Advanced Digital Communications (EQ2410)

Lecture 8, Period 3, 2016

Task 1 Go through the proof of Theorem 8.3.1.

Theorem 8.3.1 *Consider a linear time-invariant channel with impulse response $g_C(t)$ and transfer function $G_C(f)$. Then the following statements are true:*

(a) The complex exponential waveform $e^{j2\pi ft}$ is an eigenfunction of the channel with eigenvalue $G_C(f)$. That is,

$$e^{j2\pi ft} * g_C(t) = G_C(f)e^{j2\pi ft}.$$

(b) Complex exponentials at different frequencies are orthogonal.

Proof The eigenfunction property (a) is verified as follows:

$$\begin{aligned} e^{j2\pi ft} * g_C(t) &= \int_{-\infty}^{\infty} g_C(u)e^{j2\pi f(t-u)}\, du \\ &= e^{j2\pi ft}\int_{-\infty}^{\infty} g_C(u)e^{-j2\pi fu}\, du = G_C(f)e^{j2\pi ft}. \end{aligned}$$

We now verify statement (b) on orthogonality for different frequencies:

$$\langle e^{j2\pi f_1 t}, e^{j2\pi f_2 t}\rangle = \int_{-\infty}^{\infty} e^{j2\pi f_1 t}e^{-j2\pi f_2 t} = \delta(f_2 - f_1) = 0, \qquad f_1 \neq f_2,$$

using the fact that the Fourier transform of a constant is the delta function. □

Task 2 Look at the slides 4 and 5: what happens to the orthogonality of $p_m(t), p_n(t)$ if we have an additional phase shift ϕ_n, for example if $p_n(t) = \exp(j(2\pi f_n t + \phi_n))$?

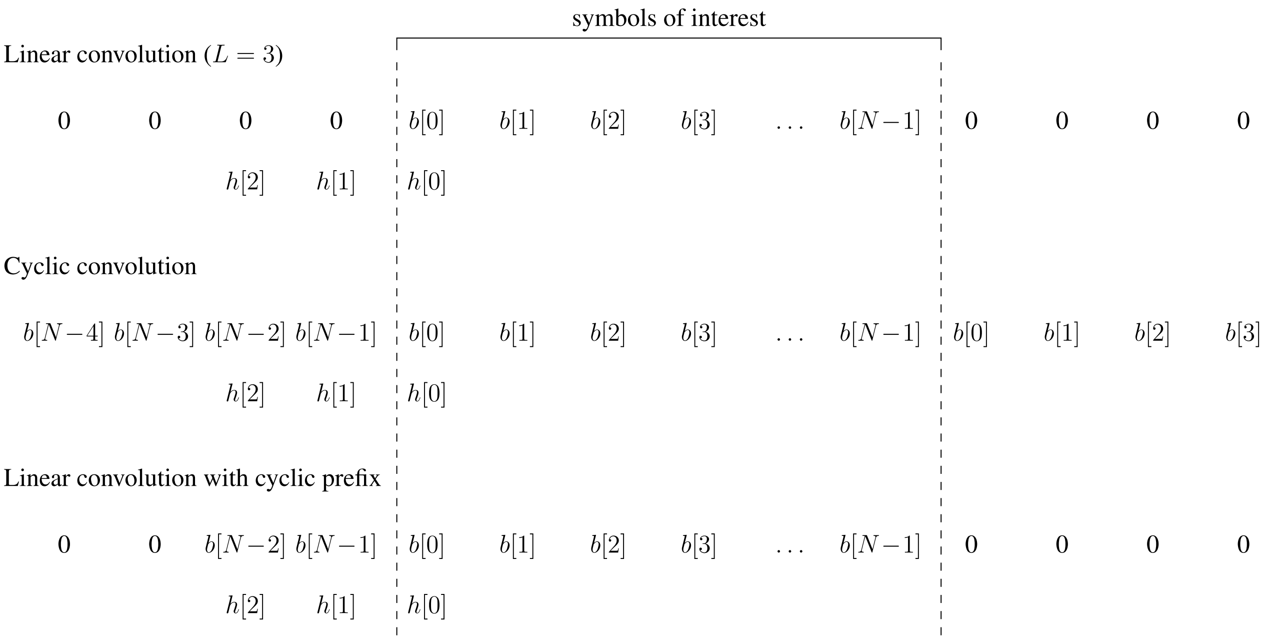
Task 3 The linear convolution of the length-N sequence $b[k]$ and the length-L sequence $h[k]$ is given as

$$y[k] = \sum_{m=0}^{L-1} h[m]b[k-m],$$

and the cyclic convolution of the length-N sequences $b[k]$ and $h'[k]$ (obtained by appending $N-L$ zeros to $h[k]$) is given as

$$z[k] = \sum_{m=0}^{N-1} h[m]b[(k-m) \mod N] = \sum_{m=0}^{L-1} h[m]b[(k-m) \mod N].$$

Discuss based on the illustration on the next page how the cyclic prefix helps to transform the linear convolution on the channel into a cyclic convolution.



Task 4 Consider two parallel AWGN channels

$$y_1 = h_1 x_1 + n_1 \quad \text{and} \quad y_2 = h_2 x_2 + n_2,$$

with the channel gains $h_1 = 2$ and $h_2 = 1$, the noise variances $N_1 = N_2 = 1$, and the symbol powers $P_1 = E[|x_1|^2]$ and $P_2 = E[|x_2|^2]$. Give the optimal power allocation (water filling) for the following two cases:

(1) Power constraint: $P_1 + P_2 = 1$,

(2) Power constraint: $P_1 + P_2 = 1/2$,