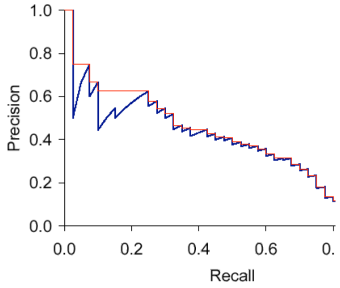
DD2476 Search Engines and Information Retrieval Systems

Lecture 5: Scoring, Weighting, Vector Space Model

Hedvig Kjellström
hedvig@kth.se
https://www.kth.se/social/course/DD2476/

Last lecture: IR beyond one shot

Precision-Recall Curve



Relevance Feedback

Really designed to come after this lecture, but has been moved before this lecture for scheduling reasons

Remember these two concepts and do the suggested reading for this lecture (5) **before** the suggested reading for last lecture (4)

Assignment 1

Thus far, Boolean queries:

BRUTUS AND CAESAR AND NOT CALPURNIA

Good for:

- Expert users with precise understanding of their needs and the collection
- Computer applications

Not good for:

- The majority of users

Problem with Boolean Search: Feast or Famine

Experienced maybe in Task 1.5?

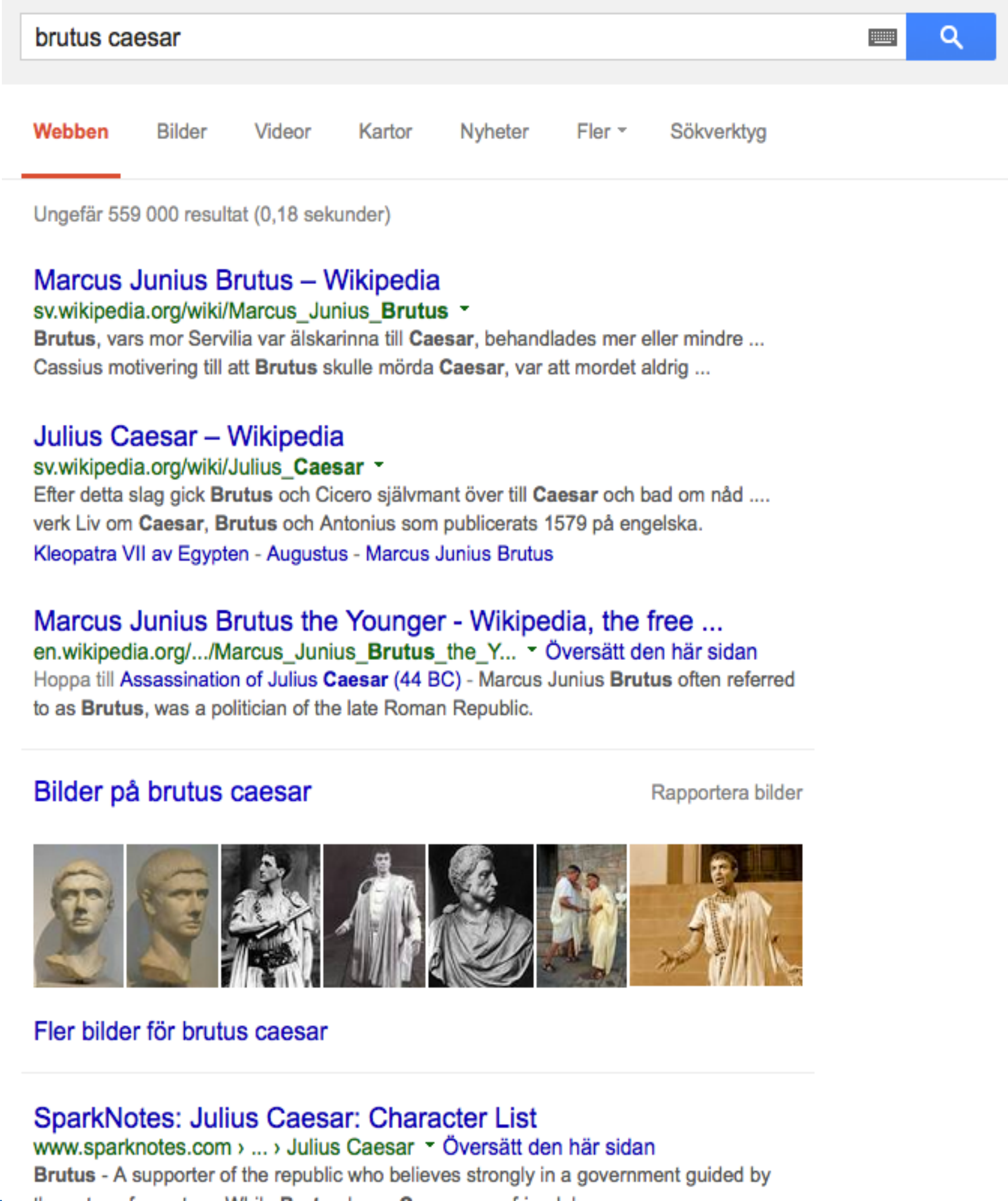
Boolean queries often result in either too few (=0) or too many (1000s) results.

