
HOMEWORK 8

Submission. The solutions should be typed and converted to .pdf. Deadline for submission is Monday February 8, 14.00. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `skjelnes@kth.se`.

Score. For each set of homework problems, the maximal score is 3 points. The total score from all twelve homeworks will be divided by four when counted towards the first part of the final exam.

Problem 1.

(a) Show that $\mathbb{Z}[x]/(x^2+1)$ equals the ring of Gaussian integers. **(1 p)**
(b) Show that (5) is not prime in $\mathbb{Z}[x]/(x^2+1)$ **(1 p)**
(c) Show that $(2+x)$ is a maximal ideal in $\mathbb{Z}[x]/(x^2+1)$. **(1 p)**

Problem 2. Let k be a field, and t, x and y independent variables over k. Show the following statements.

(a) The ring $k[t]$ is isomorphic to $k[x,y]/(y-x^2+2x)$. **(1 p)**
(b) The ring $k[t,t^{-1}]$ is isomorphic to $k[x,y]/(xy-1)$. **(1 p)**
(c) The ring $k[t^2,t^3]$ (a subring of $k[t]$) is isomorphic to $k[x,y]/(y^2-x^3)$. **(1 p)**

Problem 3. (Exercise 7.4.41) A proper ideal Q in a commutative unitary ring R is *primary* if $ab \in Q$ implies that $a \in Q$ or $b^n \in Q$ for some positive integer $n > 0$. Show the following statements.

(a) The primary ideals in $\mathbb{Z}$ are (0) and (p^n), for prime numbers p and positive integers n. **(1 p)**
(b) A proper ideal Q in R is primary if and only if every zero divisor in R/Q is nilpotent. **(1 p)**
(c) If Q is a primary ideal, then its radical $\mathrm{rad}(Q)$ is a prime ideal. **(1 p)**