KTH Electrical Engineering

# Exam in EG2050 System Planning,
# 12 January 2016, 8:00–13:00, D34

## Allowed aids

In this exam you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

## PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

## Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

**a) (1 p)** Which of the following players have the responsibility to continuously maintain the physical balance between generation and consumption?

1. Each producer and consumer.
2. The balance responsible players.
3. The system operator.

**b) (2 p)** AB Elbolaget is a balance responsible retailer of electricity. The electricity trading on the market where AB Elbolaget is active has a trading period of one hour. AB Elbolaget has only one customer, namely AB Industri. Assume that the consumption of AB Industri is varying between 800 MW and 1 250 MW with an average of 1 000 MW during the hour. In which the following cases is AB Elbolaget fulfilling its balance responsibility without any imbalance: I) Then the generation of AB Elbolaget is varying between 600 MW and 950 MW with an average of 800 MW during the hour, II) Then the generation of AB Elbolaget is varying between 600 MW and 950 MW with an average of 800 MW during the hour; moreover, Elbolaget is buying 200 MWh from the local power exchange, ElKräng, III) Then AB Elbolaget buys 800 MWh from ElKräng.

1. None of the statements is true.
2. Only I is true.
3. Only II is true.
4. Only III is true.
5. I and II are true but not III.

**c) (1 p)** What is an up-regulation bid?

1. A power company is selling electricity to a customer and the customer must in advance notify the power company about how much the customer will consume during each trading period.
2. A player offers to increase the generation (alternatively decrease the consumption) at the request of the system operator.
3. A player offers to decrease the generation (alternatively increase the consumption) at the request of the system operator.

## Problem 2 (6 p)

The electricity market in Land has perfect competition, all players have perfect information and there are neither capacity, transmission nor reservoir limitations in the system. Data for the electricity producers of Land are given in table 1. The variable operation costs are assumed to be linear within the intervals; the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

**Table 1** Data for the electricity producers in Land.

| Power source | Production capability [TWh/year] | Variable costs [¤/MWh] |
|---|---|---|
| Hydro power | 60 | 5 |
| Nuclear power | 60 | 100 |
| Biofuel | 20 | 100–300 |
| Fossil fuels | 20 | 200–400 |

**a) (3 p)** What will the electricity price be in Land if the electricity consumption is not price sensitive and amounts to 145 TWh/year?

**b) (3 p)** Assume that the electricity consumption is price sensitive. Data for the electricity consumption in Land is provided in table 2. The consumers' willingness to pay is assumed to be linear within the intervals; the consumption is zero if the price is at the higher price level and the consumption is maximal at the lower price level. What will the electricity price become in this electricity market under these conditions?

**Table 2** Data for the electricity consumption in Land.

| Type of consumer | Maximal consumption [TWh/year] | Willingness to pay [¤/MWh] |
|---|---|---|
| Base load | 140 | 1 000 |
| Price sensitive load | 10 | 210–310 |

## Problem 3 (6 p)

The small island Skäret is not connected to the national grid, but there is a local grid which is supplied by two diesel generator sets. There are two load centres on the island, Torp and Brygga, and there is one diesel generator set in each load centre; data for the diesel generator sets are provided in table 3. There is an AC line between Torp and Brygga with a maximal capacity of 150 kW. The line is equipped with a with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. The losses on the line are negligible.

**Table 3** Data for the diesel generator sets on Skäret.

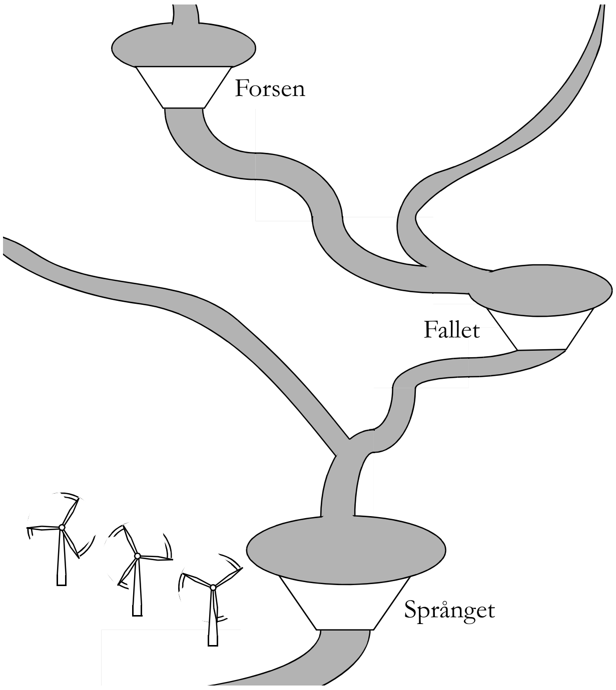
| | Torp | Brygga |
|---|---|---|
| Maximal generation [kW] | 150 | 200 |
| Minimal generation [kW] | 20 | 40 |
| Generation at 49.92 Hz | 76 | 156 |
| Generation at 50.04 Hz | 52 | 132 |

**a) (2 p)** How large is the gain of each diesel generator set?

**b) (2 p)** How large is the power flow on the AC line if the frequency is 49.96 Hz and the load in Torp is 135 kW? (Answer 0 kW if the line is disconnected due to overloading.)

**c) (2 p)** A homeowner in Brygga is connecting photovoltaic panels to the grid (via an inverter) at the time described in question b. The generation of the photovoltaic panels is 10 kW and the efficiency of the inverter is 95%. How large is the power flow on the AC line if the frequency when the primary control has stabilised the frequency of the system? (Answer 0 kW if the line is disconnected due to overloading.)

## Problem 4 (12 p)




AB Vattenkraft owns three hydro power plants and a wind farm located as in the figure above. AB Vattenkraft sells power to the local power exchange, ElKräng. The result of the trading at ElKräng is published at 1 pm and then AB Vattenkraft knows how much they will have to generate every hour the following day. Their short-term planning problem is now to maximise the value of stored water while generating the contracted load. The following symbols have been introduced for this planning problem:

Indices for the power plants: Forsen 1, Fallet 2, Språnget 3.

