Dielectric response of plasmas

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Overview

- Magnetoionic theory
  - anisotropic/gyrotropic
- Cold plasmas
  - Alfven velocity
- Warm plasmas
  - Distributions functions
    - Maxwellian distributions
  - The Vlasov equation
  - Landau resonance
  - Longitudinal and transverse response of warm plasmas

Reminder: Equations for calculating the dielectric response

E- & B-field exerts force on particles in media

$$m\dot{\mathbf{v}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \textit{solve for } \mathbf{v}\,!$$

The induced motion of charge particles form a current and a charge density

$$\mathbf{J}_{media} = \sum_{species} qn\mathbf{v} \qquad \frac{\partial}{\partial t}\rho_{media} + \nabla \circ \mathbf{J}_{media} = 0$$

(n=particle density)

The response can be quantified in e.g. the conductivity σ

$$J_i(\mathbf{k},\omega) = \sigma_{ij}(\mathbf{k},\omega) E_j(\mathbf{k},\omega)$$

Dielectric response for plasmas

- A first example of a plasma model is the Magnetoionic theory:
  - ***Assume***: ions are static; unperturbed by the wave field (no response)
  - ***Assume***: electrons are cold; they are initially static, but move in the presence of the wave field
  - ***Assume***: the plasma has a static and homogeneous magnetic field; align the coordinate system: $\mathbf{B}_0 = B_0 \mathbf{e}_z$

- What is the dielectric response of a magnetoionic media?
  - align also y-axis such that: $\mathbf{k} = k_y \mathbf{e}_y + k_z \mathbf{e}_z$
  - the response of the electrons is then given by Newtons equation

$$\dot{\mathbf{v}} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

  - Next: add a friction with the ions (a force $-m\nu_f \mathbf{v}$) and use $\mathbf{v} = \dot{\mathbf{r}}$

$$\ddot{\mathbf{r}} = \frac{q}{m}\left(\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B}\right) - \nu_f \dot{\mathbf{r}}$$

Dielectric response for plasmas (2)

- Note that the magnetic field has two components;
  a wave component and a static component

$$\mathbf{B} = \mathbf{B}_{wave} + \mathbf{B}_0$$

  - Thus the Lorentz force is non-linear: $m\dot{\mathbf{r}} \times (\mathbf{B}_{wave} + \mathbf{B}_0)$
- Assuming that the wave amplitude is small, then we can neglect $\mathbf{B}_{wave}$

$$\ddot{\mathbf{r}} - \dot{\mathbf{r}} \times \mathbf{e}_z \frac{q}{m} B_0 + \nu_f \dot{\mathbf{r}} = \frac{q}{m}\mathbf{E}$$

  - here we can identify the cyclotron frequency $\Omega = qB_0/m$
- Fourier transform: $-\omega^2 \mathbf{r} + i\omega \mathbf{r} \times \mathbf{e}_z \Omega - i\omega\nu_f \mathbf{r} = \frac{q}{m}\mathbf{E}$

$$-\omega^2 r_i + i\omega\varepsilon_{ijk} r_j \delta_{3k}\Omega - i\omega\nu_f r_i = \frac{q}{m} E_i$$

Note: $\mathbf{e}_3 = \delta_{3k}\mathbf{e}_k$

$$\underbrace{\left[(\omega + i\nu_f)\delta_{ij} - i\varepsilon_{ij3}\Omega\right]}_{\text{Matrix in the indexes } i,j} r_j = -\frac{q}{m\omega} E_i$$

Matrix in the indexes i, j

Dielectric response for plasmas (3)

- Write equation as a matrix equations:

$$\left[\left(\omega + i\nu_f\right)\delta_{ij} - i\varepsilon_{ij3}\Omega\right]\left[r_j\right] = \begin{bmatrix} \omega + i\nu_f & -i\Omega & 0 \\ i\Omega & \omega + i\nu_f & 0 \\ 0 & 0 & \omega + i\nu_f \end{bmatrix}\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \frac{q}{m\omega}\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

- Inverting the matrix

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \frac{q}{m\omega}\begin{bmatrix} M_{11} & M_{12} & 0 \\ M_{21} & M_{22} & 0 \\ 0 & 0 & M_{33} \end{bmatrix}\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

$$\begin{cases} M_{11} = M_{22} = \dfrac{\omega + i\nu_f}{(\omega + i\nu_f)^2 - \Omega^2} \\ M_{12} = -M_{21} = \dfrac{i\Omega}{(\omega + i\nu_f)^2 - \Omega^2} \\ M_{33} = \dfrac{1}{\omega + i\nu_f} \end{cases}$$

