Lecture 9
DSSS and CDMA

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Lecture 9: Direct-Sequence Spread Spectrum Techniques and CDMA
Advanced Digital Communications (EQ2410)[1]

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Monday, Feb. 22, 2016
15:00-17:00, B23

[1] Textbook: U. Madhow, *Fundamentals of Digital Communications*, 2008

Notes

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Overview

Lecture 7+8

- Wireless channels
- Diversity
- OFDM
- Capacity for parallel channels

Lecture 9: DS-SS techniques and CDMA

Notes

Motivation

Spread spectrum

- Bandwidth is significantly larger than the information rate.
- Frequency diversity
- Multiple access, the same bandwidth is shared by several users.
- Interference reduction, robustness against jamming
- Stealth signals (look like noise)

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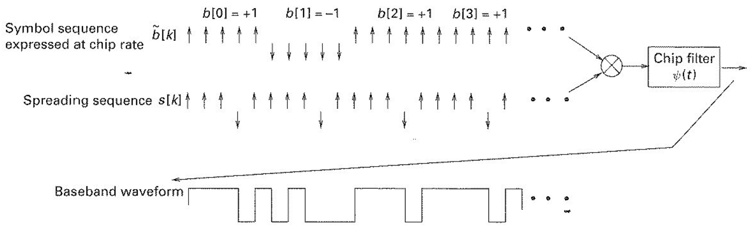
Direct-Sequence Spread Spectrum (DS-SS)



- Linear modulation using $N > 1$ complex dimensions per information symbol $b[m]$ (instead of only using one dimension); N chips.

$$u(t) = \sum_{l} \tilde{b}[l] s[l] \psi(t - lT_c)$$

- Chip-rate symbols: $\tilde{b}[l] = b[m]$, if $mN \leq l \leq (m+1)N - 1$
- Information symbols: $b[m]$
- Spreading vector/sequence: $\mathbf{s} = (s[0], \ldots, s[N-1])^T$, $\{s[l]\}$
- Chip waveform: $\psi(t)$ (modulation pulse)
- Processing gain: N

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Direct-Sequence Spread Spectrum (DS-SS)

- Alternative formulation

$$u(t) = \sum_{m} b[m]s(m; t - mT),$$

with the spreading waveform

$$s(m; t) = \sum_{l=0}^{N-1} s[mN + l]\psi(t - lT_c)$$

- Short spreading sequences
  - $\{s[l]\}$ is periodic with period N.
  - Identical spreading waveform $s(m; t) = s(t)$ for all symbols
  - $u(t)$ is cyclostationary with period T.
- Long spreading sequences
  - $\{s[l]\}$ is aperiodic or has a very long period.

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Code Division Multiple Access (CDMA)

- All users use the same channel at the same time but with different spreading codes.
- Downlink transmission (from the base station to the mobiles)
  - Base station superposes the DSSS signals for the users.
  - If orthogonal spreading codes are used, there is no interference between the signals to different users.
- Uplink transmissions (from the mobiles to the base station)
  - All users experience different channels $\rightarrow$ orthogonal design is not possible.
  - Design goal: find spreading codes which have for different time shifts a small average inner product (i.e., good auto- and crosscorrelation properties).

Notes

Correlation Properties

- Motivation: matched-filter receiver correlates the received signal with the transmitted waveforms
  - Multi-user interference can be reduced with spreading waveforms with good crosscorrelation properties.
  - Intersymbol interference can be reduced with spreading waveforms with good autocorrelation properties.
- Consider the following two spreading waveforms

$$u(t) = \sum_{l=0}^{N-1} u[l]\psi(t - lT_c) \quad \text{and} \quad v(t) = \sum_{l=0}^{N-1} v[l]\psi(t - lT_c)$$

- Continuous-time and discrete-time crosscorrelation functions

$$R_{u,v}(\tau) = \int u(t)v^*(t-\tau)dt \quad \text{and} \quad R_{u,v}[n] = \sum_l u[l]v^*[l-n]$$

