Department of Mathematics
SF2729 Groups and Rings
Period 3, 2016

HOMEWORK 9

Submission. The solutions should be typed and converted to .pdf. Deadline for submission is Monday February 15, 14.00. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `skjelnes@kth.se`.

Score. For each set of homework problems, the maximal score is 3 points, and calulated as $\min\{3, \Sigma/2\}$, where Σ is the score obtained on the homework. The total score from all twelve homeworks will be divided by four when counted towards the first part of the final exam.

Problem 1. Let V denote the real vector space of $n \times n$-matrices, for some fixed positive integer n. Let $L\colon V \to V$ be the linear map

$$L(X) = X + X^{tr},$$

where X^{tr} denotes the transpose of a matrix X. Recall that a matrix is skew symmetric if $X^{tr} = -X$.

(a) Show that skew symmetric and symmetric matrices are eigenvectors of L, and determine their eigenvalues. **(1 p)**
(b) Determine the dimension of the image of L. **(1 p)**
(c) Compute the characteristic polynomial of L. **(1 p)**

Problem 2. Let $P_1 = (0,0)$, and $P_2 = (0,1)$, and $P_3 = (2,0)$ be points in $\mathbb{R}^2$. Let L_1, L_2 and L_3 be the three lines where the each line is determined by a pair of these three points. Let $l_i(x,y) = a_i x + b_i y + c_i \in \mathbb{R}[x,y]$ be a linear form that such that its zero set $L_i = \{(p,q) \mid l_i(p,q) = 0\}$ (for $i = 1,2,3$).

(a) Show that we have an equality of ideals **(1 p)**

$$(l_1 l_2, l_2 l_3, l_3 l_1) = (x,y) \cap (x, y-1) \cap (x-2, y).$$

(b) Show that $\mathbb{R}[x,y]/(l_1 l_2, l_2 l_3, l_3 l_1) = \mathbb{R}^3$. **(1 p)**
(c) Give polynomials $e_i(x,y) \in \mathbb{R}[x,y]$ $(i = 1,2,3)$, such that their classes are the standard bases for $\mathbb{R}^3$ via the isomorphism in b). **(1 p)**
(d) Solve the system of equations **(1 p)**

$$(*) \begin{cases} l_1 l_2 = 0 \\ l_2 l_3 = 0 \\ l_3 l_1 = 0 \end{cases}$$

(e) Prove (b) with three general, non-aligned, points P_1, P_2 and P_3. **(2 p)**