

Homework Set 1, due Monday February 8, 2016.

Exercise 1 *Prove the following identities by taking limits of the Forward Euler method:*

$$\int_0^T t dW(t) = T W(T) - \int_0^T W(t) dt$$

(integration by parts)

$$\int_0^T W(t) dW(t) = \frac{W(T)^2}{2} - \frac{T}{2}$$

Exercise 2 *The Ornstein-Uhlenbeck process can be defined by*

$$X(t) = x_\infty + e^{-at}(x_0 - x_\infty) + b \int_0^t e^{-a(t-s)} dW(s),$$

where $a, b > 0$ are given constants.

Compute the expected value and the variance of $X(t)$. Then compute their limits as $t \rightarrow \infty$.

Compute the covariance between $X(t)$ and $X(t + \tau)$. Then compute its limits as $\tau \rightarrow \infty$ for t fixed.

Interpret the results.