- The current is then

Conductivity σ_{ij}!

$$\mathbf{j} = nq(-i\omega\mathbf{r}) = -i\varepsilon_0\omega_p^2\begin{bmatrix} M_{11} & M_{12} & 0 \\ M_{21} & M_{22} & 0 \\ 0 & 0 & M_{33} \end{bmatrix}\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

Dielectric response for plasmas (4)

- The dielectric tensor in the magnetoionic theory then reads:

$$\left[K_{ij}\right] = \left[\delta_{ij} + \frac{i}{\varepsilon_0\omega}\sigma_{ij}\right] = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

$$\begin{cases} S = 1 - \dfrac{\omega_p^2}{\omega}\dfrac{\omega + i\nu_f}{(\omega + i\nu_f)^2 - \Omega^2} \\ D = -\dfrac{\omega_p^2}{\omega}\dfrac{\Omega}{(\omega + i\nu_f)^2 - \Omega^2} \\ P = 1 - \dfrac{\omega_p^2}{\omega(\omega + i\nu_f)} \end{cases}$$

or

$$K_{ij} = S\left(\delta_{ij} - b_i b_j\right) + P b_i b_j - iD\varepsilon_{ijk} b_k$$

where b_k are the components of the unit vector parallel to the magnetic field

- This dielectric response tensor is:
  - ***Anisotropic***; response is different for $\mathbf{E}$ in the x, y, or z direciton.
  - ***Gyrotropic***: the off-diagonal terms (involving D) are perpendicular to a characteristic direction of the media

Hermitian part of the dielectric tensor

$$\mathbf{K}^H = \frac{1}{2}\left(\mathbf{K} + \mathbf{K}^{T*}\right) = \frac{1}{2}\left(\begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix} + \begin{bmatrix} S^* & (-iD)^* & 0 \\ (iD)^* & S^* & 0 \\ 0 & 0 & P^* \end{bmatrix}^T\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix} + \begin{bmatrix} S^* & (iD)^* & 0 \\ (-iD)^* & S^* & 0 \\ 0 & 0 & P^* \end{bmatrix}\right) =$$

Transpose make $-iD^*$ and iD^* change place

$$= \frac{1}{2}\left(\begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix} + \begin{bmatrix} S^* & -i(D^*) & 0 \\ i(D^*) & S^* & 0 \\ 0 & 0 & P^* \end{bmatrix}\right) =$$

$$= \frac{1}{2}\begin{bmatrix} S+S^* & -i(D+D^*) & 0 \\ i(D+D^*) & S+S^* & 0 \\ 0 & 0 & P+P^* \end{bmatrix} = \begin{bmatrix} \Re\{S\} & -i\Re\{D\} & 0 \\ i\Re\{D\} & \Re\{S\} & 0 \\ 0 & 0 & \Re\{P\} \end{bmatrix}$$

Antihermitian part of the dielectric tensor

$$\mathbf{K}^A = \frac{1}{2}\left(\mathbf{K} - \mathbf{K}^{T*}\right) =$$

$$= \frac{1}{2}\left(\begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix} - \begin{bmatrix} S^* & (-iD)^* & 0 \\ (iD)^* & S^* & 0 \\ 0 & 0 & P^* \end{bmatrix}^T\right) =$$

$$= i\begin{bmatrix} \Im\{S\} & -i\Im\{D\} & 0 \\ i\Im\{D\} & \Im\{S\} & 0 \\ 0 & 0 & \Im\{P\} \end{bmatrix}$$

Cold plasma dielectric response

- A commonly used representation of the plasma is the ***cold plasma***
  - Ions and electrons are initially in a stationary equilibrium, they move only in the presence of a wave field.
  - Usually the friction between ions and electrons are neglected.
  - Each species is then described by the
    - charge q^ν
    - mass m^ν
    - position $\mathbf{r}^\nu$ (or velocity $\mathbf{v}^\nu$)