$\gamma_i$ = expected future production equivalent for water stored in reservoir $i$, $i$ = 1, 2, 3,
$D_t$ = contracted load hour $t$, $t$ = 1, …, 24,
$\lambda_t$ = expected electricity price at ElKräng hour $t$, $t$ = 1, …, 24,
$\lambda_{25}$ = expected electricity price at ElKräng after the end of the planning period,
$M_{i,0}$ = contents of reservoir $i$ at the beginning of the planning period, $i$ = 1, 2, 3,
$M_{i,t}$ = contents of reservoir $i$ at the end of hour $t$, $i$ = 1, 2, 3, $t$ = 1, …, 24,
$\mu_{i,j}$ = marginal production equivalent in power plant $i$, segment $j$, $i$ = 1, 2, 3, $j$ = 1, 2,
$Q_{i,j,t}$ = discharge in power plant $i$, segment $j$, during hour $t$, $i$ = 1, 2, 3, $j$ = 1, 2, $t$ = 1, …, 24,
$S_{i,t}$ = spillage from reservoir $i$ during hour $t$, $i$ = 1, 2, 3, $t$ = 1, …, 24,
$\overline{S}_i$ = maximal spillage from reservoir $i$, $i$ = 1, 2, 3,
$V_{i,t}$ = local inflow to reservoir $i$ during hour $t$, $i$ = 1, 2, 3, $t$ = 1, …, 24,
$W_t$ = expected generation of the wind farm in hour $t$, $t$ = 1, …, 24.

**a) (4 p)** Formulate the objective function. Use the symbols defined above.

**b) (4 p)** Formulate the load balance constraint for hour $t$. Use the symbols defined above.

**c) (4 p)** The best efficiency in the hydro power plant Språnget is obtained at the discharge 100 m$^3$/s and the production equivalent is then 0.50 MWh/HE. The maximal discharge is 140 m$^3$/s and then the relative efficiency is 96%. Assume that we need a piecewise linear model of electricity generation as function of the discharge in Språnget. The model should have two segments and the breakpoint between them should be located at the best efficiency. Calculate the following parameters:

$\underline{\mu}_{3,j}$ = marginal production equivalent in Språnget, segment $j$,
$\overline{Q}_{3,j}$ = maximal discharge in Språnget, segment $j$.

## Problem 5 (12 p)

Ebbuga is a small town in Eastern Africa. The town is not connected to a national grid, but has a local system of its own. The local grid is supplied by a hydro power plant and two diesel generator sets. The hydro power plant is a run-of-the-river station and it has 300 kW capacity and the risk of failure is negligible. The natural flow in the river passing by the power plant is always sufficient to generate the installed capacity. The diesel generator sets have an installed capacity of 200 kW each, the availability is 85% and the operation cost is 10 ¤/kWh. Table 4 shows some partial results of a probabilistic production cost simulation of the power system in Ebbuga.

**Table 4** Results from a probabilistic production cost simulation of the power system in Ebbuga.

| | $x_1 = 0$ $x_2 = 200$ | $x_1 = 200$ $x_2 = 300$ | $x_1 = 300$ $x_2 = 400$ | $x_1 = 400$ $x_2 = 500$ | $x_1 = 500$ $x_2 = 600$ | $x_1 = 600$ $x_2 = 700$ | $x_1 = 700$ $x_2 = 800$ | $x_1 = 800$ $x_2 = 900$ | $x_1 = 900$ $x_2 = 1\,000$ |
|---|---|---|---|---|---|---|---|---|---|
| $\tilde{F}_0(x_2)$ | 1.000 | 0.8000 | 0.2000 | 0.1000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $\int_{x_1}^{x_2} \tilde{F}_0(\xi)d\xi$ | 200.00 | 90.00 | 50.00 | 15.00 | 5.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\tilde{F}_2(x_2)$ | 1.000 | 0.8300 | 0.3200 | 0.2050 | 0.0300 | 0.0150 | 0.0000 | 0.0000 | 0.0000 |
| $\int_{x_1}^{x_2} \tilde{F}_2(\xi)d\xi$ | 200.00 | 91.50 | 57.50 | 26.25 | 11.75 | 2.25 | 0.75 | 0.00 | 0.00 |
| $\tilde{F}_3(x_2)$ | 1.000 | 0.8555 | 0.4220 | 0.2988 | 0.0735 | 0.0435 | 0.0045 | 0.0022 | 0.0000 |
| $\int_{x_1}^{x_2} \tilde{F}_3(\xi)d\xi$ | 200.00 | 92.78 | 63.88 | 36.04 | 18.61 | 5.85 | 2.40 | 0.34 | 0.11 |

**a) (2 p)** Use probabilistic production cost simulation to calculate the *total* expected generation energy per hour in the two diesel generator sets.

**b) (2 p)** Use probabilistic production cost simulation to compute the risk of power deficit in Ebbuga.

**c) (2 p)** To improve the reliability of supply, it is considered to purchase another diesel generator set with a capacity of 200 kW, operation cost 10 ¤/kWh and the availability 85%. Calculate the risk of power deficit if this extra diesel generator set is added to the system.

**d) (2 p)** Assume that Monte Carlo simulation is used to study the power system in Ebbuga, and that complementary random numbers are applied. What is the value of the complementary random number, $D^*$, if the total load of the system is randomised to $D$ = 400 kW?

**e) (4 p)** Assume that the power system from question c is simulated using a combination of complementary random numbers and control variates. The simulation comprises 1 000 original scenarios, $y_i$, $i$ = 1, …, 1 000. The corresponding complementary scenarios, $y_i$*, $i$ = 1, …, 1 000, have also been generated. The simplified model $\tilde{g}(Y)$, corresponds to the model used in probabilistic production cost simulation, whereas the detailed model, $g(Y)$, considers factors such as the losses being dependent on how the load is varying in different parts of the system. The results are shown in table 5. Which estimate of *LOLP* is obtained?

