Advanced Digital Communications (EQ2410)

Lecture 2, Period 3, 2016

Task 1 Gather in groups of 3-4 students and verify the following example.

Running example Consider our running example of Figure 5.2, and consider a receive filter $g_{RX}(t) = I_{[0,1]}$. Note that this receive filter in this example is not matched to either the transmit filter or to the cascade of the transmit filter and the channel. The symbol interval $T = 2$, and we choose a sampling interval $T_s = 1$; that is, we sample twice as fast as the symbol rate. Note that the impulse response of the receive filter is of shorter duration than that of the transmit filter, which means that it has a higher bandwidth than the transmit filter. While we have chosen timelimited waveforms in the running example for convenience, this is consistent with the discussion in Section 5.2, in which a wideband filter followed by sampling, typically at a rate faster than the symbol rate, is employed to discretize the observation with no (or minimal) loss of information. The received samples are given by

$$r[k] = (y * g_{RX})(k) = \int_{k-1}^{k} y(t)\mathrm{d}t.$$

The sampled response to the symbol $b[0]$ can be shown to be

$$\left(\ldots, 0, 1, \frac{1}{2}, -\frac{1}{2}, 0, \ldots\right). \tag{5.20}$$

The sampled response to successive symbols is shifted by two samples, since there are two samples per symbol. This defines the signal contribution to the output. To define the noise contribution, note that the autocovariance function of the complex Gaussian noise samples is given by

$$C_w[k] = 2\sigma^2 \delta_{k0}.$$

That is, the noise samples are complex WGN. Suppose, now, that we wish to make a decision on the symbol $b[n]$ based on a block of five samples $\mathbf{r}[n]$, chosen such that $b[n]$ makes a strong contribution to the block. The model for such a block can be written as

$$\mathbf{r}[n] = b[n-1]\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix} + b[n]\begin{pmatrix} 0 \\ 1 \\ \frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix} + b[n+1]\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{2} \end{pmatrix} + \mathbf{w}_n = \mathbf{U}\mathbf{b}[n] + \mathbf{w}[n], \tag{5.21}$$

where $\mathbf{w}[n]$ is discrete-time WGN,

$$\mathbf{b}[n] = \begin{pmatrix} b[n-1] \\ b[n] \\ b[n+1] \end{pmatrix} \tag{5.22}$$

is the block of symbols making a nonzero contribution to the block of samples, and

$$\mathbf{U} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \tag{5.23}$$

is a matrix whose columns equal the responses corresponding to the symbols contributing to $\mathbf{r}[n]$. The middle column corresponds to the *desired* symbol $b[n]$, while the other columns correspond to the interfering symbols $b[n-1]$ and $b[n+1]$. The columns are acyclic shifts of the basic discrete impulse response to a single symbol, with the entries shifting down by one symbol interval (two samples in this case) as the symbol index is incremented. We use $\mathbf{r}[n]$ to decide on $b[n]$ (using methods to be discussed shortly). For a decision on the next symbol, $b[n+1]$, we simply shift the window of samples to the right by a symbol interval (i.e., by two samples), to obtain a vector $\mathbf{r}[n+1]$. Now $b[n+1]$ becomes the desired symbol, and $b[n]$ and b_{n+2} the interfering symbols, but the basic model remains the same. Note that the blocks of samples used for successive symbol decisions overlap, in general.

[Madhow, Fundamentals of Dig. Comm., 2008]

Task 2 Gather in groups of 3-4 students and show that at high SNR the MMSE equalizer converges to the ZF solution.