Lecture 7: Wireless Channels and Diversity
Advanced Digital Communications (EQ2410)[1]

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Thursday, Feb. 11, 2016
10:00-12:00, B24

[1]Textbook: U. Madhow, *Fundamentals of Digital Communications*, 2008

Overview

Lecture 1-6

- Equalization (signal processing)
- Channel Coding (information and coding theory)

Lecture 7: Wireless Channels and Diversity

Overview

Examples for wireless communications

- Radio and TV broadcast
- Point-to-point microwave links
- Satellite communications
- Cellular communications
- Wireless local area networks (WLANs), bluetooth, etc.
- Sensor networks

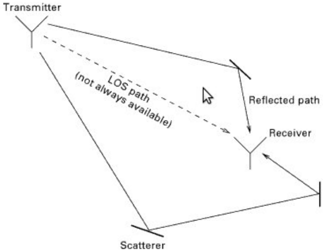
Important characteristic: broadcast nature

- All users which are close enough can listen.
- Interference from other users
- Coordination required (TDMA, FDMA, CDMA)
- Frequency planing

Channel Modeling

- Statistical models are defined based on channel measurements.
- Algorithm and system development based on channel models.
- Complex baseband model with transmitted signal $u(t)$ and received signal $y(t)$

$$y(t) = \sum_{k=1}^{M} A_k e^{j\phi_k} u(t-\tau_k) e^{-j2\pi f_c \tau_k}$$



Multipath propagation, M paths

- Amplitude of the k-th path: A_k
- Changes in the phase (e.g., due to scattering): ϕ_k
- Delay on the k-th path: τ_k
- Phase lag due to transmission delay: $2\pi f_c \tau_k$

- Impulse response and transfer function of the complex baseband channel

$$h(t) = \sum_{k=1}^{M} A_k e^{j\theta_k} \delta(t-\tau_k), \quad \text{and} \quad H(f) = \sum_{k=1}^{M} A_k e^{j\theta_k} e^{-j2\pi f \tau_k}$$

with $\theta_k = (\phi_k - 2\pi f_c \tau_k \mod 2\pi)$, uniformly distributed in $[0, 2\pi]$

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Narrowband Fading

- Channel transfer function is approximately constant over the signal band which is used; i.e., the channel impulse response is reduced to one impulse with gain

$$h \approx H(f_0) = \sum_{k=1}^{M} A_k e^{j\gamma_k} \quad \text{with}$$

$$\mathrm{Re}(h) = \sum_{k=1}^{M} A_k \cos(\gamma_k) \quad \text{and} \quad \mathrm{Im}(h) = \sum_{k=1}^{M} A_k \sin(\gamma_k)$$

with $\gamma_k = (\theta_k - 2\pi f_0 \tau_k \mod 2\pi)$ and the center frequency f_0.

- Central limit theorem: for large M, $\mathrm{Re}(h)$ and $\mathrm{Im}(h)$ can be modeled as jointly Gaussian with
  - mean $\mathrm{E}[\mathrm{Re}(h)] = \mathrm{E}[\mathrm{Re}(h)] = 0$
  - variance $\mathrm{var}[\mathrm{Re}(h)] = \mathrm{var}[\mathrm{Im}(h)] = \frac{1}{2}\sum_{k=1}^{M} A_k^2$
  - and covariance $\mathrm{cov}[\mathrm{Re}(h), \mathrm{Im}(h)] = 0$

$$h \sim CN(0, \sum_{k=1}^{M} A_k^2)$$

$$\mathrm{Re}(h) \sim N(0, \frac{1}{2}\sum_{k=1}^{M} A_k^2) \quad \text{and} \quad \mathrm{Im}(h) \sim N(0, \frac{1}{2}\sum_{k=1}^{M} A_k^2)$$

Narrowband Fading

- Rayleigh fading: for zero-mean Gaussian $\text{Re}(h)$ and $\text{Im}(h)$ it follows with $\sigma^2 = \text{var}[\text{Re}(h)] = \text{var}[\text{Im}(h)]$ that
  - $g = |h|^2$ is exponentially distributed

$$p_G(g) = \frac{1}{2\sigma^2} \exp(-g/(2\sigma^2)) I_{\{g \geq 0\}}$$

  - $r = |h|$ is Rayleigh distributed

$$p_R(r) = \frac{r}{\sigma^2} \exp(-r^2/(2\sigma^2)) I_{\{r \geq 0\}}$$

- Rice fading: one dominant multipath (line-of-sight, LOS) component, $A_1 e^{j\gamma_1}$, i.e., we have

$$h = A_1 e^{j\gamma_1} + h_{diffuse}$$

with $h_{diffuse} \sim CN(0, \sum_{k=2}^{M} A_k^2)$.

$\rightarrow$ Accordingly, $h \sim CN(A_1 e^{j\gamma_1}, \sum_{k=2}^{M} A_k^2)$, and $r = |h|$ is Rician distributed.