  Here $\nu = i$ represent the ions and $\nu = e$ represent the electrons
  NOTE: ν is not a tensor index!
- The linearlised equation of motion for species ν, when $\mathbf{B}_0 = B_0\, \mathbf{e}_z$

$$m^\nu \ddot{\mathbf{r}}^\nu - q^\nu \dot{\mathbf{r}}^\nu \times \mathbf{B}_0 = q^\nu \mathbf{E}$$

$$\ddot{\mathbf{r}}^\nu - \dot{\mathbf{r}}^\nu \times \mathbf{e}_z \Omega^\nu = \frac{q^\nu}{m^\nu} \mathbf{E}$$

  - where $\Omega^\nu = q^\nu B_0 / m^\nu$
  - this equation is solved like in the magnetoionic theory

Cold plasma dielectric response (2)

- The solution of the equation of motion for species ν is

$$\begin{bmatrix} r_1^\nu \\ r_2^\nu \\ r_3^\nu \end{bmatrix} = \frac{q^\nu}{m^\nu \omega} \begin{bmatrix} M_{11}^\nu & M_{12}^\nu & 0 \\ M_{21}^\nu & M_{22}^\nu & 0 \\ 0 & 0 & M_{33}^\nu \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

$$\begin{cases} M_{11}^\nu = M_{22}^\nu = \dfrac{\omega}{\omega^2 - \Omega^{\nu 2}} \\ M_{12}^\nu = -M_{21}^\nu = \dfrac{i\Omega^\nu}{\omega^2 - \Omega^{\nu 2}} \\ M_{33}^\nu = \dfrac{1}{\omega} \end{cases}$$

- With many species the current is a sum over the all species:

$$\mathbf{j} = \sum_\nu \mathbf{j}^\nu = \sum_\nu n^\nu q^\nu (-i\omega \mathbf{r}^\nu) = \sum_\nu -i\varepsilon_0 \omega_{p\nu}^2 \begin{bmatrix} M_{11}^\nu & M_{12}^\nu & 0 \\ M_{21}^\nu & M_{22}^\nu & 0 \\ 0 & 0 & M_{33}^\nu \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

  - thus also the conductivity is a sum over species:

$$\sigma = \sum_\nu -i\varepsilon_0 \omega_{p\nu}^2 \begin{bmatrix} M_{11}^\nu & M_{12}^\nu & 0 \\ M_{21}^\nu & M_{22}^\nu & 0 \\ 0 & 0 & M_{33}^\nu \end{bmatrix}$$

Cold plasma dielectric response (3)

- The dielectric tensor for the cold plasma reads

$$[K_{ij}] = \left[\delta_{ij} + \frac{i}{\varepsilon_0 \omega}\sigma_{ij}\right] = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

$$\begin{cases} S = 1 - \sum_\nu \dfrac{\omega_{p\nu}^2}{\omega^2 - \Omega^{\nu 2}} \\ D = -\sum_\nu \dfrac{\omega_{p\nu}^2}{\omega} \dfrac{\Omega^\nu}{\omega^2 - \Omega^{\nu 2}} \\ P = 1 - \sum_\nu \dfrac{\omega_{p\nu}^2}{\omega^2} \end{cases}$$

- Low frequency limit $\omega << \Omega^\nu , \omega_{p\nu}$

$$S = \ldots = 1 + \frac{c^2}{V_A{}^2} \approx \frac{c^2}{V_A{}^2}$$

V_A is the Alfven velocity

$$D = \ldots \approx 0$$

  – i.e. non-dispersive in S !

- Low frequency tensor:
  – compare: uniaxial crystal
  – describes Alfven wave and plasma oscillations (*see next lecture*)

$$[K_{ij}] = \begin{bmatrix} c^2/V_A{}^2 & 0 & 0 \\ 0 & c^2/V_A{}^2 & 0 \\ 0 & 0 & 1 - \sum_\nu \omega_{p\nu}^2/\omega^2 \end{bmatrix}$$

Cold plasma dielectric response (4)

- High frequency limit $\omega >> \Omega^\nu$

$$S = 1 - \sum_\nu \frac{\omega_{p\nu}^2}{\omega^2} + O\left(\frac{\Omega^\nu}{\omega}\right)$$

$$D = \sum_\nu \frac{\omega_{p\nu}^2 \Omega^\nu}{\omega^3} \sim O\left(\frac{\Omega^\nu}{\omega}\right)$$

$$K_{ij} = \left(1 - \sum_\nu \frac{\omega_{p\nu}^2}{\omega^2}\right)\delta_{ij} + O\left(\frac{\Omega^\nu}{\omega}\right)$$

Main term comes from the electrons: $\omega_{pe} \gg \omega_{pi}$

$$K_{ij} \approx \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)\delta_{ij}$$

Like an electron gas!