**Table 5** Results from a Monte Carlo simulation of the power system in Ebbuga.

| Detailed model | | Simplified model | |
|---|---|---|---|
| Number of loss of load occasions in the original scenarios, $\sum_{i=1}^{1000} g(y_i)$ | Number of loss of load occasions in the complementary scenarios, $\sum_{i=1}^{1000} g(y_i^*)$ | Number of loss of load occasions in the original scenarios, $\sum_{i=1}^{1000} \tilde{g}(y_i)$ | Number of loss of load occasions in the complementary scenarios, $\sum_{i=1}^{1000} \tilde{g}(y_i^*)$ |
| 12 | 9 | 9 | 7 |

## PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

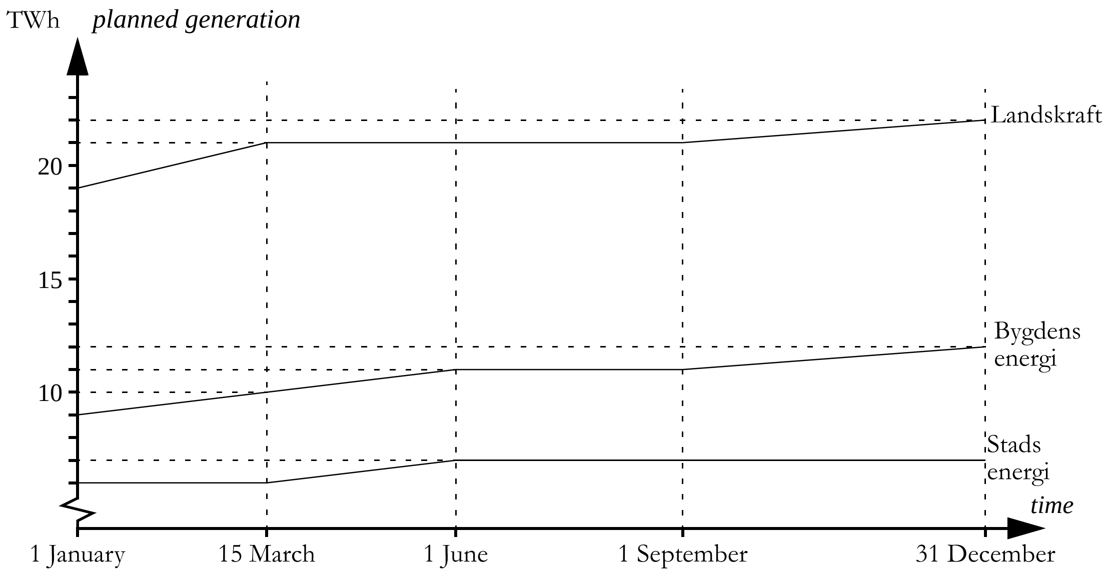
The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

## Problem 6 (10 p)

There are three hydro power producers in Land: Landskraft, Stads energi AB and Bygdens energi AB. Hydro power is an important power source in Land and contributes on average to half the annual electricity generation. However, hydro inflow is varying significantly from season to season. Fortunately, the reservoirs in the Landish hydro power system have a considerable storage capacity and can be used to balance differences between demand and inflow. However, yearly variations (i.e., differences in total inflow during wet and dry years) cannot be completely balanced, as the hydro power producers do not have perfect forecasts about the future inflow. Therefore, the hydro power producers continuously makes updated forecasts on the total inflow for the next twelve months. The forecasts are used to decide a long-term plan for hydro generation, stating how much hydro power should be generated per week. (More detailed plans for the dispatch of hydro power is decided in subsequent short-term planning problems.) Based on the weekly plans it is possible to compile how much hydro power each producer expect to generate in the next twelve months. The figure below shows how the hydro generation plans vary over one particular year for the hydro power producers. Notice that since the producers use slightly different forecasts, the plans do not vary in exactly the same way.

Once the hydro power producers have decided how much hydro they will generate, the remainder of the demand will be covered by thermal power plants and the need for thermal generation




will be determining the electricity price. Data for the thermal power plants in Land are listed in table 6. The expected thermal generation capacity does not vary during the year, except for the nuclear generation capacity. This is because on 1 August, Strålinge kraftverksgrupp makes a public announcement that during the summer maintenance, some problems were detected in one of their reactors; therefore, the maintenance period is extended to correct these problems in order to guarantee the safety of the reactor. This means that the expected generation capacity in the concerned reactor will decrease by 1 TWh during the period 1 August to 1 July next year.

**Table 6** Thermal generation in Land.

| Power source | Production capability [TWh/year] | Variable costs [¤/MWh] |
|---|---|---|
| Nuclear | 26 | 100 |
| Combined heat and power (biomass) | 14 | 100–380 |
| Coal condensing | 9 | 360–420 |
| Oil condensing | 6 | 500–560 |
| Gas turbines | 1 | 800–900 |

The electricity demand in Land has been fairly constant for the past decade and all players expect the annual consumption to be 80 TWh.

Consider a simplified model of Land, where it can be assumed that the electricity market has perfect competition and that there are neither capacity, transmission nor reservoir limitations in the system. Calculate how the electricity price in Land is going to vary during the year described above.

## Problem 7 (10 p)

The three countries Aland, Beland and Celand have a common electricity market. The countries are interconnected by AC lines as shown in the figure below; therefore, the countries also have a common system for frequency control.

The primary control is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve is available in the frequency range 49.9–50.1 Hz and is intended to manage load variations. The countries have agreed on their contribution to the normal operation reserve according to table 7.