- Query 1: "*standard user dlink 650*": 200,000 hits
- Query 2: "*standard user dlink 650 no card found*": 0 hits

It takes a lot of skill to come up with a query that produces a manageable number of hits.

- AND gives too few; OR gives too many

Ranked Retrieval



brutus caesar

Webben Bilder Videor Kartor Nyheter Fler Sökverktyg

Ungefär 559 000 resultat (0,18 sekunder)

Marcus Junius Brutus – Wikipedia
sv.wikipedia.org/wiki/Marcus_Junius_**Brutus**
Brutus, vars mor Servilia var älskarinna till **Caesar**, behandlades mer eller mindre ...
Cassius motivering till att **Brutus** skulle mörda **Caesar**, var att mordet aldrig ...

Julius Caesar – Wikipedia
sv.wikipedia.org/wiki/Julius_**Caesar**
Efter detta slag gick **Brutus** och Cicero självmant över till **Caesar** och bad om nåd ....
verk Liv om **Caesar**, **Brutus** och Antonius som publicerats 1579 på engelska.
Kleopatra VII av Egypten - Augustus - Marcus Junius Brutus

Marcus Junius Brutus the Younger - Wikipedia, the free ...
en.wikipedia.org/.../Marcus_Junius_**Brutus**_the_Y... Översätt den här sidan
Hoppa till Assassination of Julius **Caesar** (44 BC) - Marcus Junius **Brutus** often referred to as **Brutus**, was a politician of the late Roman Republic.

Bilder på brutus caesar Rapportera bilder

Fler bilder för brutus caesar

SparkNotes: Julius Caesar: Character List
www.sparknotes.com › ... › Julius Caesar Översätt den här sidan
Brutus - A supporter of the republic who believes strongly in a government guided by

Feast or Famine: Not a Problem in Ranked Retrieval

Large result sets no issue

- Show top K (≈ 10) results
- Option to see more results

Premise: the ranking algorithm works well enough

Today

Tf-idf and the vector space model (Manning Chapter 6)

- Term frequency, collection statistics
- Vector space scoring and ranking

Efficient scoring and ranking (Manning Chapter 7)

- Speeding up vector space scoring and ranking

Tf-idf and the Vector Space Model

(Manning Chapter 6)

Scoring as the Basis of Ranked Retrieval

Wish to return in order the documents most likely to be useful to the searcher

Rank-order documents with respect to a query

Assign a score – say in [0, 1] – to each document

- Measures how well document and query “match”

Query-Document Matching Scores

One-term query:

BRUTUS

Term not in document: score 0

More appearances of term in document: higher score

Recall (Lecture 1): Binary Term-Document Incidence Matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
ANTONY	1	1	0	0	0	0
BRUTUS	1	1	0	1	0	0
CAESAR	1	1	0	1	1	1
CALPURNIA	0	1	0	0	0	0
CLEOPATRA	1	0	0	0	0	0
MERCY	1	0	1	1	1	1
WORSER	1	0	1	1	1	0

Document represented by binary vector $\in \{0,1\}^{|V|}$

Term-Document Count Matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
ANTONY	157	73	0	0	0	0
BRUTUS	4	157	0	1	0	0
CAESAR	232	227	0	2	1	1
CALPURNIA	0	10	0	0	0	0
CLEOPATRA	57	0	0	0	0	0
MERCY	2	0	3	5	5	1
WORSER	2	0	1	1	1	0

Document represented by count vector $\in \mathbb{N}^{v}$

Bag of Words Model

Ordering of words in document not considered:

