**Matematik, KTH** **SF2740, Graph Theory**
**Svante Linusson** **HT 2015**

**Third set of hand in problems**

Be sure to write solutions with clear arguments that are easy to follow. You should try to have a level of details so your solution would be understandable to other students. Staple your solution together in the top left corner and write down your solutions in order. Write your name in the top right corner.

**Code of conduct(Hederskodex):** It is assumed that:
-you shall solve the problems on your own (or in cooperation with one or two fellow students) and write down your own solution. If you cooperate with someone you must mention that persons name for each problem.
- you must not use other resources
- if you in spite of this are using something you have gotten from somewhere else for some reason (a friend, a book or the internet etc.) you must give a reference to the source.

**Your solutions to these problems are due January 10, by email or regular mail or in the mail box at the entrance of the math department.**

(1) (10p) For two graphs $G$ and $H$, let $G \vee H$ be the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H) \cup \{xy : x \in V(G), y \in V(H)\}$. This is called the *join* of the two graphs. Prove that
a) $K^3 \vee C^5$ does not contain $K^6$ as induced subgraph.
b) Every coloring of the edges of $K^3 \vee C^5$ using only two colors contains a monochromatic triangle.

(2) (10p) Prove that the probabilities stated in (8.2.1) in the text by Remco van der Hofstad about the Preferential attachment model sum to 1 (this is Exercises 8.3 in that same text). Note: Only the case when $m = 1$. Note also the notation used $[t] := \{1, 2, ..., t-1, t\}$.

(3) (10p) Let $G = (V, E)$ be a graph and let $m := ||G||$ be the number of edges. Show that $G$ contains a $k$-partite subgraph with $m(k-1)/k$ edges.
(Hint: Consider a random coloring of $V(G)$ with $k$ colors and study the expected number of edges with the same color in both endvertices.)

The following two exercises might be more difficult.

(4) (10p) Define a random graph $G_n$ with vertex set $\{1, \ldots, n\}$ by choosing a random$^*$ partition $\{a_1, b_1\}, \{a_2, b_2\}, \ldots, \{a_n, b_n\}$ of $\{1, 2, \ldots, 2n\}$, say, with $a_i < b_i$ and join vertices $i$ and $j$ in $G_n$ if $[a_i, b_i] \cap [a_j, b_j] \neq \emptyset$.
a) Compute the expected number of edges in $G_n$.
b) Show that almost every $G_n$ is connected

(5) (10p) Let $G$ be a graph such that $|G| = n \geq 1$ and $||G|| = \frac{nd}{2}$, some real number $d \geq 3/2$. Prove that $\alpha(G) \geq \frac{n}{2d}$. (Hint: Think about the properties of the complement of the Turan graph.)

Lycka till!

Svante