The maximal transmission capacity between the countries is depending partly on thermal limits (i.e., the components of the transmission lines could fail due to overheating) as well as voltage stability. The players of the electricity market may however not have access to the entire transmission capacity, because the frequency control may change the flow between the countries. How large marginal must be reserved on the transmission lines if it should be possible for the system to manage the load variations listed in table 7? (Please notice that the system should be able to manage the maximal load variations occurring simultaneously in all three countries and that the load variations are considered uncorrelated.)

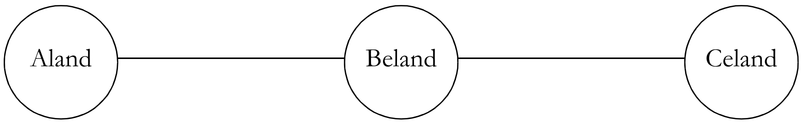



**Table 7** Data for the three countries in problem 7.

| Land | Contribution to normal operation reserve [MW/Hz] | Load variations [MW] |
|---|---|---|
| Aland | 1 500 | ±120 |
| Beland | 2 000 | ±180 |
| Celand | 500 | ±90 |

## Problem 8 (20 p)

Stads energi AB owns the thermal power plant Röksta with three blocks. Data for Röksta is given in table 8. Stads energi AB sells power to the local power exchange, ElKräng. In order to submit the bids for ElKräng in time, Stads energi AB must determine the operation plan for Wednesday already during Tuesday morning. According to the plan for Tuesday, none of the blocks in Röksta will be committed between 20–24 on Tuesday evening. When planning the operation for Wednesday, the company uses the forecasted electricity prices listed in table 9. It is also assumed that it is possible to sell unlimited quantities at the forecasted prices.

**a) (10 p)** Formulate the planning problem of Stads energi AB as a MILP problem. Use the notation in table 10 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

**Table 8** Basic data for the thermal power plant Röksta.

| | Block I | Block II | Block III |
|---|---|---|---|
| Installed capacity [MW] | 200 | 150 | 140 |
| Generation cost [SEK/MWh] | 300 | 320 | 350 |
| Minimal generation when committed [MWh/h] | 90 | 60 | 30 |
| Start-up cost [SEK/start] | 36 000 | 29 000 | 21 000 |

**Table 9** Forecast for Wednesday.

| Hour | 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 | 10-11 | 11-12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Price at ElKräng [SEK/MWh] | 225 | 220 | 220 | 220 | 230 | 235 | 255 | 325 | 435 | 345 | 435 | 340 |
| Hour | 12-13 | 13-14 | 14-15 | 15-16 | 16-17 | 17-18 | 18-19 | 19-20 | 20-21 | 21-22 | 22-23 | 23-24 |
| Price at ElKräng [SEK/MWh] | 290 | 280 | 280 | 275 | 290 | 495 | 470 | 280 | 260 | 245 | 240 | 230 |

**Table 10** Notation for the planning problem of Stads energi AB.

| Symbol | Explanation | Value |
|---|---|---|
| $\overline{G}_g$ | Installed capacity in block $g$ | See table 8 |
| $\beta_{Gg}$ | Variable generation cost in block $g$ | See table 8 |
| $\underline{G}_g$ | Minimal generation when block $g$ is committed | See table 8 |
| $C_g^+$ | Start-up cost in block $g$ | See table 8 |
| $u_{g,0}$ | Unit commitment in block $g$ before the start of the planning period | 0 |
| $\lambda_t$ | Expected electricity price at ElKräng hour $t$ | See table 9 |

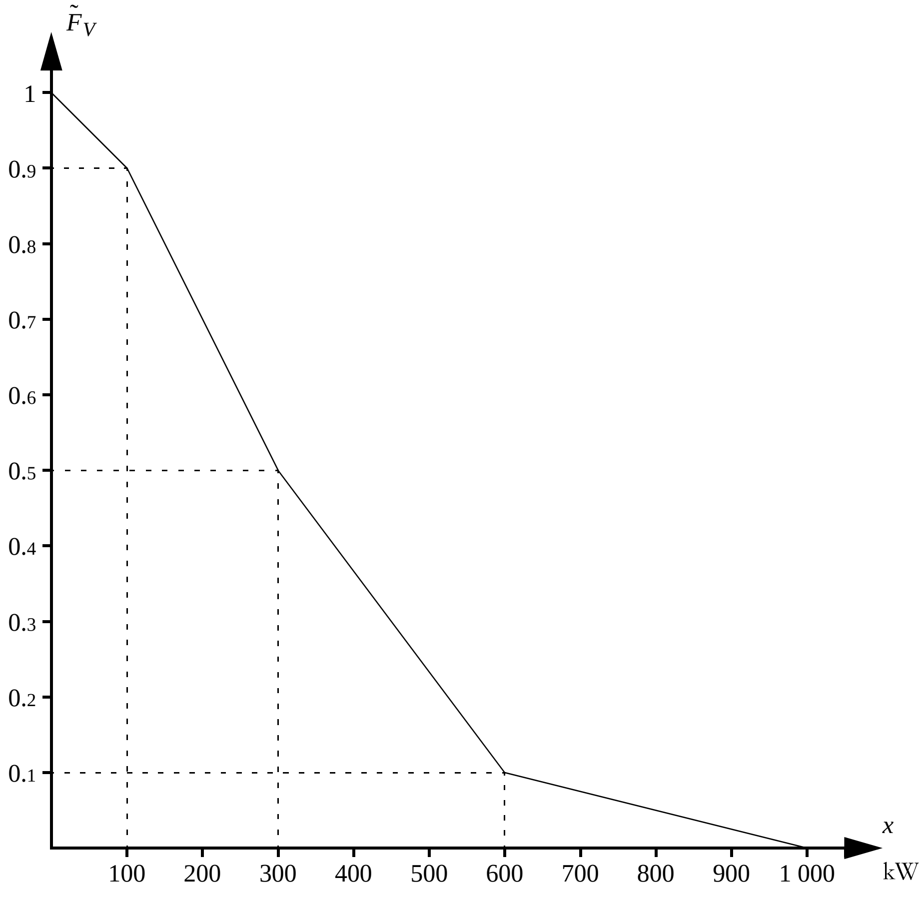
**Table 11** Ramping data for the thermal power plant Röksta.