- “John is quicker than Mary” ≅ “Mary is quicker than John”

This is called the bag of words model

In a sense, step back: The positional index (Task 1.3) was able to distinguish these two documents

Assignment 2 Ranked Retrieval: Back to Bag-of-Words

Term Frequency tf

	Antony and Cleopatra
ANTONY	$\boxed{157}$

frequency = count in IR

Term frequency $\mathrm{tf}_{t,d}$ of term t in document d ≅ number of times that t occurs in d

Log-Frequency Weighting

Raw term frequency is a bit overestimated:

- A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term
- But arguably not 10 times more relevant

Alternative is log-frequency weight of term *t* in document *d*

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

Simple Query-Document Score

Queries with >1 terms

Score for a document-query pair: sum over terms *t* in both *q* and *d*:

$$\text{score} = \sum_{t \in q \cap d} \mathrm{tf}_{t,d}$$

The score is 0 if none of the query terms is present in the document

What is the problem with this measure?

Document Frequency

Rare terms are more informative than frequent terms

Example: rare word ARACHNOCENTRIC

- Document containing this term is very likely to be relevant to query ARACHNOCENTRIC

  → High weight for rare terms like ARACHNOCENTRIC

Example: common word THE

- Document containing this term can be about anything

  → Very low weight for common terms like THE

We will use document frequency (df) to capture this.

idf Weight

df_t is the document frequency of term *t*: the number of documents that contain *t*

- df_t is an inverse measure of the informativeness of *t*
- $\mathrm{df}_t \leq N$

Informativeness idf (inverse document frequency) of *t*:

$$\mathrm{idf}_t = \log_{10} (N/\mathrm{df}_t)$$

- log (N/df_t) instead of N/df_t to “dampen” the effect
  - Mathematical reasons in lecture 7!

Exercise 2 Minutes

Suppose N = 1,000,000

$$\text{idf}_t = \log_{10}(N/\text{df}_t)$$

term	df_t	idf_t
calpurnia	1	
animal	100	
sunday	1,000	
fly	10,000	
under	100,000	
the	1,000,000	

Effect of idf on Ranking

Does idf have an effect on ranking for one-term queries, like IPHONE?

Only effect for >1 term

- Query CAPRICIOUS PERSON: idf puts more weight on CAPRICIOUS than PERSON.

Collection vs. Document Frequency

Collection frequency of *t*: total number of occurrences of *t* in the collection, counting multiple occurrences

Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

Which word is a better search term (and should get a higher weight)?

tf-idf Weighting

tf-idf weight of a term: product of tf weight and idf weight

$$w_{t,d} = \mathrm{tf}_{t,d} \times \log_{10}(N/\mathrm{df}_t)$$

Best known weighting scheme in information retrieval

- Note: the “-” in tf-idf is a hyphen, not a minus sign!
- Alternative names: tf.idf, tf x idf

Increases with the number of occurrences within a document

Increases with the rarity of the term in the collection

Binary → Count → Weight Matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
ANTONY	5.255	3.18	0	0	0	0.35
BRUTUS	1.21	6.1	0	1	0	0
CAESAR	8.59	2.54	0	1.51	0.25	1
CALPURNIA	0	1.54	0	0	0	0
CLEOPATRA	2.85	0	0	0	0	0
MERCY	1.51	0	1.9	0.12	5.25	0.88
WORSER	1.37	0	0.11	4.15	0.25	1.95

Document represented by tf-idf weight vector $\in R^{V}$

Documents as Vectors

So we have a |V|-dimensional vector space

- Terms are axes/dimensions
- Documents are points in this space

Very high-dimensional

- Order of 10^7 dimensions when for a web search engine

Very sparse vectors - most entries zero

Queries as Vectors

Key idea 1: Represent queries as vectors in same space

Key idea 2: Rank documents according to proximity to query in this space

- proximity = similarity of vectors
- proximity ≈ inverse of distance

Recall:

- Get away from Boolean model
- Rank more relevant documents higher than less relevant documents