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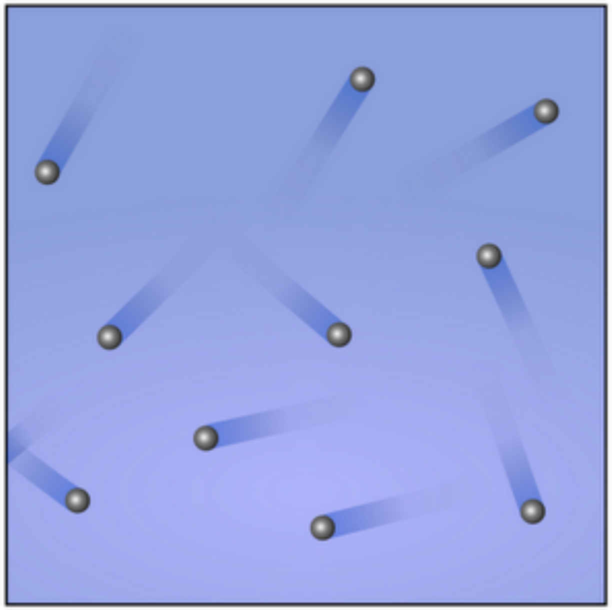
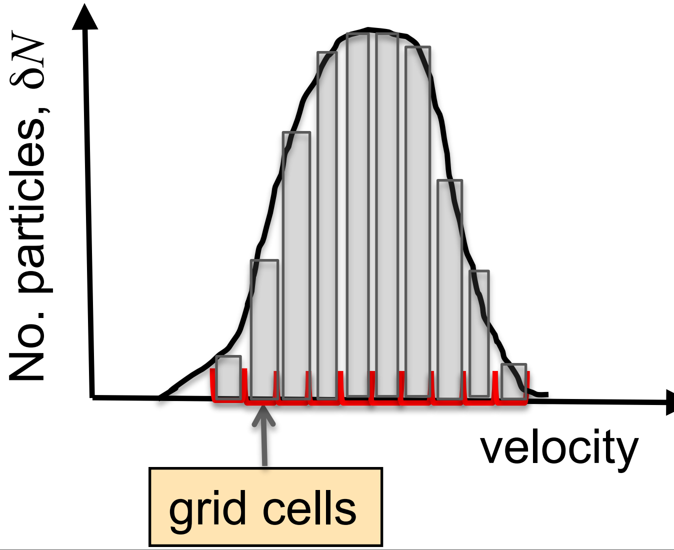
Kinetic descriptions of gases and plasmas

- Gases and plasmas are made up of particles that move "randomly"
  - This randomness makes them pratially impossible to predict exactly
- Instead: study them *statistically* :
  - Select a velocity grid:
    $v^i = i * \delta v$ for $i = 0, 1, 2 \ldots$
  - Construct histgram over particle velocity
    - counter number of particle in each grid cell
  - A density of particles in a velcity-space

$$f(v^i) = \frac{\delta N(v^i)}{\delta v}$$





Distribution function="*density in* $\mathbf{r}$ *and* $\mathbf{v}$ *space*"
- i.e. combine real and velocity space
- boxes: $(x, x + dx)$, $(v_x, v_x + dv_x)$, $(y, y + dy)$, …

$$f(\mathbf{x},\mathbf{v}) \equiv f(x,y,z,v_x,v_y,v_z) = \frac{\delta N(v_x,v_y,v_z)}{\delta x \delta y \delta z \delta v_x \delta v_y \delta v_z}$$

The Maxwellian distribution function

- The "most" important/common distribution function is called the Maxwellian distribution function. For a gas/plasma with mass per particle m, temperature T and density n

$$f^M(v) = \frac{n}{(\sqrt{2\pi m V})^3} \exp\left[-\frac{v^2}{2V^2}\right]$$

  – here V is the thermal velocity; $T = mV^2$

- E.g. when a gas or a plasma relaxed over a long time it will approach an equilibrium state. This state can be shown to be a Maxwellian!
  – The Maxwellian maximizes the entropy