| | Block I | Block II | Block III |
|---|---|---|---|
| Maximal ramp-rate when committed [MWh/h] | 50 | 50 | 60 |
| Maximal generation after start-up [MWh/h] | 100 | 75 | 50 |
| Maximal generation before stop [MWh/h] | 120 | 75 | 60 |

**b) (10 p)** The generation of a thermal power plant cannot be changed too rapidly. Therefore, Stads energi AB would like to impose so-called ramping constraints to the planning problem. The ramping constraints should limit the generation change from one hour to the next; moreover, there are limits for the maximal generation after start-up and the maximal generation before stop (see table 11 for detailed data). How must the planning problem from question a be reformulated in order to consider the ramping constraints? Do not forget to define all new variables and parameters that you introduce!

## Problem 9 (20 p)

Isla Desierta is located in the Atlantic Ocean. The electricity consumption of the island can be considered normally distributed with the mean 600 kW and the standard deviation 100 kW. THe power supply is managed by a local power company, Electricidad de Isla Desierta (EdID). The company can import electricity from the mainland via an HVDC transmission line, which is 100% available. This line is one-directional, i.e., it is only possible to transfer power from the mainland to Isla Desierta and not the other way around. The line has a capacity of 1 000 kW and EdID pays 1 ¤/kWh for imported electricity. The company also has a hydro power plant of their own on the island. This hydro power plant has a capacity of 500 kW, the availability is 100% and the operation cost is negligible. The hydro power plant has no reservoir; hence, the maximal generation is limited by the water flow, which depends on the precipitation and how much water is flowing from the volcanic wells of the island. A duration curve for the available water flow, $V$ (converted to kW), is shown in the figure below. The water flow can be assumed to be independent of the load in the system.



**a) (5 p)** Formulate a discrete model of the available generation capacity in the hydro power plant. The model should have three states: no generation capacity available, installed capacity available, and a state in between. Do not forget to motivate your calculations!

**b) (5 p)** Use the model from part a and probabilistic production cost simulation to calculate the expected operation cost of EdID. To simplify the calculations you may use the following approximation of the load duration curve:

$$\tilde{F}_0(x) = \begin{cases} 1 & x < 500, \\ 0.4 & 500 \le x < 750, \\ 0 & 750 \le x. \end{cases}$$

If you have not solved the problem in part a, then you may use the following model of the hydro power plant: 50% probability that installed capacity is available, 40% probability that half the installed capacity is available and 10% probability that no generation capacity is available.

**c) (3 p)** Assume that the EdID system is to be simulated using Monte Carlo methods. How is it then appropriate to model the available generation capacity in the hydro power plant? Do not forget to motivate your answer!

**d) (1 p)** How many random numbers are necessary to generate a scenario for a Monte Carlo simulation of the EdID system?

**e) (6 p)** Use an appropriate set of random numbers from table 12 to generate a scenario for a Monte Carlo simulation of the EdID system. Moreover, state the complementary scenarios that can be created from the same random numbers.

*Hint:* For symmetrical probability distributions, it holds that if $X = \mu + \delta$, where $\mu = E[X]$ and $\delta$ is an arbitrary number, then the complementary random number is $X^* = \mu - \delta$.

**Table 12** Some random numbers for a Monte Carlo simulation of the EdID system.

| Probability distribution | $U(0, 1)$ | $N(600, 100)$ |
|---|---|---|
| Random number | 0.95<br>0.25<br>0.60 | 595<br>725<br>675 |

KTH Electrical Engineering

## Answer sheet for part I

Name: ..........................................................................................................

Personal number: ..............................................................................................

### Problem 1

**a)** Alternative ..................................... is correct.

**b)** Alternative ..................................... is correct.

**c)** Alternative ..................................... is correct.

### Problem 2

**a)** ......................................... ¤/MWh **b)** ......................................... ¤/MWh

### Problem 3

**a)** Torp: ................................. kW/Hz Brygga: ............................... kW/Hz

**b)** The power flow is ................. kW from ........................ to...........................

**c)** The power flow is ................. kW from ........................ to...........................

### Problem 4

**a)** ..........................................................................................................

**b)** ..........................................................................................................

**c)** $\mu_{3,1}$ .................................. MWh/HE $\mu_{3,2}$ ...................................... MWh/HE

$\bar{Q}_{3,1}$ ................................. HE $\bar{Q}_{3,2}$ ..................................... HE

### Problem 5

**a)** ......................................... kWh/h **b)** ......................................... %

**c)** ......................................... % **d)** ......................................... kW

**e)** ......................................... %

## Problem 1

**a)** 3, **b)** 3, **c)** 2.

## Problem 2

**a)** Assume that the electricity price, $\lambda$, is in the range 200 to 300 ¤/MWh. Hydro power and nuclear power will generate 120 TWh; thus, the other two power sources must generate 25 TWh together. The contribution from biofuel and fossil fuels can be expressed as

$$\frac{\lambda - 100}{300 - 100}20 + \frac{\lambda - 200}{400 - 200}20.$$

Setting this expression equal to 25 and solving for $\lambda$ yields the electricity price $\lambda = 275$ ¤/MWh.

**b)** The electricity price is determined by the intersection of the supply and demand curves. Assume that the electricity price remains in the range 200 to 300 ¤/MWh. The supply at these price levels can be written as 120 (hydro & nuclear) + ($\lambda$/2 – 30) (biofuel and fossil fuels) and the demand can be written 150 – ($\lambda$ – 210)/10. These two expressions should be equal, which results in the electricity price $\lambda = 270$ ¤/MWh.

## Problem 3

**a)** In both generators, a frequency change of 0.12 Hz causes the generation to change by 24 kW; thus, the gain is $R = \Delta G/\Delta f = 24/0.12 = 200$ kW/Hz.