Formalizing Vector Space Proximity

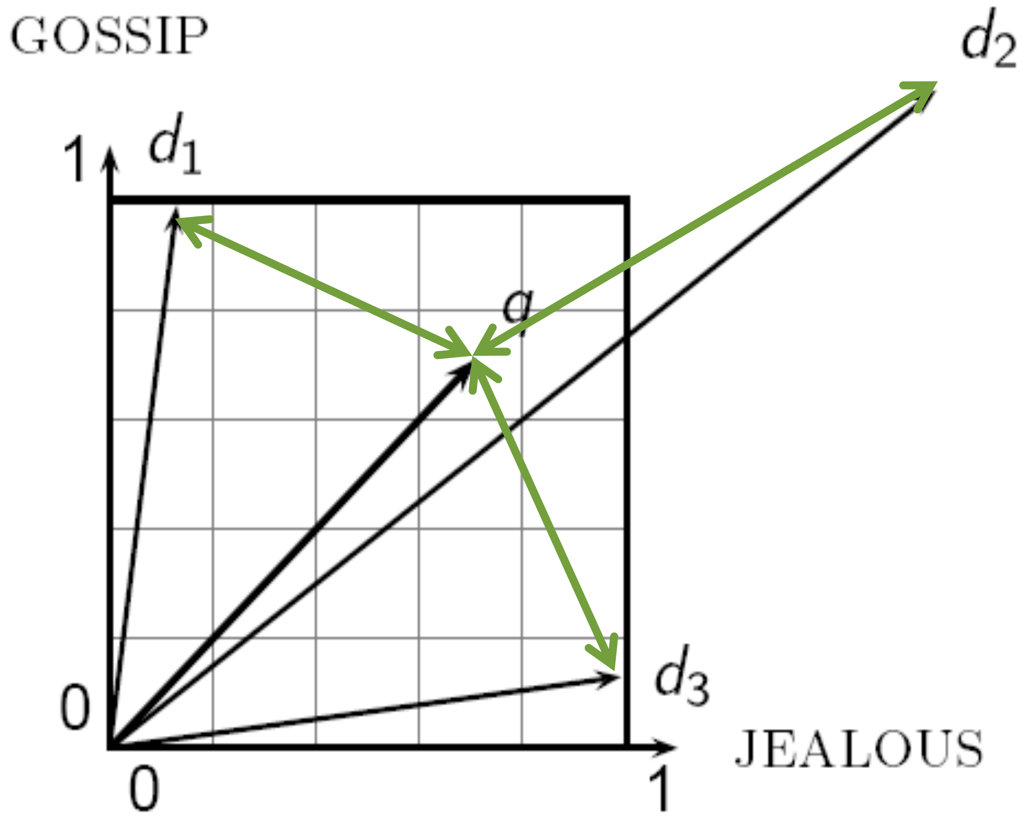
First cut:
Euclidean distance?

$$\left\| q - d_n \right\|_2$$

Euclidean distance is a bad idea . . .

. . . because Euclidean distance is large for vectors of different lengths

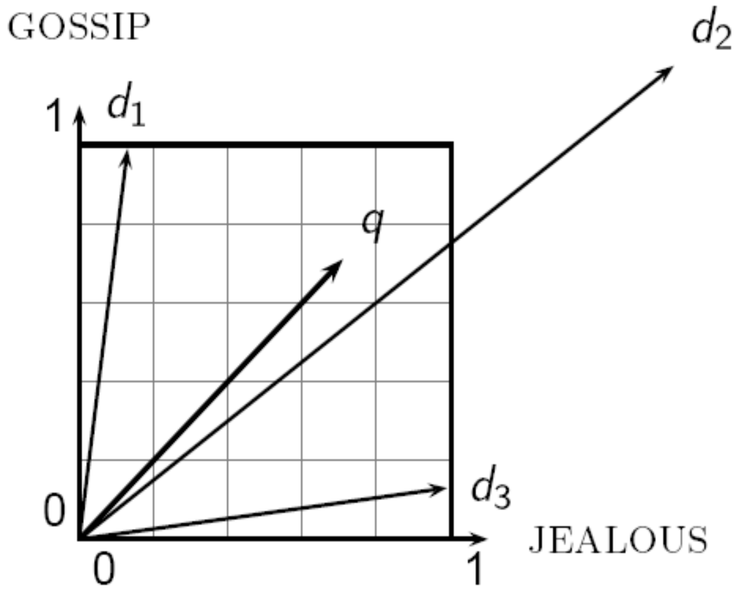
- What determines length here?