Frequency-Selective Fading

- Signal with bandwidth W; signal-spaced sampling with $T_s = 1/W$
- Tapped delay line (TDL) model (compare model for ISI channel)

$$h(t) = \sum_{i=1}^{\infty} \alpha_i \delta(t - \frac{i}{W}) \quad \text{and} \quad \alpha_i = \sum_{k:\tau_k \approx \frac{i}{W}} A_k e^{j\theta_k}$$

$\rightarrow \{\alpha_i\}$ is zero-mean, proper complex Gaussian.

- Power-delay profile (PDP, for $\tau \geq 0$)

$$P(\tau) = \frac{1}{\tau_{ms}} e^{-\frac{\tau}{\tau_{ms}}}$$

with the root mean squared delay τ_{MS}

$$\rightarrow \quad \mathrm{E}[|\alpha_i|^2] = \int_{i/W}^{(i+1)/W} P(\tau) d\tau$$

- Applications: (among others) GSM channel models

Frequency-selective vs. Narrowband Fading

Delay spread and coherence bandwidth

- Delay spread: T_m, maximum τ for which $P(\tau) > \epsilon$ ($\epsilon \to 0$)
- Coherence bandwidth: B_m, maximum bandwidth for which the channel is approximately constant in f.

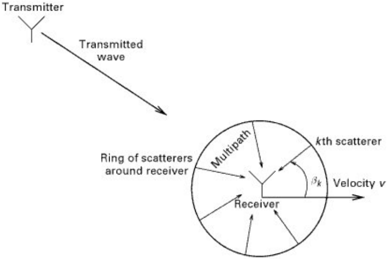
$$B_m \approx 1/T_m$$

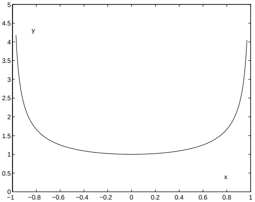
Transmitted signal $s(t)$ with bandwidth W

- $W \ll B_m \Rightarrow$ frequency-flat fading (only scaling and phase-shift, no "filtering")
- $W \gg B_m \Rightarrow$ frequency-selective fading (linear filtering, ISI)

Time-Varying Channels

- Model: TDL with time-varying coefficients $\{\alpha_i\}$
- Moving receiver with speed $v \rightarrow$ max Doppler shift $f_D = f_c v/c$; i.e., a sinusoid with frequency f_c will be shifted to frequencies $f_c \pm f_D$.
- Clarke's Model





- Time varying complex gain

$$X(t) = \sum_k e^{j(2\pi f_k t + \theta_k)}$$

$$\rightarrow y(t) = X(t) \cdot u(t)$$

- Doppler shift of the k-th component $f_k = f_D \cos(\beta_k)$
- $X(t)$: zero-mean proper complex Gaussian
- Power spectral density for rich scattering and omnidirectional antennas

$$S_X(f) = \frac{1}{\pi f_D \sqrt{1 - (f/f_D)^2}}$$

Fast/Slow Fading

- Doppler spread: f_D, a frequency impulse (sinusoid) is broadened to bandwidth f_D.
- Coherence time: T_D, the channel is approximately constant in time for T_D seconds.

$$T_D \approx 1/f_D$$

Transmitted signal $s(t)$ with bandwidth W

- $W \gg f_D \Rightarrow$ slow fading (no Doppler spread)
- $W \ll f_D \Rightarrow$ fast fading (Doppler spread)

Performance for Fading Channels

- Assumption: uncoded transmission over a slow fading channel

$$y(t) = h \cdot s(t) + n(t)$$

  - Normalized fading: $h \sim CN(0,1)$
    $\rightarrow \; G = |h|^2 \sim p_G(g) = \exp(-g) I_{g \geq 0}$
    $\rightarrow \; R = \sqrt{G} \sim p_R(r) = 2r\exp(-r^2) I_{g \geq 0}$
  - Instantaneous and average SNR: $S = E_b/N_0 = G\bar{S}$, and $\bar{S} = \bar{E}_b/N_0$
- Error Probability (averaged over fading)

$$\begin{aligned} P_e &= \mathrm{E}[P_e(G)] = \int P_e(g) p_G(g) dg \\ &= \mathrm{E}[P_e(R)] = \int P_e(r) p_R(r) dr \end{aligned}$$

  - Noncoherent FSK in Rayleigh fading

$$P_e(G) = 1/2\exp(-G\bar{S}/2) \quad \Rightarrow \quad P_e = (2 + \bar{E}_b/N_0)^{-1}$$

  - Binary DPSK in Rayleigh fading

$$P_e(G) = 1/2\exp(-G\bar{S}) \quad \Rightarrow \quad P_e = (2 + 2\bar{E}_b/N_0)^{-1}$$

  - Coherent FSK in Rayleigh fading

$$P_e(R) = Q(R\sqrt{\bar{S}}) \quad \Rightarrow \quad P_e = \frac{1}{2}(1 - (1 + 2N_0/\bar{E}_b)^{-1/2})$$