**b)** The generation in Torp can be computed for example by comparing to the generation at 49.92 Hz: $\Delta G = R \cdot \Delta f = 200 \cdot 0.04 = 8$ kW. The generation is thus 76 – 8 = 68 kW. Torp must consequently import 67 kW, which does not result in any overloading.

**c)** The primary control must decrease the generation of the diesel generator sets by 0.95 · 10 = 9.5 kW. As both generators have the same gain they will each decrease generation by 4.75 kW. In Brygga, the net increase will then be 9.5 – 4.75 = 4.75 kW; hence, the transmission must increase to 67 + 4.75 = 71.75 kW, which is still within the limits of the line.

## Problem 4

**a)** maximise $\lambda_{25}(\gamma_1 + \gamma_2 + \gamma_3)M_{1,24} + \lambda_{25}(\gamma_2 + \gamma_3)M_{2,24} + \lambda_{25}\gamma_3 M_{3,24}$.

**b)** $\sum_{i=1}^{3}\sum_{j=1}^{2} \mu_{i,j}Q_{i,j,t} + W_t = D_t$.

**c)** The following data are given in the problem text:

$\overline{Q}$ = maximal discharge in Språnget = 140,
$\hat{Q}$ = discharge in Språnget at best efficiency = 100,
$\gamma_{max}$ = maximal production equivalent in Språnget = 0.5,
$\eta(\overline{Q})$ = relative efficiency at maximal discharge in Språnget = 0.96.

To calculate the marginal production equivalents, we need the electricity generation at the corresponding discharges. These are calculated using the formula $H = \gamma_{max} \cdot \eta(V) \cdot V$:

$\hat{H}$ = electricity generation in Språnget at best efficiency = 0.5 · 1 · 100 = 50,
$\overline{H}$ = maximal electricity generation in Språnget = 0.5 · 0.96 · 140 = 67.2.

The marginal production equivalents can now be calculated according to

$$\mu_{3,1} = \frac{\hat{H}}{\hat{Q}}$$

and

$$\mu_{3,2} = \frac{\overline{H} - \hat{H}}{\overline{Q} - \hat{Q}},$$

which results in the following linear model of the power plant:

$$\mu_j = \text{marginal production equivalent in Språnget, segment } j = \begin{cases} 0.50 & j = 1, \\ 0.43 & j = 2, \end{cases}$$

$$\overline{Q}_{3,j} = \text{maximal discharge in Språnget, segment } j = \begin{cases} 100 & j = 1, \\ 40 & j = 2. \end{cases}$$

## Problem 5

**a)** The unavailability in the hydro power plant is 0%, which means that $\tilde{F}_1(x) = \tilde{F}_0(x)$. Integrals for the equivalent load duration curves are only provided up to 1 000 kW. However, as we have $\tilde{F}_1(1\,000) = \tilde{F}_3(1\,000) = 0$ we must have $\int_{1\,000}^{\infty}\tilde{F}_1(x)dx = \int_{1\,000}^{\infty}\tilde{F}_3(x)dx = 0$. The total generation is then computed by

$$EG_{23} = EENS_1 - EENS_3 = \int_{300}^{\infty}\tilde{F}_1(x)dx - \int_{700}^{\infty}\tilde{F}_3(x)dx = (50 + 15 + 5) - (2.40 + 0.34 + 0.11) =$$
$$= 67.15 \text{ kWh/h}.$$

**b)** The risk of power deficit is given by

$$LOLP = \tilde{F}_3(700) = 4.35\%.$$

**c)** The risk of power deficit is given by

$$LOLP = \tilde{F}_4(900) = 0.85\tilde{F}_3(900) + 0.15\tilde{F}_3(700) = 0.85 \cdot 0.0022 + 0.15 \cdot 0.0435 = 0.84\%.$$

**d)** The inverse transform method states that $D = F_D^{-1}(U)$, where $U$ is a $U(0, 1)$-distributed random number. Since it is the duration curve that is given in the problem, we may as well use the transform $D = \tilde{F}_D^{-1}(U)$. The original random number must then have been $U = \tilde{F}_D(400) = 0.2$. Hence, $U^* = 1 - U = 0.8$, which results in $D^* = \tilde{F}_D^{-1}(U^*) = 300$ kW.

**e)** In practice there is no need to differentiate between observations based on the original scenarios and the complementary scenarios; the expected difference between the detailed and simplified models is given by

$$m_{(LOLO-\tilde{LOLO})} = \frac{1}{2000}\left(\sum_{i=1}^{1000}(g(y_i)-\tilde{g}(y_i)) + \sum_{i=1}^{1000}(g(y_i^*)-\tilde{g}(y_i^*))\right) = 0.25\ \%.$$

The expectation value of the simplified model (which corresponds to a PPC model) was calculated to 0.84% in question c. The estimate of *LOLP* for the detailed model is thus

$$m_{LOLO} = m_{(LOLO-\tilde{LOLO})} + \mu_{\tilde{LOLO}} = 0.25 + 0.84 = 1.09\%.$$

## Problem 6

The electricity price at any given time will be a function of the expected total annual hydro power generation. As the forecasts vary linearly, it is sufficient to compute the electricity price at the breakpoints of the forecast curves (including 1 August when there is a change in the forecasted nuclear generation).

1 January

Hydro power is expected to generate 34 TWh and nuclear 26 TWh, which means that CHP and coal condensing must provide in total 20 TWh. Assume that the electricity price is at least 380 ¤/MWh (i.e., that CHP is fully utilised) ⇒ 6/9 of the coal condensing capacity is utilised ⇒ the electricity price will be 360 + 6/9·60 = 400 ¤/MWh.

