Advanced Digital Communications (EQ2410)

Lecture 1, Period 3, 2016

Task 1 Gather in groups of 3-4 students, and go through the proof of Theorem 5.2.1. Discuss the steps of the proof and try to explain it to each other.

Proof of Theorem 5.2.1 We can prove this result using either the hypothesis testing framework of Chapter 3, or the broader parameter estimation framework of Chapter 4. Deciding on the sequence $\mathbf{b}$ is equivalent to testing between all possible hypothesized sequences $\mathbf{b}$, with the hypothesis $H_{\mathbf{b}}$ corresponding to sequence $\mathbf{b}$ given by

$$H_{\mathbf{b}} : y(t) = s_{\mathbf{b}}(t) + n(t),$$

where

$$s_{\mathbf{b}}(t) = \sum_n b[n]p(t - nT)$$

is the noiseless received signal corresponding to transmitted sequence $\mathbf{b}$. We know from Theorem 3.4.3 that the ML rule is given by

$$\delta_{\text{ML}}(y) = \arg \max_{\mathbf{b}} \ \text{Re}(\langle y, s_{\mathbf{b}} \rangle) - \frac{||s_{\mathbf{b}}||^2}{2}.$$

The MPE rule is similar, except for an additive correction term accounting for the priors. In both cases, the decision rule depends on the received signal only through the term $\langle y, s_{\mathbf{b}} \rangle$. The optimal front end, therefore, should capture enough information to be able to compute this inner product for all possible sequences $\mathbf{b}$.

We can also use the more general framework of the likelihood function derived in Theorem 4.2.1 to infer the same result. For $y = s_{\mathbf{b}} + n$, the likelihood function (conditioned on $\mathbf{b}$) is given by

$$L(y|\mathbf{b}) = \exp\left(\frac{1}{\sigma^2} [\text{Re}\langle y, s_{\mathbf{b}} \rangle - \frac{||s_{\mathbf{b}}||^2}{2}] \right).$$

We have sufficient information for deciding on $\mathbf{b}$ if we can compute the preceding likelihood function for any sequence $\mathbf{b}$, and the observation-dependent part of this computation is the inner product $\langle y, s_{\mathbf{b}} \rangle$.

Let us now consider the structure of this inner product in more detail.

$$\langle y, s_{\mathbf{b}} \rangle = \langle y, \sum_n b[n]p(t - nT) \rangle = \sum_n b^*[n] \int y(t)p^*(t - nT)\, \mathrm{d}t = \sum_n b^*[n]z[n],$$

where $\{z[n]\}$ are as in (5.3). Generation of $\{z[n]\}$ by sampling the outputs of the matched filter (5.2) at the symbol rate follows immediately from the definition of the matched filter. □

While the matched filter is an analog filter, as discussed earlier, it can be implemented in discrete time using samples at the output of a wideband analog filter. A typical implementation is shown in Figure 5.3. The matched filter is implemented in discrete time after estimating the effective discrete-time channel (typically using a sequence of known training symbols) from the input to the transmit filter to the output of the sampler after the analog filter.

For the suboptimal equalization techniques that we discuss, it is not necessary to implement the matched filter. Rather, the sampled outputs of the analog filter can be processed directly by an adaptive digital filter that is determined by the specific equalization algorithm employed.

[Madhow, Fundamentals of Dig. Comm., 2008]

Task 2 Gather in groups of 3-4 students, and explain to each other based on the running example in Madhow how the Viterbi algorithm can be applied for equalization.

Running example For our running example, it is easy to see from Figure 5.1 that $p(t)$ only has nontrivial overlap with $p(t-nT)$ for $n = 0, \pm 1$. In particular, we can compute that $h[0] = 3/2$, $h[1] = h[-1] = -1/2$, and $h[n] = 0$ for $|n| > 1$.

Branch metric for running example For our running example in Figure 5.1, suppose that we employ BPSK modulation, with $b[n] \in \{-1, +1\}$. The number of states is given by $M^L = 2^1 = 2$. The state at time n is $s[n] = b[n-1]$. The branch metric in going from state $s[n] = b[n-1]$ to state $s[n+1] = b[n]$ is given by specializing (5.13), to obtain

$$\begin{aligned}\lambda_n(b[n], s[n]) &= \lambda_n(s[n] \rightarrow s[n+1]) \\ &= \mathrm{Re}(b^*[n]z[n]) - \frac{h[0]}{2}|b[n]|^2 - \mathrm{Re}[b^*[n]b[n-1]h[1]].\end{aligned}$$

Since $\{b[k]\}$ are real-valued, we see that only $y[n] = \mathrm{Re}(z[n])$ (i.e. the I component of the samples) affects the preceding metric. Furthermore, since $|b[n]|^2 \equiv 1$, the second term in the preceding equation does not depend on $b[n]$ (since $|b[n]|^2 \equiv 1$), and can be dropped from the branch metric. (This simplification applies more generally to PSK alphabets, but not to constellations with amplitude variations, such as 16-QAM.) We therefore obtain the modified metric

$$\begin{aligned}m_n(b[n], s[n]) &= m_n(s[n] \rightarrow s[n+1]) \\ &= b[n]y[n] + \frac{1}{2}b[n]b[n-1] \\ &= b[n]\left(y[n] + \frac{1}{2}b[n-1]\right). \qquad (5.16)\end{aligned}$$

[Madhow, Fundamentals of Dig. Comm., 2008]

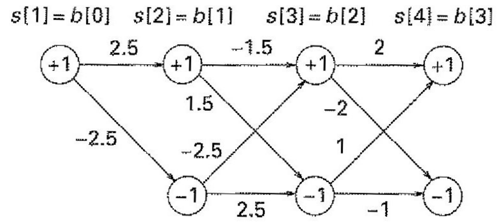
Suppose now that we know that $b[0] = +1$, and that the first few samples at the output of the matched filter are given by $y[0] = -1$, $y[1] = 2$, $y[2] = -2$, and $y[3] = 1.5$. We can now use (5.16) to compute the branch metrics for the trellis. Figure 5.4 shows the corresponding trellis, with the branches labeled by the corresponding metrics. Note that, since we know that $s[1] = b[0] = +1$, we do not need the value of $y[0]$. The first branch metric we need is

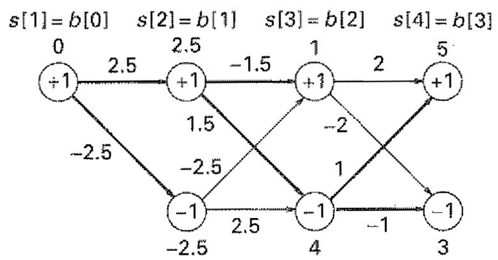
$$m_1(s[1] \to s[2]) = b[1]y[1] + \frac{1}{2}b[0]b[1].$$

We compute this for $b[0] = +1$ and for $b[1] = \pm 1$. After this, we compute the branch metrics

$$\begin{aligned} m_n(s[n] \to s[n+1]) &= b[n]y[n] - \frac{1}{2}b[n]b[n-1] \\ &= b[n](y[n] - \frac{1}{2}b[n-1]) \end{aligned}$$

for $b[n] = \pm 1$ and $b[n+1] = \pm 1$ for $n = 1, 2, 3$.





[Madhow, Fundamentals of Dig. Comm., 2008]