Lecture 6
Channel Coding 3

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Lecture 6: Channel Coding 3
Advanced Digital Communications (EQ2410)[1]

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Monday, Feb. 8, 2016
15:00-17:00, B23

[1] Textbook: U. Madhow, *Fundamentals of Digital Communications*, 2008

Notes

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Overview

Lecture 5

- Turbo codes
- Iterative decoding and BCJR algorithm
- Union bound

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Motivation

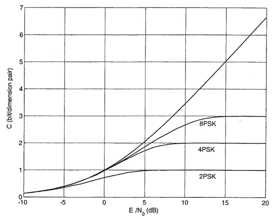

Figure 3.4: *Capacity of some two-dimensional constellations over the AWGN channel. The unconstrained capacity* $\log(1+\mathcal{E}/N_0)$ *is also shown*

[E. Biglieri, *Coding for Wireless Channels*, 2005]

- So far: binary codes with BPSK or QPSK
  $\rightarrow$ appropriate for low SNR but inefficient at high SNR
- To increase the bandwidth efficiency, we have to understand coding for higher signal constellations (e.g., 8-PSK, 16-QAM).
- Two possible techniques:
  - Bit-interleaved coded modulation
  - Trellis-coded modulation
- We can gain in two different ways:
  - For a fixed data rate we can get a coding gain.
  - For fixed SNR we can increase the data rate.

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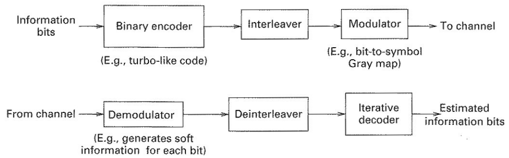
Bit-Interleaved Coded Modulation



[U. Madhow, *Fundamentals of Dig. Comm.*, 2008]

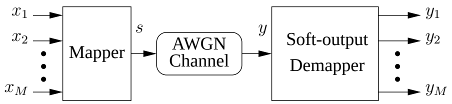
- Concatenation of a binary code (e.g., convolutional, Turbo, or LDPC code), an interleaver, and a non-binary modulator.
  - Vector of code bits which are mapped to one symbol: $\mathbf{x} = [x_1, \ldots, x_M]$.
  - Transmitted symbol: $s = s(\mathbf{x})$
  - Received symbol for the AWGN channel: $y = s(\mathbf{x}) + n$
- Conventional Decoding
  - Soft-output demapper generates LLRs for the code bits

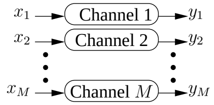
$$L(x_i) = \log\left(\frac{\Pr(x_i = 0|y)}{\Pr(x_i = 1|y)}\right) = \log\left(\frac{\sum\limits_{s\in\mathcal{S}_{x_i=0}} p(y|s)}{\sum\limits_{s\in\mathcal{S}_{x_i=1}} p(y|s)}\right)$$

  - Soft-input decoding of the outer code based on the LLRs $L(x_i)$.
- Interleaver: split bursts of errors into single error events.

Notes

Bit-Interleaved Coded Modulation
– Capacity





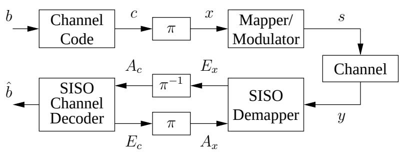
- The relation between the transmitted bits x_i and the soft outputs y_i from the demapper can be modeled by M equivalent parallel channels with capacities $C_i = I(X_i; Y_i)$.
  ($\rightarrow$ not always a correct model!)
- Capacity of the BICM scheme (or better: the sum rate)

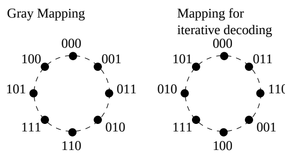
$$C_{\text{BICM}} = \sum_{i=1}^{M} C_i \leq I(S; Y)$$

  $\rightarrow$ equality can only be obtained if the channel outputs y_i are independent from the inputs x_j, with $j \neq i$.
- Observation: Gray mapping often maximizes C_{BICM} (e.g., 16QAM; exception: 32 QAM).
- Gray Mapping: bit vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ which are mapped to neighboring symbols s_i and s_j have a Hamming distance $d_H(\mathbf{x}_i, \mathbf{x}_j) = 1$.