Response of a warm plasma

- In Maxwells equations we need to know the charge density and current deinsity.
  - How can we calculate them from the distribution function?
- Note that the number density of particles

$$n(\mathbf{x}) = \sum_{\text{velocities cells}} \frac{\delta N(v_x, v_y, v_z)}{\delta x \delta y \delta z} = \sum_{\text{velocities cells}} f(\mathbf{x}, \mathbf{v}) \delta v_x \delta v_y \delta v_z$$

- How to calculate the density n and the average fluid velocity $<\mathbf{v}>$:

$$\begin{cases} n = \int f(v) d^3 v \\ n \langle \mathbf{v} \rangle = \int \mathbf{v} f(v) d^3 v \end{cases}$$

- Thus, for an ensamble of species ν (e.g. ion and electron)

$$\rho = \sum_{\nu} q^{\nu} n^{\nu} = \sum_{\nu} q^{\nu} \int f^{\nu}(v) d^3 v$$

$$\mathbf{J} = \sum_{\nu} q^{\nu} n^{\nu} \langle \mathbf{v} \rangle^{\nu} = \sum_{\nu} q^{\nu} \int \mathbf{v} f^{\nu}(v) d^3 v$$

Response of a warm plasma

- When subject to a wave field, the equation of motion reads

$$m^{\nu}\dot{v}_i(t,\mathbf{r},\mathbf{v}) = q^{\nu}\left[E_i + \varepsilon_{ijk} v_j B_k\right]$$

- The distribution then evolves according to the Vlasov equation (continuity equation in real and velocity space)

$$\left\{\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i} + \dot{v}_i(t,\mathbf{r},\mathbf{v})\frac{\partial}{\partial v_i}\right\} f^{\nu}(t,\mathbf{r},\mathbf{v}) = 0$$

$$\left\{\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i} + \frac{q^{\nu}}{m^{\nu}}\left[E_i(t,\mathbf{r}) + \varepsilon_{ijk} v_j B_k(t,\mathbf{r})\right]\frac{\partial}{\partial v_i}\right\} f^{\nu}(t,\mathbf{r},\mathbf{v}) = 0$$

- **Note**: the wave field perturbs both E, B and f, thus this equations is non-linear in the perturbation!

Response of a warm plasma (2)

- Separate unperturbed and perturbed quantities

$$\begin{cases} f(t,r,v) = f^{M\nu}(v) + f^{1\nu}(t,r,v) \\ \mathbf{E}(t,r,v) = 0 + \mathbf{E}^1(t,r,v) \\ \mathbf{B}(t,r,v) = 0 + \mathbf{B}^1(t,r,v) \end{cases}$$

- The Vlasov equation:

$$\left\{\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i} + \frac{q^\nu}{m^\nu}\left[E_i^1 + \varepsilon_{ijk} v_j B_k^1\right]\frac{\partial}{\partial v_i}\right\} f^{1\nu}(t,r,v) = -\frac{q^\nu}{m^\nu}\left[E_i^1 + \varepsilon_{ijk} v_j B_k^1\right]\frac{\partial}{\partial v_i} f^{M\nu}(v)$$

Non-linear terms

- Linearlised equations and use Faraday's law $\mathbf{B}^1 = \mathbf{k} \times \mathbf{E}^1 / w$

$$\left\{\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i}\right\} f^{1\nu}(t,r,v) = -\frac{q^\nu}{m^\nu}\left[E_i^1 + \varepsilon_{ijk}\varepsilon_{knm} v_j \frac{k_n}{\omega} E_m^1\right]\frac{\partial}{\partial v_i} f^{M\nu}(v)$$

- Fourier transform: $f^{1\nu}(\omega, k, v) = \frac{1}{i\omega}\frac{q^\nu}{m^\nu}\left[\delta_{im} - \frac{v_m k_i}{\omega - \mathbf{k}\cdot\mathbf{v}}\right] E_m^1 \frac{\partial}{\partial v_i} f^{M\nu}(v)$

Resonance when particles travel at phase velocity of the wave!

