Advanced Digital Communications (EQ2410)
Period 3, 2016

Assignment 8
Due: Wednesday, Feb. 12, 2014 M. Xiao

Reading assignment

- Madhow, Fundamentals of Digital Communication: chapters 8.3 and 6.3.2 (pp. 397-405 and 277-280)

Preparation task

Problem 8.13 (Cyclic convolution and DFT) This problem shows that cyclic convolution in the time domain corresponds to multiplication in the DFT domain. That is, if h and g are vectors of length N, with DFT H and G, respectively, and if

$$y[k] = (h \odot g)[k] = \sum_{m=0}^{N-1} h[m]b[(k-m)\mathrm{mod}\, N], \quad k = 0, 1, \ldots, N-1$$

denotes their cyclic convolution, then the DFT of y is given by

$$Y[n] = H[n]G[n]. \qquad (8.148)$$

Show this result using the following steps:

(a) Define the complex exponential functions $\{g_n, n = 0, 1, \ldots, N-1\}$ as

$$g_n[k] = e^{j2\pi nk/N}, \quad k = 0, 1, \ldots, N-1.$$

Show that

$$h \odot g_n = H[n]g_n.$$

That is, the complex exponentials $\{g_n\}$ are eigenfunctions for the operation of cyclic convolution with h, with eigenvalues equal to the DFT coefficients $\{H[n]\}$.

(b) Recognize that g can be expressed as a linear combination of the eigenfunctions $\{g_n\}$ as follows:

$$g = \frac{1}{N}\sum_{n=0}^{N-1} G[n]g_n.$$

(c) Use the linearity of cyclic convolution to infer that

$$y = h \odot g = \frac{1}{N}\sum_{n=0}^{N-1} G[n]\,(h \odot g_n)\,.$$

Plug in the result of (a) to obtain that

$$y = h \odot g = \frac{1}{N}\sum_{n=0}^{N-1} G[n]H[n]g_n.$$

Recognize that the right-hand side is an inverse DFT to infer (8.148).

[Madhow, *Fundamentals of Digital Communication*, 2008]