Department of Mathematics
SF2729 Groups and Rings
Period 3, 2016

HOMEWORK 10

Submission. The solutions should be typed and converted to .pdf. Deadline for submission is Monday February 22, 14.00. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `skjelnes@kth.se`.

Score. For each set of homework problems, the maximal score is 3 points, and calulated as $\min\{3, \Sigma/2\}$, where Σ is the score obtained on the homework. The total score from all twelve homeworks will be divided by four when counted towards the first part of the final exam.

Problem 1 (Exercise 8.1.4)**.** Let R be a Euclidean Domain.

(a) Show that if $(a, b) = 1$ and a divides bc, then a divides c. More generally, show that if a divides bc (with $a \neq 0, b \neq 0$) then $a/(a, b)$ divides c. **(1 p)**

(b) Consider the Diophantine Equation $ax + by = N$, with integer N, and non-zero integers a and b. Suppose x_0, y_0 is a solution. Prove that the full set of solutions is given by

$$x = x_0 + m \cdot \frac{b}{(a,b)}, \quad y = y_0 - m \cdot \frac{a}{(a,b)},$$

where m ranges over the integers. **(2 p)**

Problem 2. The discriminant D of a monic polynomial $f(x) = x^2 + bx + c \in \mathbb{Z}[x]$ is the integer $D = b^2 - 4c$. Let $\xi \in \mathbb{C}$ be a root of such a monic $f(x)$.

(a) If the discriminant $D = d^2$ is a square, show that ξ is an integer. **(1 p)**

(b) If the discriminant D is not a square, show that ξ is irrational. **(1 p)**

(c) If the discriminant D is not a square, show that the polynomial $f(x)$ is uniquely determined by ξ. **(1 p)**

Problem 3. Show that $\mathbb{Z}[\sqrt{2}] = \mathbb{Z}[x]/(x^2 - 2)$ is a P.I.D. **(3 p)**