Landau-resonance

- The resonance in the solution to the linearised Vlasov equation is related to a damping

$$f^{1,\nu}(\omega, \mathbf{k}, \mathbf{v}) = \frac{1}{\omega - \mathbf{k} \cdot \mathbf{v}}$$

  - This was first realised by Lev Landau in 1946
  - This type of damping is called Landau damping.

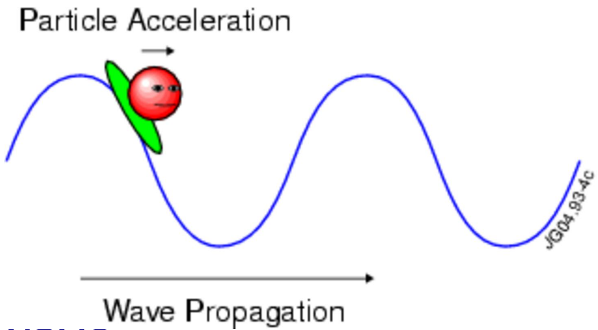
- What is the physics of this resonance?

The physics of the Landau-resonance

- Consider a plane wave $E \sim \exp(i\mathbf{k}\bullet\mathbf{x} - i\omega t)$
- Let a particle travel with the constant velocity $\mathbf{x} = \mathbf{v}t$

$$E \sim \exp(i\mathbf{k}\bullet\mathbf{v}t - i\omega t) = \exp(i[\mathbf{k}\bullet\mathbf{v} - \omega]t) = \exp(-i\omega' t)$$

- Thus, the particles will *see* a field oscillating with the frequency ω'
  - ω' is the Doppler shifted velocity!



- The resonance condition $\omega - \mathbf{k}\bullet\mathbf{v} = 0$,
  - i.e. the Doppler shifted frequency is zero
  - i.e. particle travels with the *same* speed as the wave
  - i.e. the E-field will *accelerate the particle forever* – the wave is damped!
- **Note**: we have linearised the equations, thus we assume that changes in particle velocity are small no matter now long the acceleration time!
  - in reality non-linear effects come in play - damping remains unchanged only if the dissipation (Γ) is more important than non-linearity

Response of a warm plasma (3)

- The current is obtained from the integral over velocity space

$$j_n(\omega,k) = \sum_\nu q^\nu \int v_n f^{1\nu}(\omega,k,v)d^3v$$

  - using the perturbed distribution from the previous page

$$f^{1\nu}(\omega,k,v) = \frac{1}{i\omega}\frac{q^\nu}{m^\nu}\left[\delta_{im} - \frac{v_m k_i}{\omega - \mathbf{k}\cdot\mathbf{v}}\right] E_m^1 \frac{\partial}{\partial v_i} f^{M\nu}(v)$$

- The current can be written as

Add a weak dissipation to allow for use of Plemej formula

$$j_n(\omega,k) = \left\{ -i\varepsilon_0 \sum_\nu \omega_{p\nu}{}^2 \int \left[\delta_{im} + \frac{v_m k_i}{\omega - \mathbf{k}\bullet\mathbf{v} + i0}\right] \frac{v_n v_i}{n^\nu V^\nu} f^{M\nu}(v)d^3v \right\} E_m{}^1$$

The conductivity tensor!

Plemei formula in kinetic plasmas

- In cold plasmas the Plemej formula appear for resonances like:

$$\sim \frac{1}{\omega - \Omega + i0}$$

  - where Ω is a natural frequency of the system.
  - i.e. only at *exactly the correct resonance* is there an antihermitian tensor component
  - In many practical situation the ω-spectrum is discrete and no mode match the resonance condition exactly – no damping.
- In warm plasmas the resonance condition is

$$\sim \frac{1}{\omega - \mathbf{k} \bullet \mathbf{v} + i0}$$

  - i.e. all particles travelling with *the right speed* are in resonance.
  - For smooth distributions there are always some particle with the right speed.
  - Thus, there is an antihermitian part of the dielectric tensor, and thus damping, for a wide range of frequencies!

