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Tutorial #4: Basics of Quantization

Recap Useful info

If x denotes the input at the quantizer, Q, then $y = Q[x]$ denotes its output.

- Q is a midtread quantizer if $Q[x] = 0$ for $x = 0$.
- Q is a midrise quantizer if $Q[x] \neq 0$ for $x = 0$.

It is quite convenient to express some results in terms the Q-function (since it is tabulated)

$$\mathcal{Q}(t) = \frac{1}{2\pi} \int_t^\infty e^{-\gamma^2} d\gamma$$

Note that $\mathcal{Q}(t)$ is monotonically decreasing in t.

Let Q be a quantizer with L levels. Then $\{x_1, ..., x_L\}$ are intervals for the input x, and $\{y_1, ..., y_L\}$ the corresponding quantized outputs. The variance of quantization noise is defined as

$$\sigma_q^2 = \sum_{k=1}^{L} \int_{x_{k-1}}^{x_k} (x - Q(x))^2 f_X(x) dx = \sum_{k=1}^{L} \int_{x_{k-1}}^{x_k} (x - y_k)^2 f_X(x) dx$$

Problem 1

A Gaussian random variable with zero mean and variance 1, is to be quantized. A uniform midtread quantizer with step-size Δ and $L = 7$ levels

1. Compute the probability mass function (pmf) at the output of the quantizer.

Problem 2

A stochastic signal X has the following pdf,

$$f_X(x) = \begin{cases} k(1 - |x|), & -1 \leq x \leq 1 \\ 0\,, & \text{otherwise} \end{cases}$$

1. Calculate the constant k

2. Calculate the step-size Δ for a uniform 2-bit midrise quantizer. The step-size should be chosen as small as possible such as to minimized the overload distortion

3. Calculate the quantization error σ_q^2 for the quantizer in 2.

4. Derive the step-size that minimizes σ_q^2. Compare the resulting quantization error to that of 3.

Problem 3

Consider a signal X that has the following pdf,

$$p_X(x) = \begin{cases} 3/4, & -1 \leq x \leq 0 \\ 1/4, & 0 \leq x \leq 1 \end{cases}$$

We wish to design a 16-bit scalar quantizer. Since the pdf of is piecewise uniform (i.e. uniform over two intervals: $[-1, 0]$ and $[0, 1]$) it seems reasonable to construct the quantizer as two separate ones, one for each interval. Each quantizer has L_1 and L_2 levels respectively, and they satisfy $L_1 + L_2 = 2^{16}$.

1. What values of L_1 and L_2 minimize the average distortion ?