Notes

Bit-Interleaved Coded Modulation
– Iterative Decoding





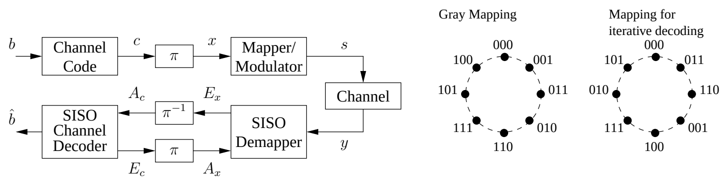
- Soft-input/soft-output demapper

$$E_{x\,i} = \log\left(\frac{\Pr(x_i = 0|y, \mathbf{A}_{\backslash i})}{\Pr(x_i = 1|y, \mathbf{A}_{\backslash i})}\right) = \log\left(\frac{\sum\limits_{s\in\mathcal{S}_{x_i=0}} p(y|s)\Pr(s|\mathbf{A}_{\backslash i})}{\sum\limits_{s\in\mathcal{S}_{x_i=1}} p(y|s)\Pr(s|\mathbf{A}_{\backslash i})}\right)$$

  with $\mathbf{A}_{\backslash i} = [A_1, \ldots, A_{i-1}, A_{i+1}, \ldots, A_M]$
- Mapping is important
  - Gray mapping: no gain by iterative decoding since the outputs y_i of the sub-channels are independent of the bits x_j, with $j \neq i$; i.e., having *a priori* information on x_j, with $j \neq i$, does not improve the quality of y_i.
  - $\rightarrow$ Mapping has to be optimized for iterative decoding (see example above) and adapted to the channel code.

Notes

Bit-Interleaved Coded Modulation
– Iterative Decoding



2 practical design strategies

(1) Capacity-approaching channel code (e.g., LDPC or Turbo code), Gray mapping, no iterative decoding between channel decocder and demapper.
$\rightarrow$ Good performance (low error floor) but higher complexity.

(2) Convolutional code, optimized mapping, and iterative decoding.
$\rightarrow$ Low complexity but higher error-floor.

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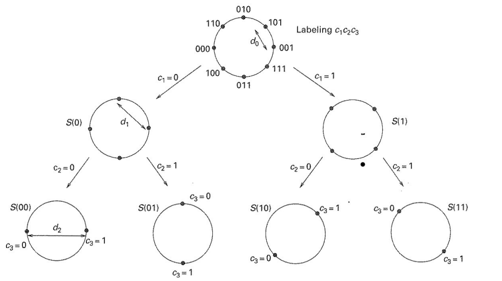
Trellis-Coded Modulation

- Ungerböck (early 1980s): combining coding and modulation
- Goal: increase reliability for a fixed spectral efficiency
- Solution (TCM)
  - For spectral efficiency of k bits per channel use choose a $(k+n)$-bit constellation and insert redundancy.
  - Use a hierarchical partitioning of the constellation.
  - Use a trellis code and maximize the Euclidean distance between paths in the trellis.

Notes

Trellis-Coded Modulation
– Set partitioning by Ungerböck (example 8-PSK)

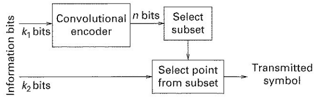


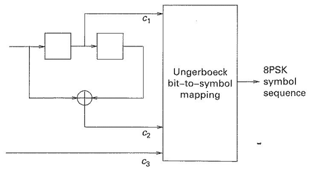
[U. Madhow, *Fundamentals of Dig. Comm.*, 2008]

- The mapping from the bits $[c_1, c_2, c_3]$ to the symbols of the constellation is hierarchical such that
  - c_1 splits the constellation into 2 QPSK constellations,
  - (c_1, c_2) split the constellation into 4 BPSK constellations.
- Observation
  - Performance of 8-PSK is limited by the minimum distance d_0.
  - Knowing c_1 increases the minimum distance from d_0 to d_1.
  - Knowing (c_1, c_2) increases the minimum distance from d_0 to d_2.

