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EQ2320: Speech Signal Processing (Fall 2015)
Tutorial #2

1 Problem 1.12: Upsampling

Problem

We study in this problem upsampling. Upsampling by a factor of two can be implemented by inserting a zero between every two samples of the original signal. Then low-pass filtering with (normalized) cutoff frequency of 0.25 is performed.

1. Calculate the DtFT of the signal after zero insertion but before low-pass filtering. Make use of the sum of a geometric series,

$$\sum_{n=0}^{N-1} r^n = \frac{r^n - 1}{r - 1},\ for\ |r| < 1$$

2. Assume the signal has the following spectrum spectrum. Sketch the DtFT (magnitude) after zero insertion.

$$X(e^{j\omega}) = \begin{cases} \pi - \omega, & 0 \leq \omega \leq \pi \\ \pi + \omega, & -\pi \leq \omega \leq 0 \end{cases}$$

3. If the original signal was sampled at F_s, what will it sounds like is we play the upsampled signal at F_s

4. If we want to upsample by an integer factor of M we inset $M - 1$ zeros between each two samples of the original. What should be cut-off frequency of the low-pass filter to avoid aliasing

Solution

2 Problem 1.16: Effect of windowing

Problem

For short time signal analysis we cut the signal into pieces using a windowing function (a signal with a finite number of non-zero elements). This problem illustrates the behavior of different window function, in the time domain.

A rectangular window is defined as,

$$w_R(n) = \begin{cases} 1, \ 0 \leq n \leq N \\ 0, \ otherwise \end{cases} \tag{1}$$

and a Hamming window as,

$$w_H(n) = \begin{cases} 0.54 - 0.46\cos(\frac{2\pi n}{N-1}), \ 0 \leq n \leq N \\ 0, \ otherwise \end{cases} \tag{2}$$

1. Show a rectangular window has the following spectrum

$$W_R(e^{j\omega}) = \frac{\sin(N\omega/2)}{\sin(\omega/2)} e^{-j\omega\frac{N-1}{2}}$$

2. Sketch the magnitude spectrum of $W_R(e^{j\omega})$ as a function of ω
3. Derive an expression for $W_H(e^{j\omega})$
4. Sketch $W_H(e^{j\omega})$ as a function of ω
5. What are the advantages and disadvantages of using the rectangular or Hamming window.

Solution