Department of Mathematics
SF2729 Groups and Rings
Period 2, 2015

HOMEWORK 4

Solving homework problems is an important practice and can improve your grade. It is therefore especially important that your work is:

- legible and written in an understandable way with full sentences and complete arguments,
- original, i.e. not copied or paraphrased from another source, and
- handed in on time.

Submission. The solutions can be hand-written or typed and should be submitted before the deadline, Monday November 30, 2pm. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `boij@kth.se`. If submitted by email, the homework should be in one pdf-file and typed or scanned with high contrast.

Scoring. The maximal total score from all twelve sets of homework is 36 and the total score will be divided by four and rounded up when counted towards the first part of the final exam. For each set of homework problems, the maximal score is 3 which correponds to $2/3$ of the points of the problems, i.e., $\min\{3, \Sigma/2\}$.

Problem 1.

(a) Show that a non-trivial group homomorphism from a simple group has to be injective. **(2 p)**

(b) Show that the center of any simple non-abelian group is trivial. **(1 p)**

Problem 2. Let N be a normal subgroup of G contained in the kernel of the homomorphism $\Phi\colon G \longrightarrow H$. Show that there is a homomorphism $\Psi\colon G/N \longrightarrow H$ such that $\Phi = \Psi \circ \Theta$, where $\Theta\colon G \longrightarrow G/N$ is the natural quotient homomorphism. **(3 p)**

Problem 3. Let $\mathbb{F}$ be a field and let G be the group of matrices in $\mathrm{GL}_4(\mathbb{F})$ of the form

$$\begin{bmatrix} A & B \\ 0 & C \end{bmatrix}$$

where A, B and C are 2×2-matrices. Find a normal subgroup N of G such that G/N is isomorphic to $\mathrm{GL}_2(\mathbb{F}) \times \mathrm{GL}_2(\mathbb{F})$. **(3 p)**