IK1611 Dimensioning of communication systems
Solutions to the exam on March 17th 2011, 14-19

Problem 1

Consider two queuing systems with Poisson arrivals and an exponentially distributed service time. The arrival rate to the both systems is λ arrivals per time unit. The first system consists of two servers each of which is characterized by the mean service time $1/\mu$ and one queue place. The second system is with one server and two queuing places and can be modeled as a birth-death process with $\lambda_k=\lambda$ (k=0,1,2) and $\mu_k=k\mu$ (k=0,1,2,3). Assume $\lambda = 1$ arrival/s and $\mu = 1$/s. For these two systems determine and compare:

a) Probability that a customer has to wait (2p)

b) Blocking probability (2p)

c) Mean waiting time for an arbitrary customer (4p)

d) Mean waiting time for a customer who gets service (2p)

Solution

System I

p0 = 4/11; p1 = 4/11; p2 = 2/11; p3 = 1/11

a) P(wait) = p2 = 2/11
b) Blocking probability Pb = p3 = 1/11
c) $W = N_q/\lambda = (0p0 + 0p1 + 0p2 + 1p3)/\lambda = p3/\lambda = 1/11$ s
Another way to calculate W:
W = W(meets0) p0 + W(meets1) p1 + W(meets2) p2 + W(meets3) p3
W(meets0) = W(meets1) = W(meets3) = 0
W(meets2) = 1/2 s → W = 1/11s
d) W(gets service) = W/(1-Pb) = 1/10 s

System II

p0 = 3/8; p1 = 3/8; p2 = 3/16; p3 = 1/16

a) P(wait) = p1 + p2 = 9/16
b) Blocking probability Pb = p3 = 1/16
c) $W = N_q/\lambda = (0p0 + 0p1 + 1p2 + 2p3)/\lambda = (1p2 + 2p3)/\lambda = 5/16$ s
d) W(gets service) = W/(1-Pb) = (5/16) / (1 – 1/16) = (5/16)(16/15) = 1/3 s

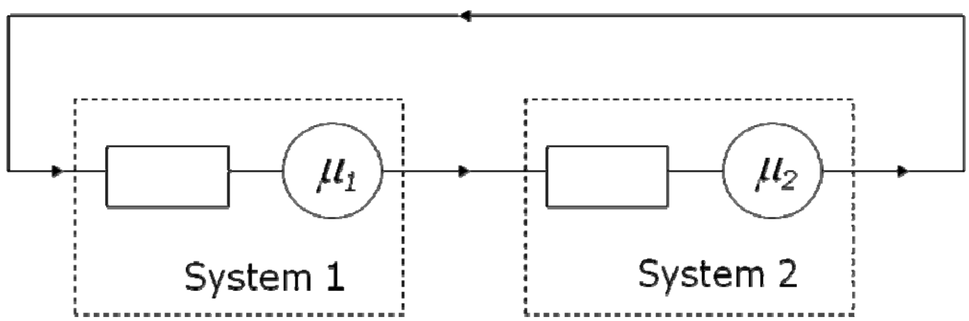
Problem 2

Consider the computer system shown in Fig. 1. Each time a job leaves System 2 it is replaced by a new job that arrives at System 1. Thus, number of circulating jobs is constant and equal to M. Systems 1 and 2 are queuing systems with one server and at least M-1 places in the queue. Service times are exponentially distributed with mean values $1/\mu_1$ and $1/\mu_2$ respectively. As soon as job departures from System 1 it arrives immediately at System 2. Service times are independent. Determine:

a) Average number of customers in system 1 (4p)

b) Utilization of system 1 (4p)

c) Utilization of system 2 (2p)



Solution

$\mu_2/\mu_1 = A$

$P_k = A^k(1 - A)/(1 - A^{M+1})$

a) $N = A(1 + MA^{M+1} - (M + 1)A^M)/ ((1 - A)(1 - A^{M+1}))$
b) U_1 = P(server 1 is busy) = $1 - p_0 = A (1 - A^M)/(1 - A^{M+1})$
c) U_2 = P(server 2 is busy) = $U_1/A = (1 - A^M)/(1 - A^{M+1})$