Exercise 5 Minutes

Euclidean distance bad for

- vectors of different length (documents with different #words)
- high-dimensional vectors (large dictionaries)



Discuss in pairs:

- Can you come up with a better difference measure?

Use Angle Instead of Distance

Thought experiment: take a document d and append it to itself. Call this document d′

“Semantically” d and d′ have the same content

The Euclidean distance between the two documents can be quite large

The angle between the two documents is 0, corresponding to maximal similarity

Key idea:

- Length unimportant
- Rank documents according to angle from query

Problems with Angle

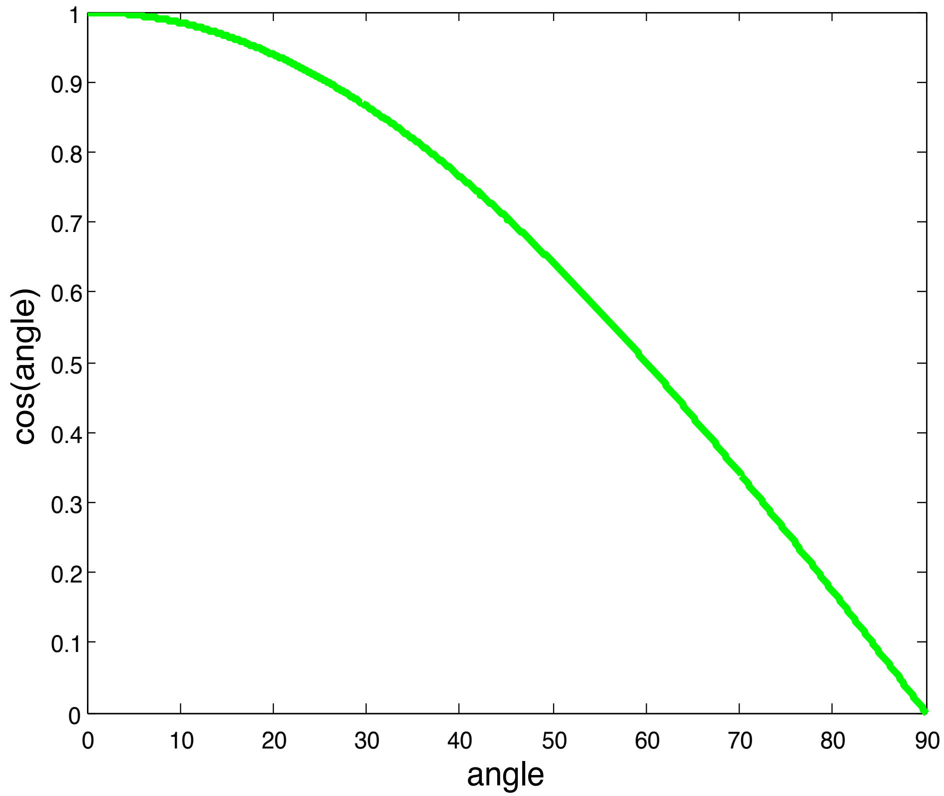
Angles expensive to compute – arctan

Find a computationally cheaper, equivalent measure

- Give same ranking order ≅ monotonically increasing/decreasing with angle

Any ideas?

Cosine More Efficient Than Angle



Monotonically decreasing

Length Normalization

Computing cosine similarity involves length-normalizing document and query vectors

L_2 norm: $$\left\|\vec{x}\right\|_2 = \sqrt{\sum_i x_i^2}$$

Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)

Recall:

- Length unimportant
- Rank documents according to angle from query

Cosine Similarity

dot / scalar / inner product

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}}{\|\vec{q}\|_2} \bullet \frac{\vec{d}}{\|\vec{d}\|_2} = \frac{\vec{q} \bullet \vec{d}}{\|\vec{q}\|_2 \|\vec{d}\|_2} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term *i* in the query

d_i is the tf-idf weight of term *i* in the document

$\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$

= the cosine of the angle between $\vec{q}$ and $\vec{d}$.

