Overview

Lecture 7+8
- Wireless channels
- Diversity
- OFDM
- Capacity for parallel channels

Lecture 9: DS-SS techniques and CDMA

Motivation

Spread spectrum
- Bandwidth is significantly larger than the information rate.
- Frequency diversity
- Multiple access, the same bandwidth is shared by several users.
- Interference reduction, robustness against jamming
- Stealth signals (look like noise)

Direct-Sequence Spread Spectrum (DS-SS)
- Linear modulation using $N > 1$ complex dimensions per information symbol $b[m]$ (instead of only using one dimension); $N$ chips.
  $$u(t) = \sum_i \tilde{b}[l]s[l]\psi(t - lT_c)$$
- Chip-rate symbols: $\tilde{b}[l] = b[m]$, if $mN \leq l \leq (m + 1)N - 1$
- Information symbols: $b[m]$
- Spreading vector/sequence: $s = (s[0], \ldots, s[N-1])^T$, $\{s[l]\}$
- Chip waveform: $\psi(t)$ (modulation pulse)
- Processing gain: $N$
Direct-Sequence Spread Spectrum (DS-SS)

- Alternative formulation
  \[ u(t) = \sum_{m} b[m]s(m; t - mT), \]
  with the spreading waveform
  \[ s(m; t) = \sum_{l=0}^{N-1} s[mN + l]\psi(t - lT_c) \]

- Short spreading sequences
  - \( \{s[l]\} \) is periodic with period \( N \).
  - Identical spreading waveform \( s(m; t) = s(t) \) for all symbols
  - \( u(t) \) is cyclostationary with period \( T \).

- Long spreading sequences
  - \( \{s[l]\} \) is aperiodic or has a very long period.

Code Division Multiple Access (CDMA)

- All users use the same channel at the same time but with different spreading codes.
- Downlink transmission (from the base station to the mobiles)
  - Base station superposes the DSSS signals for the users.
  - If orthogonal spreading codes are used, there is no interference between the signals to different users.
- Uplink transmissions (from the mobiles to the base station)
  - All users experience different channels → orthogonal design is not possible.
  - Design goal: find spreading codes which have for different time shifts a small average inner product (i.e., good auto- and crosscorrelation properties).
Correlation Properties

- Motivation: matched-filter receiver correlates the received signal with the transmitted waveforms
  - Multi-user interference can be reduced with spreading waveforms with good crosscorrelation properties.
  - Intersymbol interference can be reduced with spreading waveforms with good autocorrelation properties.

- Consider the following two spreading waveforms
  \[ u(t) = \sum_{l=0}^{N-1} u[l] \psi(t - lT_c) \quad \text{and} \quad v(t) = \sum_{l=0}^{N-1} v[l] \psi(t - lT_c) \]

- Continuous-time and discrete-time crosscorrelation functions
  \[ R_{u,v}(\tau) = \int u(t)v^*(t-\tau)dt \quad \text{and} \quad R_{u,v}[n] = \sum_{l} u[l]v^*[l-n] \]

- Continuous-time and discrete-time autocorrelation functions
  \[ R_u(\tau) = \int u(t)u^*(t-\tau)dt \quad \text{and} \quad R_u[n] = \sum_{l} u[l]u^*[l-n] \]

- Crosscorrelation function can be rewritten as
  \[ R_{u,v}(\tau) = \sum_{l} \sum_{k} u[l]v^*[k]r_{\psi}(\tau) \]
  \[ = \sum_{l} \sum_{k} u[l]v^*[k]r_{\psi}(\tau) = \sum_{l} \sum_{k} u[l]v^*[k]r_{\psi}(\tau) \]

  and with \( D = \lfloor \tau / T_c \rfloor \) and \( \delta = \tau / T_c - D \) (i.e., \( \tau = DT_c + \delta T_c \)) as
  \[ R_{u,v}(\tau) = \sum_{l} \sum_{k} u[l]v^*[k]r_{\psi}(\tau) = \sum_{l} \sum_{k} u[l]v^*[k]r_{\psi}(\tau) \]

- For rectangular chips we get
  \[ r_{\psi}(k + D - 1)T_c + \delta T_c) = \begin{cases} 1 - \delta, & k + D - l = 0 \\ \delta, & k + D - l = 0 \\ 0, & \text{else} \end{cases} \]

  and
  \[ R_{u,v}(\tau) = (1 - \delta)R_{u,v}[D] + \delta R_{u,v}[D+1] \]

  \[ \rightarrow \text{Note that for } \tau = \kappa T_c, \text{ we get } D = \kappa, \delta = 0, \text{ and } R_{u,v}(\kappa T_c) = R_{u,v}[\kappa]. \]

  \[ \rightarrow \text{Continuous-time crosscorrelation function can be made small by making the discrete-time crosscorrelation function small.} \]

  \[ \rightarrow \text{Pseudo-random spreading sequences (aperiodic or period with long period)} \]
Performance Conventional Receiver

