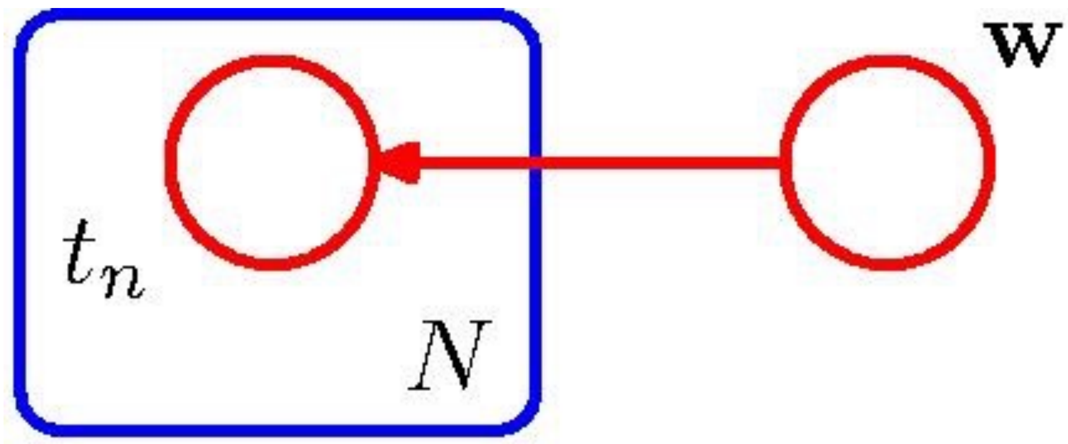
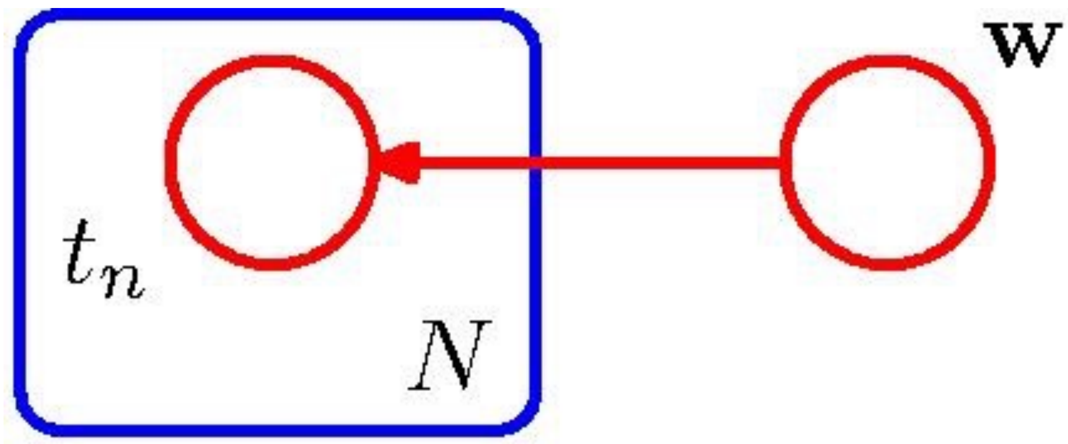


