
**Department of Mathematics**

SF1626
Several Variable Calculus
Academic year 2015/2016, Period 2

# Seminar 6

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a quiz on a variant of one of the recommended exercises from the text book Calculus by Adams and Essex (8th edition) which are marked by boldface in the following list:

| Section | Recommended exercises |
|---|---|
| 16.1 | 3, 7, 11 |
| 16.2 | 9, 15, 17 |
| 16.3 | **3**, **5**, 9 |
| 16.4 | **5**, **11**, **15** |
| 16.5 | 1, **3**, 5 |

In the seminar the following problems will be discussed.

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## PROBLEMS

**Problem 1.** Let $\mathbf{F}$ be the vector field given by

$$\mathbf{F}(x, y, z) = (x^2 - yz,\ y^2 - xz,\ z^2 - xy).$$

for all $(x, y, z)$ in $\mathbb{R}^3$.

(a) Determine $\operatorname{rot} \mathbf{F} = \nabla \times \mathbf{F}$ and $\operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F}$.

(b) Determine whether there is a potential, i.e., a function $g$ such that $\mathbf{F} = \operatorname{grad} g$.

(c) Compute the flux of $\mathbf{F}$ out from the sphere given by

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2.$$

**Problem 2.** Let $\mathbf{F} = \operatorname{rot} \mathbf{G} = \nabla \times \mathbf{G}$ where

$$\mathbf{G}(x, y, z) = (z^2 - y^2,\, x^2 - z^2,\, y^2 - x^2)$$

(a) Compute the flux of $\mathbf{F}$ through the triangle with verices in $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$. (The normal direction is chosen so that it points away from the origin.)

(b) Use Stoke'e Theorem in order to relate the flux from part (a) to a line integral and compute this line integral by means of a parametrization.