- Conventional receiver: ignore effect of ISI and multi-user interference
- Received signal for $K$ users
  \[ y(t) = \sum_{k=1}^{K} b_k A_k s_k(t) + n(t) \]
  with the spreading waveforms $s_k(t) = \sum_{i=0}^{N-1} s_k[i] \psi(t - iT_c)$, the BPSK symbols $b_k$, and the channel gains $A_k$ for the $k$-th user.
- Matched-filter statistic for user 1
  \[ Z_1 = \int y(t) s_1(t) dt = \sum_{k=1}^{K} A_k b_k \int s_k(t) s_1(t) dt + \int n(t) s_1(t) dt \]
  For normalized waveforms with $r_\psi(0) = 1$ we have
  \[ Z_1 = \sum_{k=1}^{K} A_k b_k R_{s_k, s_1}[0] + N_1 = A_1 b_1 N + \sum_{k=2}^{K} A_k b_k R_{s_k, s_1}[0] + N_1 \]
  with $N_1 \sim N(0, \sigma^2 N)$.

- Exact error probability: average $P_e(b_2, \ldots, b_K)$ over all realizations of the interfering symbols $b_2, \ldots, b_K$.
- Gaussian approximation
  - For large $N$, $R_{s_k, s_1}[0] = \sum_{i=0}^{N-1} s_k[i] s_1[i]$ can be modeled as zero-mean Gaussian RV with variance $N$.
  - Interference plus noise term is zero-mean Gaussian with variance
    \[ \nu^2 = \sum_{k=2}^{K} A_k^2 N + \sigma^2 N. \]
  - Error probability estimate for hard decision on $Z_1$
    \[ P_e \approx Q \left( \frac{1}{\sqrt{(2E_b/N_0)^{-1} + \frac{1}{N} \sum_{k=2}^{K} A_k^2 / A_1^2}} \right) \]
    with $E_b = A_1^2 N$ and $\sigma^2 = N_0/2$.
    \[ \rightarrow \text{System performance is interference limited!} \]
- Power control required:
  - Near/far problem: interference dominates the decision if $|A_1 R_{s_1, s_k}[0]| \gg |A_k N|$
  - Perfect power control: all amplitudes are equal.
Rake Receiver

- Channel model: multi-path channel ($L$ paths with gains $\alpha_i$ and delays $\tau_i$)

$$ h(t) = \sum_{i=1}^{L} \alpha_i \delta(t - \tau_i) $$

- Effective spreading waveform seen by symbol $m$

$$ \tilde{s}(m; t) = s(m; t) * h(t) = \sum_{i=1}^{L} \alpha_i s(m; t - \tau_i) $$

- Received signal

$$ y(t) = (u * h)(t) + n(t) = \sum_{m} b[m] \tilde{s}(m; t - mT) + n(t) $$

- For DS spreading waveforms with good autocorrelation and crosscorrelation properties ISI and interference from other users can be ignored; reduced model:

$$ y(t) = b[m] \tilde{s}(m; t - mT) + n(t) $$

Rake Receiver

- Optimal decision statistics (correlating with $\tilde{s}(m; t - mT)$)

$$ Z[m] = \int y(t) \tilde{s}^*(m; t - mT) dt = \sum_{i=1}^{L} \alpha_i \int y(t) \tilde{s}^*(m; t - mT - \tau_i) dt $$

$$ (y*_{sCF,m}(mT + \tau); \cdot_{sCF,m}(t) = s^*(m, t - t) $$

→ Rake receiver sums up ("rakes up") the energy from the multi-path components (given that good spreading codes are used).

- Example for $L = 2$

$$ b(m)(m; t - mT) $$

- Timing of the of the sampler is important!
Rake Receiver

- Chip rate implementation of the rake receiver
  - Short spreading sequence: \( s(m; t - mT) = s(t) = \sum_{l=0}^{N-1} s[l] \psi(t - l T_c) \)
  - Chip matched filter: \( \psi_{MF}(t) = \psi^*(t) \)
  - Decision statistics can be rewritten as
    \[
    Z[m] = \sum_{i=1}^{L} \alpha_i \sum_{l=0}^{N-1} s[l] \int y(t) \psi(t - mT - \tau_i - l T_c) dt
    = \sum_{i=1}^{L} \alpha_i \sum_{l=0}^{N-1} s[l] Y_{\tau_i}[l]
    \]
    with
    \[
    Y_{\tau_i}[l] = (y * \psi_{MF})((l + mN + D_i + \delta_i) T_c)
    \]
    and \( T = NT_c \), \( D_i = \lfloor \tau_i / T_c \rfloor \) and \( \delta_i = (\tau_i / T_c) - D_i \).

Problem: sampling offsets \( \delta_i T_c \) vary across multi-path components

Possible solutions
- Sampling at chip rate and interpolation
- Fractionally sampling and use the set of samples which are closest to the multi-path component in time.
- Track \( \delta_i T_c \) and use \( L \) chip rate samplers; i.e., one for each multi-path component.