  - Coherent BPSK in Rayleigh fading

$$P_e(R) = Q(R\sqrt{2\bar{S}}) \quad \Rightarrow \quad P_e = \frac{1}{2}(1 - (1 + N_0/\bar{E}_b)^{-1/2})$$

Capacity

Fast fading (ideal interleaving and long blocks)

- Model: $y[n] = h[n] \cdot s[n] + w[n]$
    - Normalized fading: $h[n] \sim CN(0, 1)$
    - Transmit power constraint: $\mathrm{E}[|x[n]|^2] \leq P$
    - AWGN: $w[n] \sim CN(0, 2\sigma^2)$
    - Signal-to-noise ratio: $SNR = \mathrm{E}[|h|^2]P/(2\sigma^2)$
- Ergodic capacity for Gaussian $s[n] \sim CN(0, P)$ (averaged over G)

$$C_{Rayleigh} = \mathrm{E}[\log(1 + G\ SNR)] = \int_0^\infty \log(1 + g\ SNR) p_G(g) dg$$

Slow fading

- Model: $y[n] = h \cdot s[n] + w[n]$
- Capacity for a given $G = |h|^2$

$$C(h) = \log(1 + |h|^2\ SNR) = \log(1 + g\ SNR) = C(g)$$

    - Capacity can become $C(h) = 0$.
    - No rate R which guarantees error-free transmission for all h.
    - For a given R, system outage if $C(h) < R$
      $\rightarrow$ outage probability $\Pr(C(h) < R)$
- Outage capacity and outage probability

$$C_{out} = \log(1 + G_{out}\ SNR) \quad \text{with} \quad \Pr(G < G_{out}) = \epsilon$$

Receive Diversity

- Problem with fading: strong SNR fluctuations

  "The channel can be bad for some time!"

- Solution: diversity; i.e., provide the receiver with different copies of the same signal (create parallel channels).
  - Spatial diversity, multiple antennas
  - Temporal diversity (e.g., repetition coding in time)
  - Frequency diversity (e.g., select carriers with independent fading)

- Model: N received branches $y_1, \ldots, y_N$ for the same symbol s with

$$y_i = h_i \cdot s + n_i$$

  - Independent zero-mean complex AWGN terms n_i with variance σ^2
  - Complex fading gains h_i

- Diversity combining
  - Selection combining (choose the strongest path)
  - Maximum ratio combining (optimal linear combination of branches)
  - Equal gain combining (sum of all branches)
  - Switched diversity (pick one branch at random)

Coherent Diversity Combining

- Coherent maximum ratio combining (with $\mathbf{h} = (h_1, \ldots, h_N)$ and $\mathbf{y} = (y_1, \ldots, y_N)$)

$$L(\mathbf{y}|s, \mathbf{h}) = \prod_{i=1}^{N} L(y_i|s, h_i) \quad \text{with}$$

$$L(y_i|s, h_i) = \exp\left(\frac{1}{\sigma^2}[\mathrm{Re}(\langle y_i, h_i s\rangle) - \frac{1}{2}\|h_i s\|^2]\right)$$

- Decision variable (ignoring the energy term)

$$Z = \sum_{i=1}^{N} \mathrm{Re}(\langle y_i, h_i s\rangle) = \sum_{i=1}^{N} h_i^* \langle y_i, s\rangle$$

$\rightarrow$ Coherent matched filter

Coherent Diversity Combining

- Matched filter output

$$r = \sum_{i=1}^{N} h_i^* y_i = (\sum_{i=1}^{N} \|h_i\|^2) \cdot s + (\sum_{i=1}^{N} h_i^* n_i)$$

$$\rightarrow \text{SNR}= (\sum_{i=1}^{N} \|h_i\|^2)^2 P / (\sum_{i=1}^{N} \|h_i\|^2 \sigma^2); \text{ i.e., SNR gain } G = \sum_{i=1}^{N} \|h_i\|^2$$

- Error probability for BPSK (averaged over all realizations of $\mathbf{h}$)

$$\begin{aligned} Pe &= \Pr(r < 0 | s = +1) = \mathrm{E}[P_h(\mathbf{h})] \\ &= \left(\frac{1-\mu}{2}\right)^N \sum_{i=0}^{N-1} \binom{N-1+i}{i} \left(\frac{1+\mu}{2}\right)^i \end{aligned}$$

with $\mu = \sqrt{\bar{S}/(1+\bar{S})}$ and the average SNR $\bar{S}$

- High SNR: $Pe = K(N)(1/(4\bar{S}))^N$, with $K(N) = \binom{2N-1}{N}$
- Diversity gain:

$$\lim_{\bar{S}\rightarrow\infty} \frac{\ln P_e}{\ln \bar{S}} = -N$$

- Similar analysis for selection combining, equal gain combining etc.