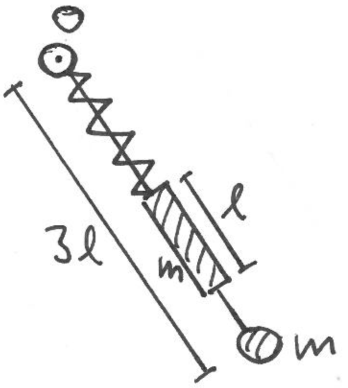
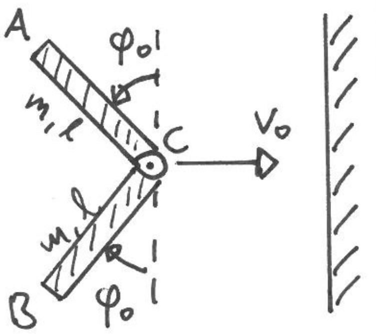
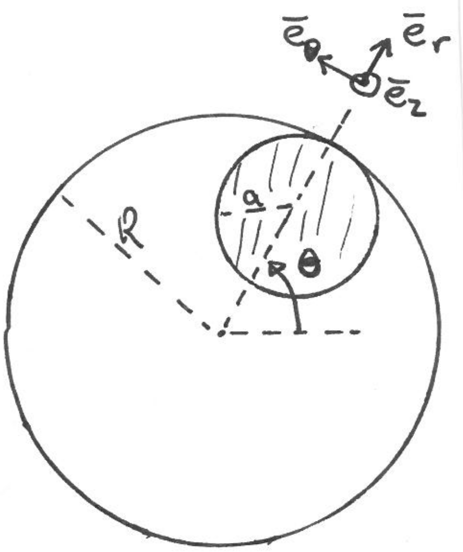
Rigid Body Dynamics (SG2150)
Exam, 2016-01-23, 9.00-13.00



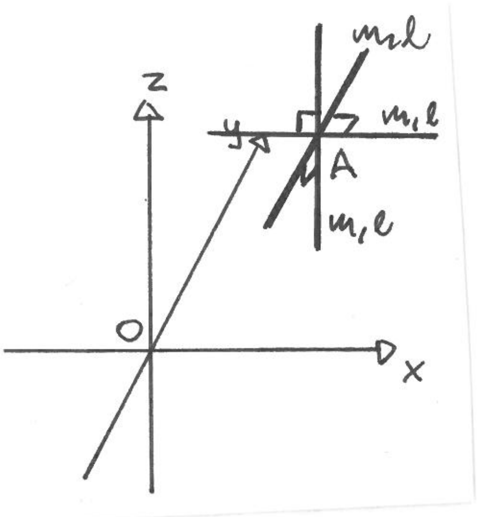
Problem 1. A particle pendulum in a vertical plane consists of a smooth light rod of length $3l$ and particle of mass m attached to the end point. A narrow homogeneous pipe of length l and mass m can slide on the rod. A linear spring with spring constant $k = 2mg/l$ and natural length l connects the pipe with the pendulum suspension point O. Find a stable equilibrium point for the system, and find the frequencies of small oscillations around that equilibirum point.



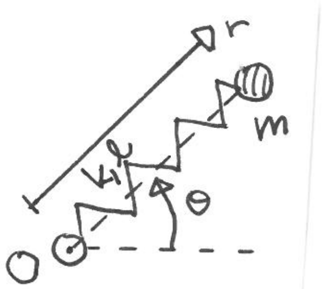
Problem 2. Two thin homogeneous rods of mass m and length l are connected at the point C through a smooth hinge joint. Initially, the rods have translational motion on a smooth horisontal surface with velocity v_0 to the right, and both rods makes an angle φ_0 with a line perpendicular to the motion. The point C then collides elastically with a wall. Compute the velocity of the end point A immediately after the collision.
Hint:Remember that an elastic collision preserves the kinetic energy, but also means that the normal velocity component of the impacting point is reversed.



Problem 3. A homogeneous ball of mass m and radius a rolls (without slipping) on the rough inside of a verical cylindrical pipe of radius R. Derive the equations of motion (with contact force components eliminated). Show that there are solutions where the ball center of mass keeps constant height. You may try to find a general solution to the equations of motion and decribe the resulting motion.
Hint: Use cylindrical coordinates for the ball center of mass, and cylindrical components $\boldsymbol{\omega} = (\omega_r \boldsymbol{e}_r + \omega_\theta \boldsymbol{e}_\theta + \omega_z \boldsymbol{e}_z)$ for the angular velocity.



Problem 4. Three thin homogeneous rods of length l and mass m, each one parallel to a coordinate axis, are orthogonally connected at their mid points. The common mid point A is displaced from the origin O by $\boldsymbol{r}_{OA} = (l/2)(\boldsymbol{e}_x + \boldsymbol{e}_y + \boldsymbol{e}_z)$. Compute the 3×3 inertia matrix $\boldsymbol{J}_O$ for the axes indicated in the figure. Also find at least one pricipal axis and at least one principal moment of inertia.



Problem 5. A particle of mass m is sliding on a smooth horisontal plane, and is connected to a fixed point O through a linear spring with spring constant k and natural length l. Find the Lagrange function. Take advantage of θ being a *cyclic* coordinate to find a corresponding conserved quantity. Show that for each fixed $r_0 > l$, there exists a *relative* equilibirum point, where both $r = r_0$ and $\dot{\theta}$ are constant in time. For the relative equilibirum point, express the value of the conserved quantity as a function of r_0. Finally, use that value of the conserved quantity to eliminate $\dot{\theta}$ from the r equation of motion, and find the frequency (as a function of r_0) of small radial oscillations about $r = r_0$.

Problem 6. Let $q_a(t)$ and $p_a(t)$ (for $a \in 1..n$) be *independent* functions of time. Suppose also that the first order variation of the integral

$$\int_{t_0}^{t_1} \left[\left(\sum_a \dot{q}_a p_a \right) - H(q, p, t) \right] dt$$

is zero when the values of t_0, t_1, $q_a(t_0)$, and $q_a(t_1)$ are kept fixed. Show that $q_a(t)$ and $p_a(t)$ then must satisfy Hamilton's equations:

$$\dot{q}_a = \frac{\partial H}{\partial p_a} \qquad \dot{p}_a = -\frac{\partial H}{\partial q_a}.$$

Each problem gives a maximum of 3 points, so that the total maximum is 18. Grading: 1–3 F; 4–5 FX; 6: E; 7–9 D; 10–12 C; 13–15 B; 16–18 A.

Allowed equipment: Handbook of mathematics and physics. One A4 page with your own compilation of formulae.

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