Cosine Similarity

In reality:

- Length-normalize when document added to index:

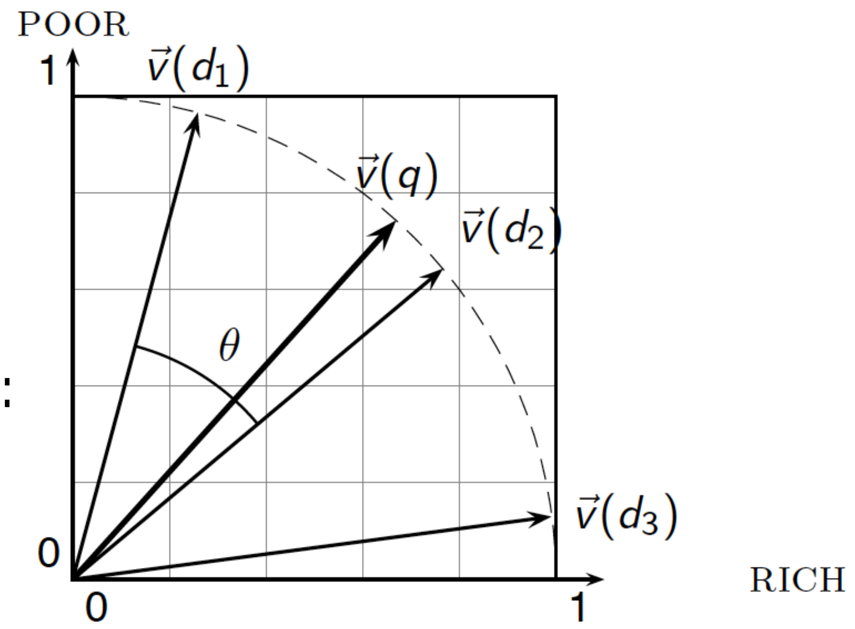
$$\vec{d} \leftarrow \frac{\vec{d}}{\left\|\vec{d}\right\|_2}$$

- Length-normalize query:

$$\vec{q} \leftarrow \frac{\vec{q}}{\left\|\vec{q}\right\|_2}$$

- Fast to compute cosine similarity:

$$\cos(\vec{q},\vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$



Cosine Similarity Example

How similar are the novels

- SaS: Sense and Sensibility
- PaP: Pride and Prejudice
- WH: Wuthering Heights?

No idf weighting!

Term frequency tf_t

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Cosine Similarity Example

Log frequency weights $w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

Cosine Similarity Example

After length normalization $\vec{d} = \left[w_{1,d}, \ldots, w_{4,d} \right]$, $\vec{d} \leftarrow \dfrac{\vec{d}}{\left\| \vec{d} \right\|_2}$

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈ 0.789*0.832 + 0.515*0.555 + 0.335*0 + 0*0 ≈ 0.94
cos(SaS,WH) ≈ 0.79
cos(PaP,WH) ≈ 0.69

Why is cos(SaS,PaP) > cos(*,WH)?

Computing Cosine Scores

```
COSINESCORE(q)
float Scores[N] = 0
float Length[N]
for each query term t
do calculate w_t,q and fetch postings list for t
    for each pair(d, tf_t,d) in postings list
    do Scores[d] += w_t,d × w_t,q
Read the array Length
for each d
do Scores[d] = Scores[d]/Length[d]
return Top K components of Scores[]
```

Summary – vector space ranking

Represent the query as a tf-idf vector

Represent each document as a tf-idf vector

Compute the cosine similarity score for the query vector and each document vector

Rank documents with respect to the query by score

Return the top K (e.g., $K = 10$) to the user

Efficient Scoring and Ranking

(Manning Chapter 7)

Efficient Cosine Ranking

Find the K docs in the collection "nearest" to the query $\Rightarrow$ K largest query-document cosine scores

Up to now: Linear scan through collection

- Did not make use of sparsity in term space
- Computed all cosine scores

Efficient cosine ranking:

- Computing each cosine score efficiently
- Choosing the K largest scores efficiently

Computing Cosine Scores Efficiently

Approximation:

- Assume that terms only occur once in query

$$w_{t,q} \leftarrow \begin{cases} 1, & \text{if } w_{t,q} > 0 \\ 0, & \text{otherwise} \end{cases}$$

Works for short querys (|q| << *N*)

Works since ranking only relative

Computing Cosine Scores Efficiently

```
FastCosineScore(q)
  float Scores[N] = 0
  for each d
  do Initialize Length[d] to the length of doc d
  for each query term t
  do calculate w_{t,q} and fetch postings list for t
       for each pair(d, tf_{t,d}) in postings list
       do add wf_{t,d} to Scores[d]
  Read the array Length[d]
  for each d
  do Divide Scores[d] by Length[d]
  return Top K components of Scores[]
```

Figure 7.1 A faster algorithm for vector space scores.