15 March

Hydro power is expected to generate 37 TWh and nuclear 26 TWh, which means that CHP and coal condensing must provide in total 17 TWh. Assume that the electricity price is at least 380 ¤/MWh (i.e., that CHP is fully utilised) ⇒ 3/9 of the coal condensing capacity is utilised ⇒ the electricity price will be 360 + 3/9·60 = 380 ¤/MWh.

1 June

Hydro power is expected to generate 39 TWh and nuclear 26 TWh, which means that CHP and coal condensing must provide in total 15 TWh. Assume that the electricity price is less than 380 ¤/MWh ⇒

$$\frac{\lambda-100}{380-100}14 + \frac{\lambda-360}{420-360}9 = 15 \Rightarrow \lambda = 370\ ¤/\text{MWh}.$$

1 August

Hydro power is expected to generate 39 TWh and nuclear 25 TWh, which means that CHP and coal condensing must provide in total 16 TWh. Assume that the electricity price is less than 380 ¤/MWh ⇒

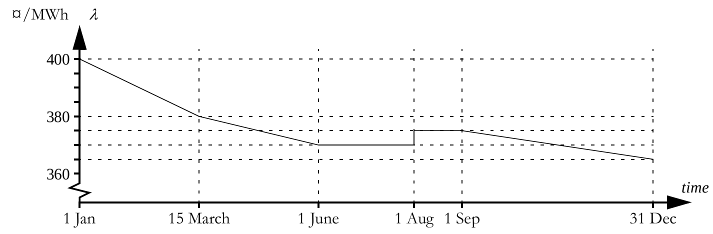
$$\frac{\lambda-100}{380-100}14 + \frac{\lambda-360}{420-360}9 = 15 \Rightarrow \lambda = 375\ ¤/\text{MWh}.$$

31 December

Hydro power is expected to generate 41 TWh and nuclear 25 TWh, which means that CHP and coal condensing must provide in total 14 TWh. Assume that the electricity price is less than 380 ¤/MWh ⇒

$$\frac{\lambda-100}{380-100}14 + \frac{\lambda-360}{420-360}9 = 15 \Rightarrow \lambda = 365\ ¤/\text{MWh}.$$

From the results above it can be conluded that the electricity price in Land is going to vary as in the figure below.




## Problem 7

The largest transmission change on the line between Aland and Beland occurs when the load decreases in Aland while it increases in Beland and Celand. This results in a net load increase for the system, which means that there is less load in Aland while at the same time the generation in the power plants participating in primary control have increased; the total excess power must be exported on the transmission line between Aland and Beland. The size of the transmission change is computed below. (Notice that the direction of the load change does not have any importance, as long as the load in Aland is changed in a different direction compared to the load in Beland and Celand.)

$$\Delta D_{tot} = \Delta D_A + \Delta D_B + \Delta D_C = -120 + 180 + 90 = 150\ \text{MW}.$$

$$\Delta G_A = \frac{R_A}{R_{tot}}\Delta D_{tot} = 1\,500/4\,000 \cdot 150 = 56.25\ \text{MW}.$$

$$\Delta P_{AB} = \Delta G_A - \Delta D_A = 56.25 + 120 = 176.25\ \text{MW}.$$

Similarly, the largest change on the interconnection between Celand and Beland occurs when the load in Celand is changing in the opposite direction compared to the load in Aland and Beland:

$$\Delta D_{tot} = \Delta D_A + \Delta D_B + \Delta D_C = 120 + 180 - 90 = 210\ \text{MW}.$$

$$\Delta G_C = \frac{R_C}{R_{tot}}\Delta D_{tot} = 500/4\,000 \cdot 210 = 26.25\ \text{MW}.$$

$$\Delta P_{CB} = \Delta G_C - \Delta D_C = 26.25 + 90 = 116.25\ \text{MW}.$$

Thus, the conclusion is that at least 176.25 MW transmission capacity must be reserved between Aland and Beland for the normal operation reserve, and between Beland and Celand, at least 116.25 MW must be reserved.

## Problem 8

**a)** The problem we want to solve is

maximise *income of sold electricity – generation costs – start-up costs,*
subject to *relation between unit committment, start and stop,*

*generation limitations.*

### *Indices for the power plants*

Block I - 1, Block II - 2, Block III - 3.

### *Parameters*

The parameters are defined in table 10 in the problem text.

### *Optimisation variables*

$G_t$ = generation in Bränninge hour $t$, $t = 1, \ldots, 24$,
$s^+_{g,t}$ = start-up variable for power plant $g$, hour $t$, $g = 1, 2, 3$, $t = 1, \ldots, 24$,
$s^-_{g,t}$ = stop variable for power plant $g$, hour $t$, $g = 1, 2, 3$, $t = 1, \ldots, 24$,
$u_t$ = unit commitment in Bränninge during hour $t$, $t = 1, \ldots, 24$.

### *Objective function*

$$\text{maximise} \quad \sum_{t=1}^{24} \sum_{g=1}^{3} (\lambda_t G_{g,t} - \beta_{Gg} G_{g,t} - C^+_g s^+_{g,t}).$$

### *Constraints*

Relation between unit commitment, start-up and stop in the thermal power plants:

$$u_{g,t} - u_{g,t-1} = s^+_{g,t} - s^-_{g,t}, \qquad g = 1, 2, 3, t = 1, \ldots, 24.$$

Maximal generation in the thermal power plants:

$$G_{g,t} \le u_{g,t}\overline{G}_g, \qquad g = 1, 2, 3, t = 1, \ldots, 24.$$

Minimal generation in the thermal power plants:

$$G_{g,t} \ge u_{g,t}\underline{G}_g, \qquad g = 1, 2, 3, t = 1, \ldots, 24.$$

### *Variable limits*

$$s^+_{g,t} \in \{0, 1\}, \qquad g = 1, 2, t = 1, \ldots, 24,$$
$$s^-_{g,t} \in \{0, 1\}, \qquad g = 1, 2, t = 1, \ldots, 24,$$
$$u_{g,t} \in \{0, 1\}, \qquad g = 1, 2, t = 1, \ldots, 24.$$