Notes

Trellis-Coded Modulation
– Encoder Structure



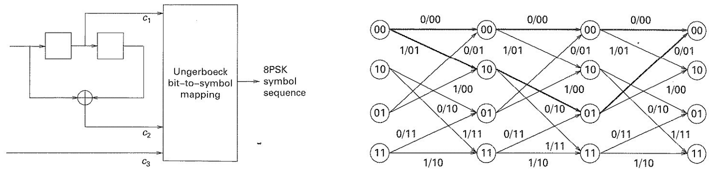


[U. Madhow, *Fundamentals of Dig. Comm.*, 2008]

- TCM encoder for spectral efficiency $(k_1 + k_2)$ bits using a $(k_2 + n)$-bit constellation.
- Convolutional code with rate $R_c = k_1/n$ selects the subsets/sub-constellations (i.e., the first n bits $[c_1, \dots, c_n]$).
- The remaining k_2 bits are mapped directly to the sub-constellation (the last k_2 bits $[c_{n+1}, \dots, c_{n+k_2}]$).
- Example (right): $k_1 = k_2 = 1$, $n = 2$, $R_c = 1/2$, $k_2 + n = 3$, spectral efficiency 2 bpcu.

Notes

Trellis-Coded Modulation
– Trellis



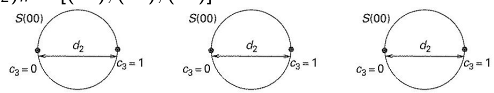
[U. Madhow, *Fundamentals of Dig. Comm.*, 2008]

- The TCM trellis is defined by the trellis states of the employed convolutional code.
  - States are given by the content of memory cells of the encoder.
  - State transitions are driven by the k_1 input bits to the convolutional code.
  - Each state transition is associated with one realization of the vector $[c_1, \dots, c_n]$ and selects hence the sub-constellation for the remaining k_2 uncoded bits.
  - Note that the impact of the k_2 uncoded bits $[c_{n+1}, \dots, c_{n+k_2}]$ is not shown in the trellis above.
  - In fact there are 2^{k_2} parallel branches for each state transition which are labeled by all possible realizations of the bits $[c_{n+1}, \dots, c_{n+k_2}]$.
- Decoding: Viterbi or BCJR algorithm on the TCM trellis.

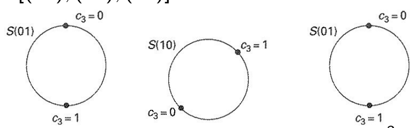
Notes

Trellis-Coded Modulation
– Performance

- Error probability at high SNR is characterized by the minimum Euclidean distance between any two TCM sequences.
- Example in the book:
  - Path 1: $(c_1, c_2)_n = [(00), (00), (00)]$



  - Path 2: $(c_1, c_2)_n = [(01), (10), (01)]$



  - Minimum squared Euclidean distance between any of the 2^3 bit symbol sequences (due to the uncoded bit c_3) transmitted via Path 1 or Path 2

$$\begin{aligned} d^2_{subset} &= d^2_{min}(S(00), S(01)) + d^2_{min}(S(00), S(10)) + d^2_{min}(S(00), S(01)) \\ &= d_1^2 + d_0^2 + d_1^2 = 4.586 \end{aligned}$$

  - Minimum squared Euclidean distance between parallel state transitions: $d^2_{branch} = d_2^2 = 4$

Notes

Trellis-Coded Modulation
– Performance

Comments

- Generally, the decoding problem can be split into 2 parts:
  1. Find the correct sub-constellation by exploiting the trellis.
  2. For a given sub-constellation, make the correct decision.
- The error probability for (1) is limited by the impact of d_{subset}; the error probability for (2) is limited by the minimum distance in the sub-constellation d_{branch}.
- In this example, since $d_{subset} > d_{branch}$, it is more likely to make a wrong decision in the sub-constellations compared to selecting a wrong constellation.
- Therefore the performance is limited by $d_{branch} = d_2 = 2$.
- Compared to uncoded QPSK, we gain 3 dB since

$$d_{QPSK}^2 = d_1^2 = 1/2 \cdot d_2^2 = 1/2 \cdot d_{TCM}^2$$

Notes

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