Response of a warm plasma (3)

- After *some* algebra it is possible to rewrite the dielectric tensor as a tensor with different longitudinal and transverse responses.

$$K_{ij} = K^L \kappa_i \kappa_i + K^T \left(\delta_{ij} - \kappa_i \kappa_i \right)$$

  – The key parameter in the reponse is the ratio between the phase velocity and the thermal velocity: $y_\nu = \omega / \sqrt{2} k V^\nu$

  – Thus, the thermal velocity is at the Landau resonance if $y_\nu = 1/\sqrt{2}$

- The tensor components reads:

$$K^L = 1 + \sum_\nu \left(\frac{\omega_{p\nu}}{kV^\nu} \right)^2 \left[1 - \phi(y_\nu) + i\sqrt{\pi} y_\nu \exp(-y_\nu{}^2) \right]$$

$$K^T = 1 + \sum_\nu \left(\frac{\omega_{p\nu}}{\omega} \right)^2 \left[\phi(y_\nu) - i\sqrt{\pi} y_\nu \exp(-y_\nu{}^2) \right]$$

$$\phi(z) = 2ze^{-z^2} \int_0^z e^{-t^2} dt$$

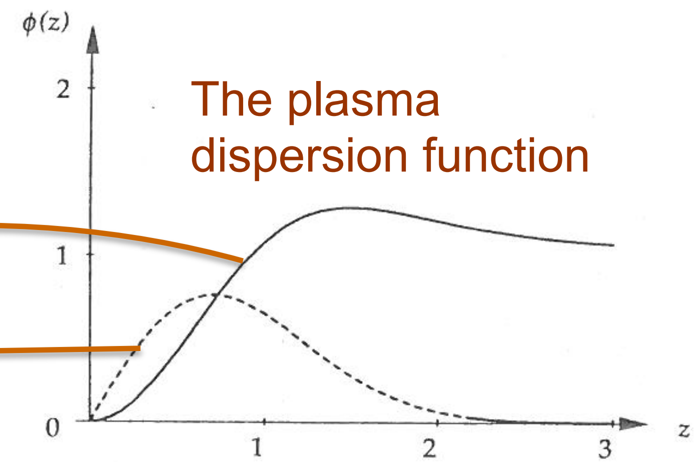


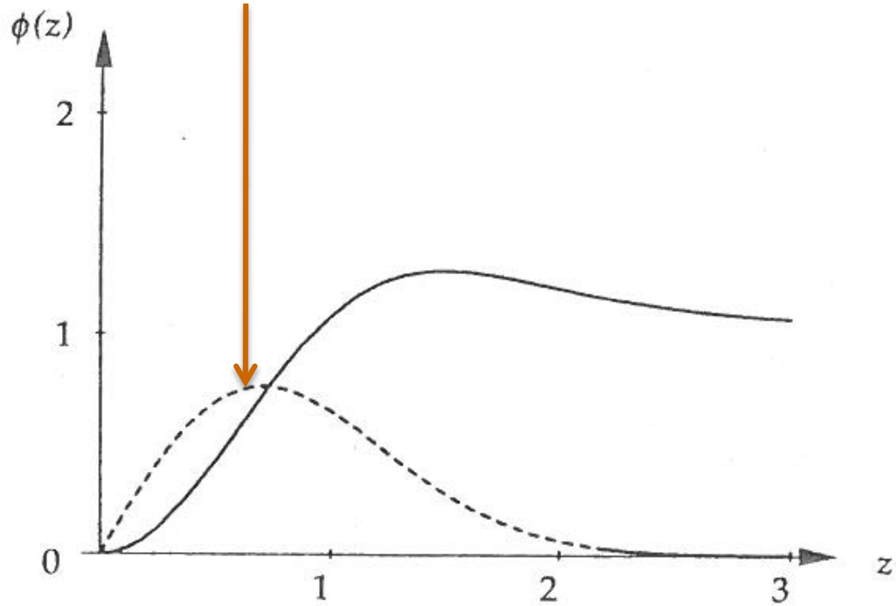
Fig. 10.1 The real part (solid curve) and the imaginary part (dotted curve) of the plasma dispersion function (10.27).

Damping in warm plasma

- Consider longitudinal waves
  - the damping is then proportional to (see later lectures for details)

$$\Im\{K^L\} = \sum_\nu \left(\frac{\omega_{p\nu}}{kV^\nu}\right)^2 \sqrt{\pi}\, y_\nu \exp(-y_\nu{}^2)$$

- The maximum damping is for $y_\nu = 1/\sqrt{2} \sim 0.7$, or $\omega/k \sim V_{th}$,
  i.e. when the phase velocity of the wave is similar to the thermal velocity
  - this the when the Landau resonance is most effective



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