Department of Mathematics

SF1626
Several Variable Calculus
Academic year 2015/2016, Period 2

Seminar 5

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a hand-in of one of the problems. Solve problems 1-4 below and write down your solutions on separate sheets. Write your name and personal number on the top of each page. When the seminar starts you will be informed about which problem to hand in. Before starting with the seminar problems you should solve the recommended exercises from the text book Calculus by Adams and Essex (8th edition). These exercises are:

Section	Recommended problems
15.1	3, 5, 17,
15.2	3, 5, 7, 21
15.3	7, 11
15.4	1, 5, 7, 15
15.5	1, 7, 13
15.6	5, 9, 13, 15

PROBLEMS

Problem 1. Let $\mathbf{F}$ be the vector field given by

$$\mathbf{F}(x, y) = \left(x^3 y, xy^3\right)$$

for (x, y) in $\mathbb{R}^2$.

(a) Determine the field lines of $\mathbf{F}$.
(b) Determine whether $\mathbf{F}$ is konservative.
(c) Compute the line integral $\int_{C_R} \mathbf{F} \cdot d\mathbf{r}$ where C_R is a circle with radius R centered at the origin, which is traversed counter-clockwise.

Problem 2. Let g be the function given by

$$g(x, y) = x^2 + 4y^2$$

for all (x, y) in $\mathbb{R}^2$.

(a) Find a vector field $\mathbf{F}$ such that the field lines of $\mathbf{F}$ are level curves of g.
(b) Are there several such vector fields? Is it possible to determine all such vector fields?
(c) Is there a conservative vector field with these field lines?

Problem 3. Let $\mathbf{F}$ be the vector field that outside the origin in $\mathbb{R}^2$ is given by

$$\mathbf{F}(x, y) = \left(\frac{ax - by}{x^2 + y^2}, \frac{bx + ay}{x^2 + y^2} \right)$$

where x and y are coordinates measured in meters in an orthogonal coordinate system and $a = 1{,}0$ Nm, $b = 2{,}0$ Nm. A particle travels along a curve C according to

$$\mathbf{r}(t) = \left(Re^{-kt} \cos \omega t, Re^{-kt} \sin \omega t \right),$$

where $R = 6{,}1$ m, $k = 3{,}2$ s^{-1}, $\omega = 1{,}6$ rad/s och $0{,}0 \text{ s} \leq t \leq 4{,}5$ s.

(a) Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

(b) Is $\mathbf{F}$ conservative?

Problem 4. Let S be the surface that in spherical coordinates is given by $r = 2$ and $0 \leq \theta \leq \pi/3$. The orientation of the surface is given the the normal vector is pointing away from the origin.

Compute the flux of the vector field $\mathbf{F} = (x^2, y^2, z^2)$ through S.