Problem 3

In a post office at lunch time and there are only two counters in service. Arriving customers receive a numbered tag and wait until one of the counters is free. Customers arrive in a Poisson fashion, in average 6 per hour. Each counter serves in average 6 customers per hour and the service time is exponentially distributed.

a) Calculate mean waiting time for a customer who needs to wait (2p)

b) Calculate probability that a customer needs to wait longer than 10 min (4p)

c) At your arrival you receive a numbered tag which shows that you are third in the queue. Calculate your mean waiting time in the queue and the mean total time you spend in the post office (4p)

Solution

M/M/2

Arrival rate λ = 6/h, mean service time $1/\mu$ = 1/6 h → offered load ρ = 1 Erlang

a) W(wait) = W/P(wait) = 1/(12 – 6) = 1/6 h = 10 min

b) P(w > 10min) = $D_2(1)\ e^{-1}$ = **1/(3e)**

$D_2(1) = 2E_2(1)/(2 – 1 + E_2(1)) = 2E_2(1)/(1 + E_2(1)) = 0.4/1.2 = 1/3$

$E_2(1) = 0.2$

c) W_3 = 10/2 + 10/2 +10/2 = 15 min ; T_3 = 15 + 10 = 25 min

Problem 4

Consider a network with three nodes (see Fig. 2). The first and second nodes are M/M/1 systems. The third node is single server system without buffer. Packets arrive to the first node with arrival rate λ packets/s. Packets have exponentially distributed length with a mean value x. The servers at node 1, 2 have capacity of C bit/s. Service in the third node consists of two exponentially distributed steps, each step with double intensity compared to nodes 1 and 2.

Assume: C = 8Mbit/s, λ = 200 packets/s, x = 600 bytes.

Calculate:

a) Blocking probability at Node 3 (2p)

b) Mean time an arbitrary packet spends in the network (4p)

c) Mean time in the network for a packet which will get service at Node 3 (4p)

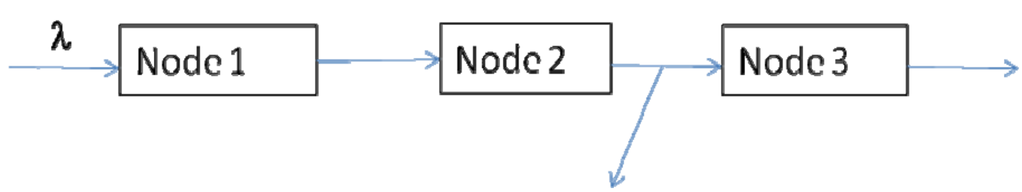


Fig. 2: Network with three nodes

Solution

N3: M/E2/1/1

p0 = 100/112 = 50/56; p1 = p2 = 6/112 = 3/56

a) Pb = 1 – p0 = 6/56

b) $T = T1 + T2 + T3$

$T1 = T2 = 1/(\mu - \lambda) = 3/4400s$; $T3 = 1/\mu\ (1-Pb) = (6/10000)\ (4400/56) =$
$= (1/500)\ (11/6) = 11/3000s \rightarrow T = 3/2200 + 11/3000$ s

c) T(gets service) = $3/2200 + T3/(1- Pb) = 3/2200 + T3\ (56/50)$ s

Problem 5

To a system with one server and two places in the queue customers arrive in groups with two customers. Time between arrivals of the groups is exponentially distributed with arrival rate λ. Customers are served independently and the service time for each customer is exponentially distributed. The mean service time for each customer is $1/\mu$.

Assume: λ = 1arrival/min and $1/\mu$ = 1min

Calculate:

a) Mean number of customers in the system (3p)

b) Utilization (3p)

c) Blocking probability (2p)

d) Call blocking probability (2p)

Solution

$p0 = p1 = p3 = 1/5$; $p2 = 2/5$

a) $N = 0p0 + 1p1 + 2p2 + 3p3 = 8/5$
b) $Ns = 0p0 + 1p1 + 1p2 + 1p3 = 4/5 \rightarrow U = 80\%$
c) $Pb = p3 = 1/5$
d) Call blocking = P(arrival is blocked) = $p2 + p3 = 3/5$