**b)** Introduce the following new parameters:

$$\Delta\overline{G}_g = \text{maximal ramp rate generation when block } g \text{ is committed} = \begin{cases} 50 & g = 1, \\ 50 & g = 2, \\ 60 & g = 3, \end{cases}$$

$$\Delta\overline{G}^+_g = \text{maximal generation after start-up of block } g = \begin{cases} 100 & g = 1, \\ 75 & g = 2, \\ 50 & g = 3, \end{cases}$$

$$\Delta\overline{G}^-_g = \text{maximal generation before stopping block } g = \begin{cases} 120 & g = 1, \\ 75 & g = 2, \\ 60 & g = 3. \end{cases}$$

We can now introduce a constraint limiting the generation increase when a block is committed or starting:

$$G_{g,t} - G_{g,t-1} \le \Delta\overline{G}_g + (\Delta\overline{G}^+_g - \Delta\overline{G}_g)s^+_{g,t}, \qquad g = 1, 2, 3, t = 1, \ldots, 24.$$

Similarly, we will need a constraint limiting the generation decrease when a block is committed or stopping:

$$G_{g,t-1} - G_{g,t} \le \Delta\overline{G}_g + (\Delta\overline{G}^-_g - \Delta\overline{G}_g)s^-_{g,t}, \qquad g = 1, 2, 3, t = 1, \ldots, 24.$$

**a)** In reality the available generation capacity can assume any value between 0 and 500 kW, but in our model we only consider three possible states: 0, 250 and 500 kW respectively. The question is which probability should be assigned to each of these states. In other words, we need a discrete approximation of the continuous duration curve for the water flow. A simple and good solution is to represent flows between 0 and 125 kW in the continuous model by the discrete state $\overline{H} = 0$ kW, flows between 125 kW and 375 kW in the continuous model corresponds to the discrete state $\overline{H} = 250$ kW, and finally all flows larger than 375 kW are represented by the discrete state $\overline{H} = 500$ kW. Consequently, we get

$$P(\overline{H} = 0) = P(V < 125) = 1 - \tilde{F}_V(125) = \{\text{see figure}\} = 0.15,$$

$$P(\overline{H} = 250) = P(V > 375) - P(V < 125) = \tilde{F}_V(125) - \tilde{F}_V(375) = \{\text{see figure}\} = 0.45,$$

$$P(\overline{H} = 500) = P(V > 375) = \tilde{F}_V(375) = \{\text{see figure}\} = 0.4.$$

By that, we have identified three states for the hydro power plant and calculated the probability of each state, which is what is needed for a power plant model in a probabilistic production cost simulation.

**b)** *ETOC* depends on the cost to import electricity from the mainland. Therefore, we need to calculate *EENS* with and without the import. As the hydro power plant has the least operation cost, we add this unit first:

$$\tilde{F}_1(x) = 0.4\tilde{F}_0(x) + 0.45\tilde{F}_0(x - 250) + 0.15\tilde{F}_0(x - 500) =$$
$$= \begin{cases} 0.4 \cdot 1 + 0.45 \cdot 1 + 0.15 \cdot 1 = 1 & x < 500, \\ 0.4 \cdot 0.4 + 0.45 \cdot 1 + 0.15 \cdot 1 = 0.76 & 500 \le x < 750, \\ 0.4 \cdot 0 + 0.45 \cdot 0.4 + 0.15 \cdot 1 = 0.33 & 750 \le x < 1\,000, \\ 0.4 \cdot 0 + 0.45 \cdot 0 + 0.15 \cdot 0.4 = 0.06 & 1\,000 \le x < 1\,250, \\ 0 & 1\,250 \le x. \end{cases}$$

The electricity import can be considered as a 100% reliable power plant with a capacity of 1 000 kW, i.e., $\tilde{F}_2(x) = \tilde{F}_1(x)$. Thus, we get

$$EENS_1 = \int_{500}^{\infty} \tilde{F}_1(x)dx = 250 \cdot 0.76 + 250 \cdot 0.33 + 250 \cdot 0.06 = 287.5 \text{ kWh/h},$$

$$EENS_2 = \int_{1\,500}^{\infty} \tilde{F}_2(x)dx = 0 \text{ kWh/h}.$$

The expected import is $EENS_1 - EENS_2 = 237.5$ kWh/h, which results in $ETOC = 287.5$ ¤/h.

**c)** We do not need a discrete approximation of the hydro power plant in the Monte Carlo simulation. To calculate the available generation capacity in a specific scenario, we just randomise a water flow from the given duration curve.

**d)** In each scenario there are two scenario parameters (i.e., values that vary in a random manner from scenario to scenario): water flow and electricity consumption. Hence, we need two random numbers to create a scenario.

**e)** Assume that the first random number from the $U(0, 1)$-distribution is used to generate the water flow and that the first random number from the $N(600, 100)$-distribution is used to generate the total load.

The water flow is calculated using the inverse transform method: $V = F_V^{-1}(0.95)$ = {see figure} = 50 kW. The complementary random number is then $V^* = F_V^{-1}(1 - 0{,}95)$ = {see figure} = 800 kW. Hence, we get the available generation capacities $\overline{H} = 50$ kW and $\overline{H}^* = 500$ kW respectively. (In the second case we cannot exceed the installed capacity of the power plant, even though the water flow is large enough to generate more than 500 kW).

The load is given directly by the first random number form the normal distribution: $D = 595$ kW. As the normal distribution is symmetrical, we then have $D^* = 605$ kW.

These random numbers and complementary random numbers can be combined into four scenarios:

- **Scenario 1.** Available hydro power capacity: 50 kW. Load: 595 kW.
- **Scenario 2.** Available hydro power capacity: 50 kW. Load: 605 kW.
- **Scenario 3.** Available hydro power capacity: 500 kW. Load: 595 kW.
- **Scenario 4.** Available hydro power capacity: 500 kW. Load: 605 kW.