Speedup here

Computing Cosine Scores Efficiently

Downside of approximation: sometimes get it wrong

- A document not in the top *K* may creep into the list of *K* output documents

How bad is this?

Cosine similarity is only a proxy (Task 1.5)

- User has a task and a query formulation
- Cosine matches documents to query
- Thus cosine is anyway a proxy for user happiness
- If we get a list of *K* documents "close" to the top *K* by cosine measure, should be ok

Choosing K Largest Scores Efficiently

Retrieve top *K* documents wrt query
- Not totally order all documents in collection

Do selection:
- avoid visiting all documents

Already do selection:
- Sparse term-document incidence matrix, |d| << *N*
- Many cosine scores = 0
- Only visits documents with nonzero cosine scores (≥1 term in common with query)

Choosing K Largest Scores Efficiently Generic Approach

Find a set A of contenders, with $K < |A| << N$

- A does not necessarily contain the top *K*, but has many docs from among the top *K*
- Return the top *K* documents in A

Think of A as pruning non-contenders

Same approach used for any scoring function!

Will look at several schemes following this approach

Choosing K Largest Scores Efficiently Index Elimination

Basic algorithm FastCosineScore only considers documents containing at least one query term

- All documents have ≥1 term in common with query

Take this further:

- Only consider high-idf query terms
- Only consider documents containing many query terms

Choosing K Largest Scores Efficiently Index Elimination, only high-idf

Example:

CATCHER IN THE RYE

Only accumulate scores from CATCHER and RYE

Intuition:

- IN and THE contribute little to the scores – do not alter rank-ordering much
- Compare to stop words

Benefit:

- Posting lists of low-idf terms have many documents → eliminated from set A of contenders

Choosing K Largest Scores Efficiently Index Elimination, several query terms

Example:

CAESAR ANTONY CALPURNIA BRUTUS

Only compute scores for documents containing ≥3 query terms

Antony	3	4	8	16	32	64	128	
Brutus	2	4	8	16	32	64	128	
Caesar	1	2	3	5	8	13	21	34
Calpurnia	13	16	32					

Choosing K Largest Scores Efficiently Champion Lists

Precompute for each dictionary term *t,* the *r* documents of highest tf-idf$_{td}$ weight

- Call this the champion list (fancy list, top docs) for *t*

Benefit:

- At query time, only compute scores for documents in the champion lists – fast

Issue:

- *r* chosen at index build time
- Too large: slow
- Too small: $r < K$

Exercise 5 Minutes

Index Elimination: consider only high-idf query terms and only documents with many query terms

Champion Lists: for each term *t*, consider only the *r* documents with highest tf-idf$_{td}$ values

Think quietly and write down:

- How do Champion Lists relate to Index Elimination? Can they be used together?
- How can Champion Lists be implemented in an inverted index?

Choosing K Largest Scores Efficiently Static Quality Scores

Develop idea of champion lists

We want top-ranking documents to be both relevant and authoritative

- Relevance – cosine scores
- Authority – query-independent property

Examples of authority signals

- Wikipedia pages (qualitative)
- Articles in certain newspapers (qualitative)
- A scientific paper with many citations (quantitative)
- PageRank (quantitative)

More in Lecture 6

Choosing K Largest Scores Efficiently
Static Quality Scores

Assign query-independent quality score g(*d*) in [0,1] to each document *d*

net-score(*q*,*d*) = g(*d*) + cos(*q*,*d*)

- Two “signals” of user happiness
- Other combination than equal weighting

Seek top *K* documents by net score

Choosing K Largest Scores Efficiently Champion Lists + Static Quality Scores

Can combine champion lists with *g(d)*-ordering

Maintain for each term *t* a champion list of the *r* documents with highest *g(d)* + tf-idf$_{td}$

Seek top *K* results from only the documents in these champion lists

Next

Assignment 1 left?

- You can present it at the session for Assignment 2
- Reserve two slots, one for each assignment!

Lecture 6 (February 23, 13.15-15.00)

- B3
- Readings: Manning Chapter 21
  Avrachenkov Sections 1-2

Lecture 7 (February 24, 10.15-12.00)

- B3
- Readings: Manning Chapters 11, 12