ED2210: Assignment 3

Your answers should be handed in 2016-02-11. Discussions between students are allowed, but you may ***not*** copy each others results. The answers may be written down on paper, or submitted electronically. Please, take small step between each line in your derivation to make it easy to understand your algebra. The text has to be easily readable – your credits will be reduced if I cannot read your answers.

Question (an extension of question 10.3 in Melrose and McPhedran):
This question concerns the dielectric response of a cold homogeneous magnetised plasma in the low frequency limit. The plasma is neutral, consisting of electron and N different ions species. The ion species have charge $q_i = Z_i e$, mass $m_i = A_i m_{proton}$, and density n_i for $i = 1,2,3,\dots,N$, while the electron have charge $q_e = -e$, mass m_e, and density n_e.

a) Show that the charge neutrality condition, $n_e = \sum_{i=1}^{N} Z_i n_i$, implies

$$\frac{\omega_{pe}^2}{\Omega_e} = \sum_{i=1}^{N} \frac{\omega_{pi}^2}{\Omega_i}$$

where we've used the plasma frequencies and cyclotron frequencies as defined in equation (10.11) in Melrose and McPhedran. (*1 credit*)

b) With the Alfvén speed defined by $V_A = B/\sqrt{\mu_0 \rho_M}$, where ρ_M is the mass density, show that if the mass of the electrons is neglected compared to that of the ions, then one has

$$\sum_{i=1}^{N} \frac{\omega_{pi}^2}{\Omega_i} = \frac{c^2}{V_A^2}$$

(*1 credit*)

c) Now consider the dielectric tensor for a cold magnetised plasma, as described in equations (10.20) and (10.21) in Melrose and McPhedran. Show that in the limit of low frequencies ($\omega \to 0$) the tensor components have the following limits

$$S \to 1 + \frac{c^2}{V_A^2}$$
$$D \to 0$$
$$P \to \infty$$

(*1 credit*)

d) To model the dielectric response of the plasma described above we will assume that the motion of the plasma particles can be composed into the following components (for a detailed derivation, see the basic course on Plasma Physics):
   i) a free streaming parallel motion, similar to an unmagnetised plasma, i.e. like an electron gas;
   ii) gyromotion, $\mathbf{v}_{gyro}$, which is periodic with a frequency much higher than the wave frequency;
   iii) an $\mathbf{E}\times\mathbf{B}$ drift;
   iv) a polarisation drift, $\propto d\mathbf{E}_\perp/dt$.

   Consequently, the velocity of each electron and ion can be written as:

$$\mathbf{v}(t) = \mathbf{v}_\parallel(t) + \mathbf{v}_{gyro}(t) + \frac{\mathbf{E}(t)\times\mathbf{B}}{B^2} + \frac{m}{qB^2}\frac{d\mathbf{E}_\perp(t)}{dt}$$

   Here $\mathbf{E}_\perp$ is the part of the electric field that is perpendicular to the magnetic field and the parallel motion can be approximated by an electron gas:

$$\frac{d\mathbf{v}_\parallel(t)}{dt} = \frac{q}{m}\mathbf{E}_\parallel(t)$$

   Derive the cold plasma conductivity tensor and the corresponding dielectric tensor of this plasma? In this derivation you may use equation (6.17) from Melrose and McPhedran. Also note that the conductivity only includes terms driven by the electro-magnetic field. (*2 credits*)

e) Write the tensor in terms of the Alfvén speed and compare with the results from c). (*0.5 credit*)

f) Of the four terms in equation for $\mathbf{v}(t)$, which one affects the dielectric tensor element S and which one affect D? (*0.5 credit*)