PLATE NOTATION



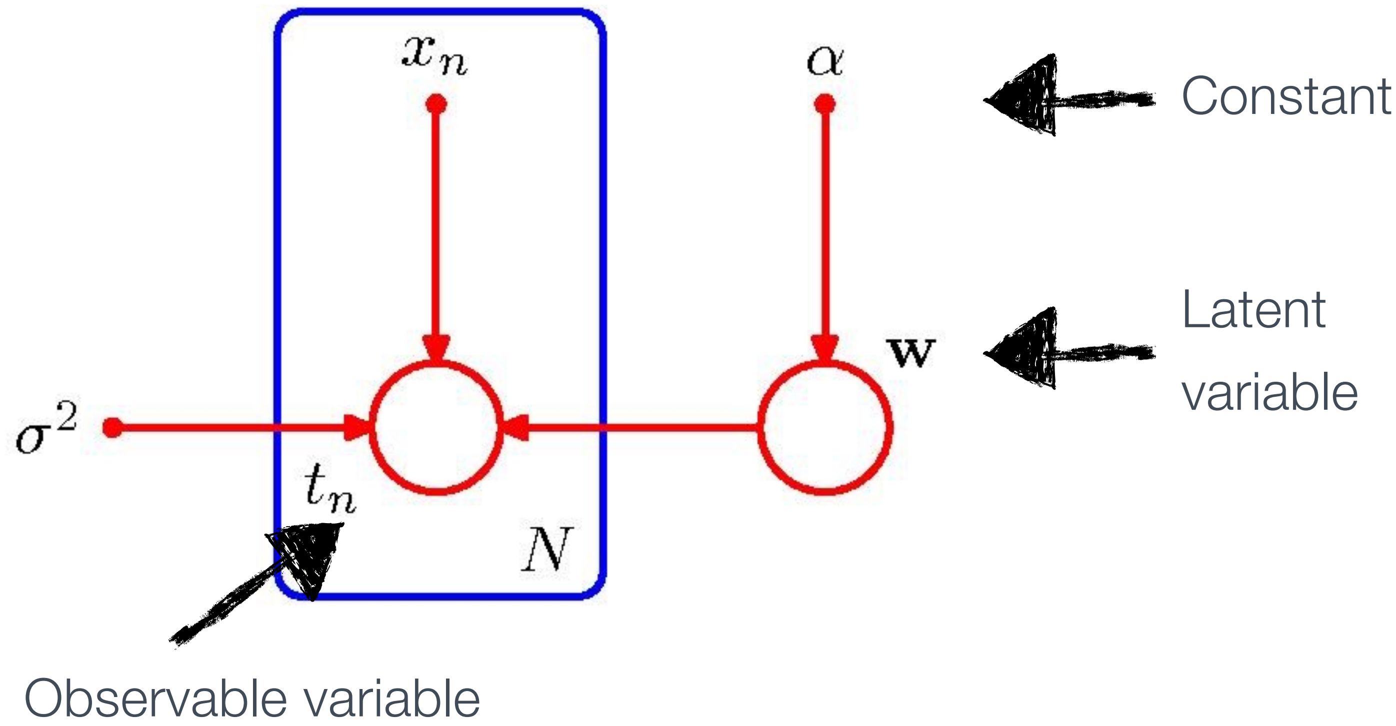
THE JOINT

$$p(\mathbf{t}, \mathbf{w}) = p(\mathbf{w}) \prod_{n=1}^{N} p(t_n|\mathbf{w})$$



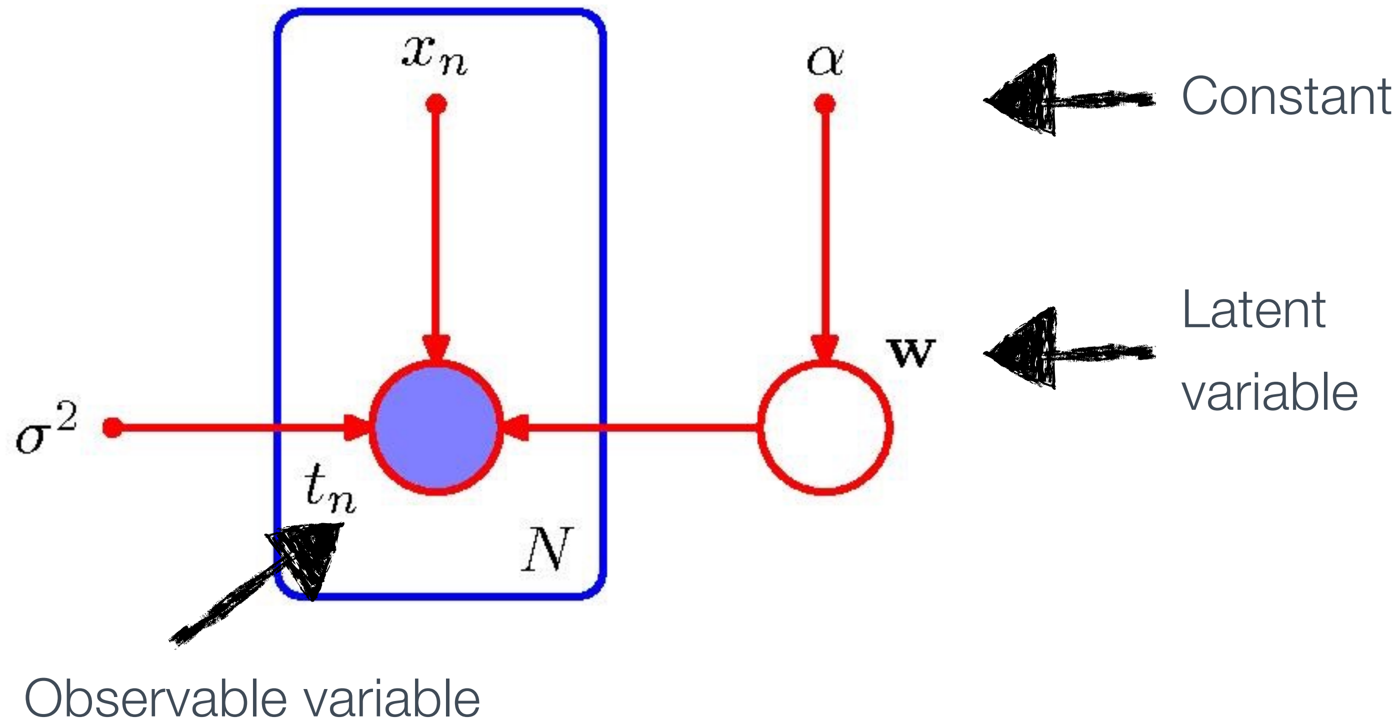
INCLUDING PARAMETERS

$$p(\mathbf{t}, \mathbf{w}|\mathbf{x}, \alpha, \sigma^2) = p(\mathbf{w}|\alpha) \prod_{n=1}^{N} p(t_n|\mathbf{w}, x_n, \sigma^2)$$



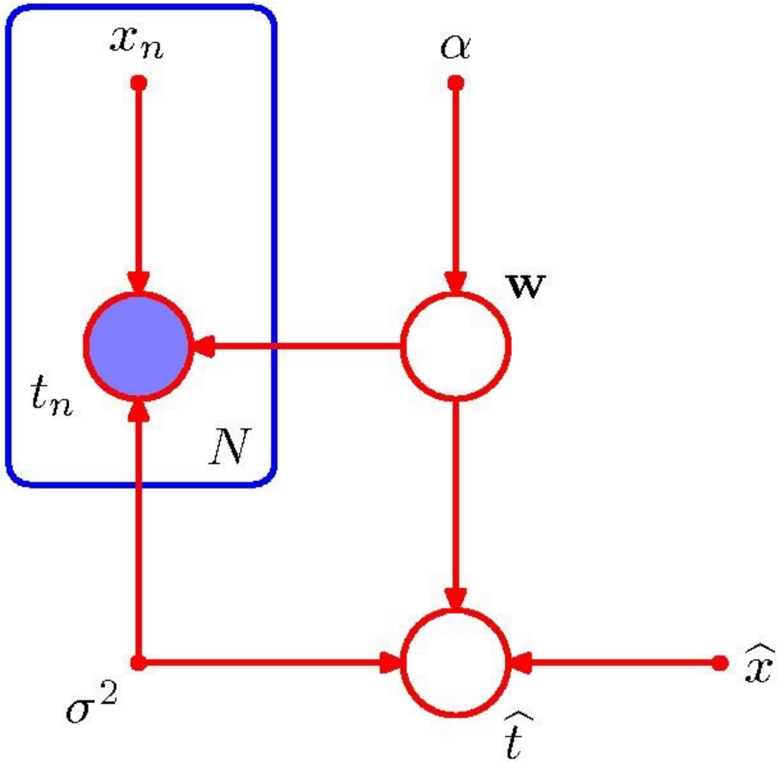
INCLUDING PARAMETERS

$$p(\mathbf{t}, \mathbf{w}|\mathbf{x}, \alpha, \sigma^2) = p(\mathbf{w}|\alpha) \prod_{n=1}^{N} p(t_n|\mathbf{w}, x_n, \sigma^2)$$



JOINT WITH NEW POINT

$$p(\widehat{t}, \mathbf{t}, \mathbf{w}|\widehat{x}, \mathbf{x}, \alpha, \sigma^2) = \left[\prod_{n=1}^{N} p(t_n|x_n, \mathbf{w}, \sigma^2)\right] p(\mathbf{w}|\alpha) p(\widehat{t}|\widehat{x}, \mathbf{w}, \sigma^2)$$



THE END