- Continuous-time and discrete-time autocorrelation functions

$$R_u(\tau) = \int u(t)u^*(t-\tau)dt \quad \text{and} \quad R_u[n] = \sum_l u[l]u^*[l-n]$$

Notes

Correlation Properties

- Crosscorrelation function can be rewritten as

$$R_{u,v}(\tau) = \sum_l \sum_k u[l]v^*[k]r_\psi((k-l)T_c+\tau) \quad \text{with} \quad r_\psi(\tau) = (\psi \star \psi_{MF})(\tau)$$

and with $D = \lfloor \tau/T_c \rfloor$ and $\delta = \tau/T_c - D$ (i.e., $\tau = DT_c + \delta T_c$) as

$$R_{u,v}(\tau) = \sum_l \sum_k u[l]v^*[k]r_\psi((k + D - l)T_c + \delta T_c)$$

- For rectangular chips we get

$$r_\psi((k + D - l)T_c + \delta T_c) = \begin{cases} 1-\delta, & k + D - l = 0 \\ \delta, & k + D - l = -1 \\ 0, & \text{else} \end{cases}$$

and

$$R_{u,v}(\tau) = (1-\delta)R_{u,v}[D] + \delta R_{u,v}[D+1]$$

→ Note that for $\tau = \kappa T_c$ we get $D = \kappa$, $\delta = 0$, and $R_{u,v}(\kappa T_c) = R_{u,v}[\kappa]$.

→ Continuous-time crosscorrelation function can be made small by making the discrete-time crosscorrelation function small.

→ Pseudo-random spreading sequences (aperiodic or period with long period)

Notes

Performance Conventional Receiver

- Conventional receiver: ignore effect of ISI and multi-user interference
- Received signal for K users

$$y(t) = \sum_{k=1}^{K} b_k A_k s_k(t) + n(t)$$

with the spreading waveforms $s_k(t) = \sum_{l=0}^{N-1} s_k[l]\psi(t - lT_c)$, the BPSK symbols b_k, and the channel gains A_k for the k-th user.

- Matched-filter statistic for user 1

$$Z_1 = \int y(t)s_1(t)dt = \sum_{k=1}^{K} A_k b_k \int s_k(t)s_1(t)dt + \int n(t)s_1(t)dt$$

- For normalized waveforms with $r_\psi(0) = 1$ we have

$$Z_1 = \sum_{k=1}^{K} A_k b_k R_{s_k,s_1}[0] + N_1 = A_1 b_1 N + \sum_{k=2}^{K} A_k b_k R_{s_k,s_1}[0] + N_1$$

with $N_1 \sim N(0, \sigma^2 N)$.

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Performance Conventional Receiver

- Exact error probability: average $P_e(b_2, \ldots, b_K)$ over all realizations of the interfering symbols $b_2, \ldots, b_K$.
- Gaussian approximation
  - For large N, $R_{s_k,s_1}[0] = \sum_{l=0}^{N-1} s_k[l]s_1[l]$ can be modeled as zero-mean Gaussian RV with variance N.
  - Interference plus noise term is zero-mean Gaussian with variance $v^2 = \sum_{k=2}^{K} A_k^2 N + \sigma^2 N$.
  - Error probability estimate for hard decision on Z_1

$$P_e \approx Q\left(\sqrt{\frac{1}{(2E_b/N_0)^{-1} + \frac{1}{N}\sum_{k=2}^{K} A_k^2/A_1^2}}\right)$$

  with $E_b = A_1^2 N$ and $\sigma^2 = N_0/2$.

  → System performance is interference limited!
- Power control required:
  - Near/far problem: interference dominates the decision if $|A_k R_{s_k,s_1}[0]| \gg |A_1 N|$
  - Perfect power control: all amplitudes are equal.

