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HOMWORK 6

Solving homework problems is an important practice and can improve your grade. It is therefore especially important that your work is:

- legible and written in an understandable way with full sentences and complete arguments,
- original, i.e. not copied or paraphrased from another source, and
- handed in on time.

**Submission.** The solutions can be hand-written or typed and should be submitted before the deadline, Monday December 14, 2pm. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `boij@kth.se`. If submitted by email, the homework should be in one pdf-file and typed or scanned with high contrast.

**Scoring.** The maximal total score from all twelve sets of homework is 36 and the total score will be divided by four and rounded up when counted towards the first part of the final exam. For each set of homework problems, the maximal score is 3 which corresponds to  $2/3$  of the points of the problems, i.e.,  $\min\{3, \Sigma/2\}$ .

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**Problem 1.** Determine all the Sylow  $p$ -subgroups of  $D_{2n}$  for all  $p$  dividing  $2n$ , when  $n$  is odd. (3 p)

**Problem 2.** Determine the number of Sylow  $p$ -subgroups of  $GL_2(\mathbb{F}_5)$  for all primes  $p$  dividing the order of the group. (3 p)

**Problem 3.**

(a) Determine the order of the group of inner automorphisms of  $S_4$ . (1 p)

(b) Use the presentation of  $S_4$  as

$$S_4 = \langle s_1, s_2, s_3 \mid s_1^2 = s_2^2 = s_3^2 = (s_1 s_2)^3 = (s_2 s_3)^3 = (s_1 s_3)^2 = 1 \rangle$$

in order to determine  $\text{Aut}(S_4)$ . (2 p)