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Rake Receiver

- Channel model: multi-path channel (L paths with gains α_i and delays τ_i)

$$h(t) = \sum_{i=1}^{L} \alpha_i \delta_i(t - \tau_i)$$

- Effective spreading waveform seen by symbol m

$$\tilde{s}(m; t) = s(m; t) \star h(t) = \sum_{i=1}^{L} \alpha_i s(m; t - \tau_i)$$

- Received signal

$$y(t) = (u \star h)(t) + n(t) = \sum_{m} b[m]\tilde{s}(m; t - mT) + n(t)$$

- For DS spreading waveforms with good autocorrelation and crosscorrelation properties ISI and interference from other users can be ignored; reduced model:

$$y(t) = b[m]\tilde{s}(m; t - mT) + n(t)$$

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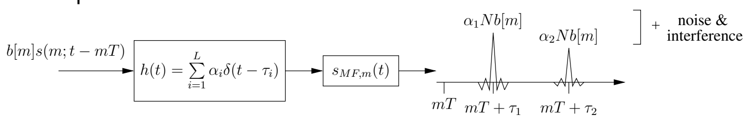
Rake Receiver

- Optimal decision statistics (correlating with $\tilde{s}(m; t - mT)$)

$$Z[m] = \int y(t)\tilde{s}^*(m; t{-}mT)dt = \sum_{i=1}^{L} \alpha_i^* \underbrace{\int y(t)s^*(m; t - mT - \tau_i)dt}_{(y \star s_{MF,m})(mT+\tau_i),\ s_{MF,m}(t)=s^*(m;-t)}$$

$\rightarrow$ Rake receiver sums up ("rakes up") the energy from the multi-path components (given that good spreading codes are used).

- Example for $L = 2$



$L = 2$ copies of the symbol $b[m]$ are received; rake performs maximum ratio combining.

- Timing of the of the sampler is important!

Notes

Rake Receiver

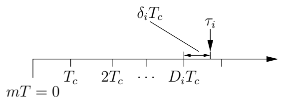
- Chip rate implementation of the rake receiver
  - Short spreading sequence: $s(m; t - mT) = s(t) = \sum_{l=0}^{N-1} s[l]\psi(t - lT_c)$
  - Chip matched filter: $\psi_{MF}(t) = \psi^*(-t)$
  - Decision statistics can be rewritten as

$$
\begin{aligned}
Z[m] &= \sum_{i=1}^{L} \alpha_i^* \sum_{l=0}^{N-1} s^*[l] \int y(t)\psi(t - mT - \tau_i - lT_c)dt \\
&= \sum_{i=1}^{L} \alpha_i^* \sum_{l=0}^{N-1} s^*[l] Y_{\tau_i}[l]
\end{aligned}
$$

with

$$Y_{\tau_i}[l] = (y \star \psi_{MF})((l + mN + D_i + \delta_i)T_c)$$

and $T = NT_c$, $D_i = \lfloor \tau_i / T_c \rfloor$ and $\delta_i = (\tau_i/T_c) - D_i$.



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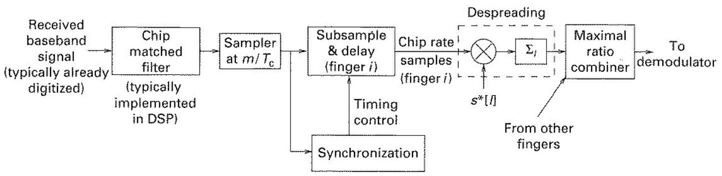
Rake Receiver

- Block diagram for the chip rate implementation of the rake receiver



- Problem: sampling offsets $\delta_i T_c$ vary across multi-path components
- Possible solutions
  - → Sampling at chip rate and interpolation
  - → Fractionally sampling and use the set of samples which are closest to the multi-path component in time.
  - → Track $\delta_i T_c$ and use L chip rate samplers; i.e., one for each multi-path component.

Notes